

Solve the system of linear equations using substitution.

$$1. \begin{cases} 2x + 4y + z = 1 & \textcircled{1} \\ x - 2y - 3z = 2 & \textcircled{2} \\ x + y - z = -1 & \textcircled{3} \end{cases}$$

Solve eqn 1 for z: $z = 1 - 2x - 4y$

Sub. into eqn 2:

$$x - 2y - 3(1 - 2x - 4y) = 2$$

$$x - 2y - 3 + 6x + 12y = 2$$

$$7x + 10y = 5 \text{ new 2}$$

Sub into eqn 3:

$$x + y - (1 - 2x - 4y) = -1$$

$$x + y - 1 + 2x + 4y = -1$$

$$3x + 5y = 0 \text{ new 3}$$

Solve new system:

$$\begin{array}{r} 7x + 10y = 5 \\ -2(3x + 5y = 0) \Rightarrow -6x - 10y = 0 \\ \hline 1x = 5 \\ \boxed{x = 5} \end{array}$$

Sub. $x = 5$ into new 3:

$$3(5) + 5y = 0$$

$$15 + 5y = 0$$

$$5y = -15$$

$$\boxed{y = -3}$$

$$\boxed{\text{Solution: } (5, -3, 3)}$$

Sub $x = 5$ and $y = -3$ into eqn 1:

$$z = 1 - 2(5) - 4(-3)$$

$$z = 1 - 10 + 12$$

$$\boxed{z = 3}$$

Solve the system of linear equations using elimination.

$$2. \begin{cases} 3x - 5y + 5z = 1 & \textcircled{1} \\ 5x - 2y + 3z = 0 & \textcircled{2} \\ 7x - y + 3z = 0 & \textcircled{3} \end{cases}$$

* Eliminate y

Equations $\textcircled{1}$ and $\textcircled{2}$

$$\begin{array}{r} -2(3x - 5y + 5z = 1) \\ 5(5x - 2y + 3z = 0) \end{array} \Rightarrow \begin{array}{r} -6x + 10y - 10z = -2 \\ 25x - 10y + 15z = 0 \end{array}$$

$$19x + 5z = -2 \text{ new 1}$$

Equations $\textcircled{2}$ and $\textcircled{3}$

$$\begin{array}{r} 5x - 2y + 3z = 0 \\ -2(7x - y + 3z = 0) \end{array} \Rightarrow \begin{array}{r} 5x - 2y + 3z = 0 \\ -14x + 2y - 6z = 0 \end{array}$$

$$-9x - 3z = 0 \text{ new 2}$$

Solve new system:

$$\begin{array}{r} 3(19x + 5z = -2) \\ 5(-9x - 3z = 0) \end{array} \Rightarrow \begin{array}{r} 57x + 15z = -6 \\ -45x - 15z = 0 \end{array}$$

$$12x = -6$$
$$x = -\frac{1}{2}$$

Sub $x = -\frac{1}{2}$ into new 2:

$$-9(-\frac{1}{2}) - 3z = 0$$

$$4.5 - 3z = 0$$

$$-3z = -4.5$$

$$z = 1.5$$

$$\text{Solution: } (-\frac{1}{2}, 1, 1.5)$$

Sub $x = -\frac{1}{2}$ and $z = 1.5$ into eqn 1:

$$3(-\frac{1}{2}) - 5y + 5(1.5) = 1$$

$$-1.5 - 5y + 7.5 = 1$$

$$-5y + 6 = 1$$

$$-5y = -5$$

$$y = 1$$

$c = \text{coffee}$; $d = \text{donut}$; $t = \text{tip}$

3. On Monday, Ms. Rabinko's favorite customer at Dunkin Donuts paid \$1.70 for two cups of coffee and one donut, including the tip. On Tuesday, he paid \$1.65 for two donuts and a cup of coffee, including the tip. On Wednesday, he paid \$1.30 for one coffee and one donut, including the tip. If he always tips the same amount, then what is the amount of each item?

$$\begin{cases} 2c + d + t = 1.70 & \textcircled{1} \\ c + 2d + t = 1.65 & \textcircled{2} \\ c + d + t = 1.30 & \textcircled{3} \end{cases}$$

Solve eqn 1 for d

$$d = 1.70 - 2c - t$$

sub into eqn 2:

$$c + 2(1.70 - 2c - t) + t = 1.65$$

$$c + 3.40 - 4c - 2t + t = 1.65$$

$$-3c - t = -1.75 \text{ new 2}$$

sub into eqn 3:

$$c + (1.70 - 2c - t) + t = 1.30$$

$$-c = -0.4$$

$$c = \$0.40$$

sub $c = 0.40$ into new eqn 2:

$$-3(0.40) - t = -1.75$$

$$-1.20 - t = -1.75$$

$$-t = -.55$$

$$t = 0.55$$

Coffee: \$0.40

donut: \$0.35

Tip: \$0.55

sub $c = 0.40$ and $t = 0.55$ into eqn 1:

$$d = 1.70 - 2(0.40) - 0.55$$

$$d = 0.35$$

$n = \text{nickels}$
 $d = \text{dimes}$
 $q = \text{quarters}$

4. Harry has \$2.25 in nickels, dimes, and quarters. If he had twice as many nickels, half as many dimes, and the same number of quarters, he would have \$2.50. If he has 27 coins all together, then how many of each coin does he have?

$$\begin{cases} n + d + q = 27 \\ .05n + .10d + .25q = 2.25 \\ 2(.05)n + \frac{1}{2}(.10d) + .25q = 2.50 \end{cases} \Rightarrow \begin{cases} n + d + q = 27 \\ .05n + .10d + .25q = 2.25 \\ .10n + .05d + .25q = 2.50 \end{cases}$$

Solve eqn 1 for n:

$$n = 27 - d - q$$

Sub into eqn 2:

$$.05(27 - d - q) + .10d + .25q = 2.25$$

$$1.35 - .05d - .05q + .10d + .25q = 2.25$$

$$.05d + .20q = 0.90 \text{ new 2}$$

Sub into eqn 3:

$$.10(27 - d - q) + .05d + .25q = 2.50$$

$$2.70 - .10d - .10q + .05d + .25q = 2.50$$

$$-.05d + .15q = -.20 \text{ new 3}$$

Solve new system:

$$.05d + .20q = 0.90$$

$$-.05d + .15q = -.20$$

$$.35q = 0.70$$

$$q = 2$$

Sub $q = 2$ into new 2:

$$.05d + .20(2) = 0.90$$

$$.05d + .40 = 0.90$$

$$.05d = 0.50$$

$$d = 10$$

Sub $q = 2$ and $d = 10$ into ①

$$n = 27 - 10 - 2$$

$$n = 15$$

15 nickels
10 dimes
2 quarters

Answer Key:

1. (5, -3, 3) ✓

3. Coffee: \$0.40, Donut: \$0.35, Tip: \$0.55 ✓

2. (-1/2, 1, 1.5) ✓

4. 15 nickels, 10 dimes, 2 quarters ✓