Pre-Calculus A
Exponential & Logarithmic Applications HW

Name : Key
Date : ______ Period : ______

1. **Typing Speed** In typing class, the average number of words per minute N typed after t weeks of lessons was found to be modeled by $N = \dfrac{158}{1+5.4e^{-0.12t}}$. Find the numbers of weeks necessary to type:

$N=50$

(a) 50 words per minute

$\frac{50}{1} = \frac{158}{1+5.4e^{-0.12t}}$

$50(1+5.4e^{-0.12t}) = 158$

$1+5.4e^{-0.12t} = 3.16$

$5.4e^{-0.12t} = 2.16$

$e^{-0.12t} = \frac{2.16}{5.4}$

$-0.12t = \ln\left(\frac{2.16}{5.4}\right)$

$\boxed{t = 7.6 \text{ weeks}}$

$N=75$

(b) 75 words per minute

$\frac{75}{1} = \frac{158}{1+5.4e^{-0.12t}}$

$75(1+5.4)e^{-0.12t} = 158$

$75 + 405e^{-0.12t} = 158$

$405e^{-0.12t} = 83$

$e^{-0.12t} = \frac{83}{405}$

$-0.12t = \ln\left(\frac{83}{405}\right)$

$\boxed{t = 13.2 \text{ weeks}}$

2. A deposit of $10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 12 years.

$A = Pe^{rt}$

(a) What is the annual interest rate for this account?

$A = 2(10000) = 20,000$
$P = 10,000$
$t = 12$

$20000 = 10000e^{12r}$

$2 = e^{12r}$

$\ln 2 = 12r$

$r = .057$

$\boxed{5.7\%}$

(b) Find the balance after 1 year.

$P = 10,000$
$t = 1$
$r = .057$

$A = Pe^{rt}$

$A = 10,000e^{.057(1)}$

$\boxed{A = \$10,594.63}$

3. **Sales** The sales S (in thousands of units) of a cleaning solution after x hundred dollars is spent on advertising and is given by the equation $S = 10(1-e^{kx})$. When $500 is spent on advertising, 2500 units are sold.

↳ equivalent to 2.5 thousand

$S = 2.5$ thousand
$x = 5$ hundred

(a) Complete the model by solving for k

$2.5 = 10(1-e^{5x})$

$.25 = 1-e^{5x}$

$-0.75 = -e^{5x}$

$0.75 = e^{5x}$

$\ln(0.75) = 5x$

$x = \frac{\ln 0.75}{5}$

$\boxed{x = -0.0575}$

(b) Estimate the number of units that will be sold if advertising expenditures are raised to $700.

$S = ?$
$k = -.0575$
$x = 7$ hundred

$S = 10(1-e^{-0.575(7)})$

$S = 3.32 \approx \boxed{3,320 \text{ units}}$

4. **Population** The populations P (in thousands) of Cameron County, Texas, from 2006 through 2012 can be modeled by $P = 339.2e^{kt}$ where t is the year, with t = 6 corresponding to 2006. In 2011, the population was 412,600.

$t = 0$: 2000 $t = 11$

(a) Find the value of k for the model. Round your answer to four decimal places. $t = 11$; $P = 412.6$ thousand

$412.6 = 339.2e^{k(11)}$

$\frac{412.6}{339.2} = e^{11k}$

$\ln\left(\frac{412.6}{339.2}\right) = 11k$

$k = \frac{\ln\left(\frac{412.6}{339.2}\right)}{11}$

$\boxed{k = .0178}$

(b) Use your model to predict the population in 2018. $t = 18$; $P = ?$; $k = 0.0178$

$P = 339.2e^{.0178(18)}$

$P = 467.3$ thousand $= \boxed{467{,}300}$

5. **Demography** The populations P (in thousands) of Antioch, California, from 2006 through 2012 can be modeled by $P = 90e^{0.013t}$, where t is the year, with t = 6 corresponding to 2006.

(a) According to the model, was the population of Antioch increasing or decreasing from 2006 through 2012?

$t = 6$: 2006: $P = 90e^{.013(6)}$ $t = 9$: 2009: $P = 90e^{.013(9)}$ $t = 12$: 2012: $P = 90e^{.013(12)}$

$P = 97.3$ thousand $P = 101.2$ thousand $P = 106.2$ thousand

$\boxed{\text{increasing}}$

(b) What were the populations of Antioch in 2006, 2009, and 2012?

2006: 97,300

2009: 101,200

2012: 106,200

(c) According to the model, when will the population of Antioch be approximately 116,000?

$P = 116$ thousand

$t = ?$

116000 = 116 thousand

$116 = 90e^{.013t}$

$\frac{116}{90} = e^{.013t}$

$\ln\left(\frac{116}{90}\right) = .013t$

$t = 19.5$

$t = 0$: 2000
$+ 19.5$

$\boxed{2019}$