Name Key

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Exponential Models (3.5)

LEARNING TARGETS *I can use exponential growth and decay functions to model and solve problems.*

Exponential Growth Model: $y = ae^{bx}$

Exponential Decay Model: $y = ae^{-bx}$

The number N of bacteria in a culture is given by the model $N = 175e^{kt}$ where t is time in hours. If $N = 420$ when $t = 8$, estimate the time required for the population to double in size.

First: find the value of k using N=420 and t=8

$420 = 175e^{k(8)}$

$2.4 = e^{8k}$

$\ln(2.4) = \cancel{\ln e}^{8k}$

$\ln(2.4) = 8k$

$k = \frac{\ln(2.4)}{8}$

$k \approx .1094$

$t = ?$ $N = (175)2 = 350$, $k = .1094$

$350 = 175e^{.1094t}$

$2 = e^{.1094t}$

$\ln(2) = \cancel{\ln e}^{.1094t}$

$\ln(2) = .1094t$

$t = \frac{\ln(2)}{.1094}$

$\boxed{t \approx 6.3 \text{ hours}}$

The populations P (in thousands) of Pineville, NC from 2006 through 2012 can be modeled by $P = 5.4e^{kt}$, where t is the year, with $t = 6$ corresponding to 2006. In (t=8) → 2008, the population was 7000 (7 thousand). Find the value of k in the model and use the model to predict the population in 2018. (t=18)

First: find the value of K using P=7 and t=8

$7 = 5.4e^{k(8)}$

$\frac{7}{5.4} = e^{8k}$

$\ln\left(\frac{7}{5.4}\right) = \cancel{\ln e}^{8k}$

$\ln\left(\frac{7}{5.4}\right) = 8k$

$k = \frac{\ln\left(\frac{7}{5.4}\right)}{8}$

$k = 0.0324$

$t = 18$, $P = ?$, $k = 0.0324$

$P = 5.4e^{.0324(18)}$

$P = 5.4e^{.5839}$

$\boxed{P = 9{,}680 \text{ people}}$

The populations P (in thousands) of Pittsburgh, PA from 1990 through 2004 can be modeled by $P = 372.55e^{-0.01052t}$, where t is the year, with $t = 0$ corresponding to 1990. According to the model, was the population increasing or decreasing from 1990 to 2004? According to the model, when was the population approximately 300,000? $(t=14)$

$t=?\ P=300$

$P(1990)$: $t=0$ $P = 372.55e^{0}$

$P = 372.55$

$P = 372{,}550$

$P(2004)$: $t=14$ $P = 372.55e^{-0.01052(14)}$

$P = 321.53$

$P = 321{,}530$

$300 = 372.55e^{-0.01052t}$

$\frac{300}{372.55} = e^{-0.0105t}$

$\ln\left(\frac{300}{372.55}\right) = -0.01052t$

$t = \frac{\ln\left(\frac{300}{372.55}\right)}{-0.01052} \approx 20.6$

$1990 + 20.6 = 2010$

The population was decreasing from 1990 to 2004

$\boxed{\text{In 2010 the population was 300,000}}$

A certain type of bacteria, given a favorable growth medium, doubles in population every 6.5 hours, following the law of exponential growth, $f(t) = ae^{bt}$. Given that there were approximately 100 bacteria to start with, how many bacteria will there be in 3 days? $(a=100)$

$t = 72$ hrs

$200 = 100e^{6.5b}$

$2 = e^{6.5b}$

$\ln 2 = 6.5b$

$b = \frac{\ln 2}{6.5}$

$b \approx 1.066$

$f(72) = 100e^{\left(\frac{\ln 2}{6.5}\right)(72)}$

$= \boxed{216{,}016 \text{ bacteria}}$