Writing Polynomial Functions With Given Zeros (2.5)

LEARNING ◎ TARGETS

I can write a polynomial function given real or non-real zeros.

↗ 3 zeros, 3 factors

Example 1: Find a <u>third degree</u> polynomial function with real coefficients that has 0, -1, and 2 as zeros.

First, take zeros and turn into factors :

$x=0 : (x-0) = x$

$x=-1 : (x+1)$

$x=2 : (x-2)$

Then, take factors and FOIL out

$x(x+1)(x-2)$

$x(x^2-x-2)$

$$\boxed{x^3 - x^2 - 2x = f(x)}$$

↗ 4 zeros, 4 factors

Example 2: Find a <u>fourth degree</u> polynomial function with real coefficients that has -2, -2, and $2i$ as zeros.

↳ To get fourth zero, take the conjugate of $2i$

$0+2i , 0-2i$

$x=-2 : (x+2)$

$x=-2 : (x+2)$

$x=2i : (x-2i)$ ⇒

$x=-2i : (x+2i)$

$(x+2)(x+2)(x-2i)(x+2i)$ ← FOIL real parts together ⋮ FOIL complex parts together

$(x^2+4x+4)(x^2+2ix-2ix-4i^2)$

$(x^2+4x+4)(x^2-4(-1))$

$(x^2+4x+4)(x^2+4)$

$x^4+4x^2+4x^3+16x+4x^2+16$

$$\boxed{g(x) = x^4+4x^3+8x^2+16x+16}$$

Try these!

1. Find a third degree polynomial function with real coefficients that has 1, 2, and -3 as zeros.

$x = 1 : (x-1)$
$x = 2 : (x-2)$
$x = -3 : (x+3)$

$(x-1)(x-2)(x+3)$

$(x-1)(x^2+x-6)$

$x^3+x^2-6x-x^2-x+6$

$$\boxed{f(x) = x^3-7x+6}$$

2. Find a third degree polynomial function with real coefficients that has 2 and i as zeros.

$x = 2 : (x-2)$
$x = i : (x-i)$
$x = -i : (x+i)$

$(x-2)(x-i)(x+i)$

$(x-2)(x^2+ix-ix-i^2)$

$(x-2)(x^2-(-1))$

$(x-2)(x^2+1)$

x^3+x-2x^2-2

$$\boxed{g(x) = x^3-2x^2+x-2}$$

Challenge: Find a third degree polynomial function with real coefficients that has -1 and $2+i$ as zeros.

$x = -1 : (x+1)$
$x = 2+i : (x-(2+i)) = (x-2-i)$
$x = 2-i : (x-(2-i)) = (x-2+i)$

$(x+1)(x-2-i)(x-2+i)$

$(x+1)(x^2-2x+ix-2x+4-2i-ix+2i-i^2)$

$(x+1)(x^2-4x+4-(-1))$

$(x+1)(x^2-4x+5)$

$x^3-4x^2+5x+x^2-4x+5 \Rightarrow \boxed{j(x) = x^3-3x^2+x+5}$

Assignment: Polynomial Quiz Review Packet