

1.
$$\begin{cases} 4x + 4y + z = 24 & \textcircled{1} \\ 2x - 4y + z = 0 & \textcircled{2} \\ 5x - 4y - 5z = 12 & \textcircled{3} \end{cases}$$
 *Eliminate y

Equations $\textcircled{1}$ & $\textcircled{2}$

$$\begin{array}{r} 4x + 4y + z = 24 \\ 2x - 4y + z = 0 \\ \hline 6x + 2z = 24 \text{ new 1} \end{array}$$

Equations $\textcircled{1}$ & $\textcircled{3}$

$$\begin{array}{r} 4x + 4y + z = 24 \\ 5x - 4y - 5z = 12 \\ \hline 9x - 4z = 36 \text{ new 2} \end{array}$$

$$\begin{array}{r} 2(6x + 2z = 24) \\ 9x - 4z = 36 \\ \hline 12x + 4z = 48 \\ 9x - 4z = 36 \\ \hline 21x = 84 \\ \hline x = 4 \end{array}$$

Solution:
(4, 2, 0)

Sub into new 1:

$$\begin{array}{r} 6(4) + 2z = 24 \\ 24 + 2z = 24 \\ 2z = 0 \\ \hline z = 0 \end{array}$$

Sub into orig eqn 1:

$$\begin{array}{r} 4(4) + 4y + 0 = 24 \\ 16 + 4y = 24 \\ 4y = 8 \\ \hline y = 2 \end{array}$$

2.
$$\begin{cases} 6r - s + 3t = -9 & \textcircled{1} \\ 5r + 5s - 5t = 20 & \textcircled{2} \\ 3r - s + 4t = -5 & \textcircled{3} \end{cases}$$
 *Eliminate s

Equations $\textcircled{1}$ & $\textcircled{2}$

$$\begin{array}{r} 5(6r - s + 3t = -9) \\ 5r + 5s - 5t = 20 \\ \hline 30r - 5s + 15t = -45 \\ 5r + 5s - 5t = 20 \\ \hline 35r + 10t = -25 \text{ new 1} \end{array}$$

Equations $\textcircled{2}$ & $\textcircled{3}$

$$\begin{array}{r} 5r + 5s - 5t = 20 \\ 5(3r - s + 4t = -5) \\ \hline 5r + 5s - 5t = 20 \\ 15r - 5s + 20t = -25 \\ \hline 20r + 15t = -5 \text{ new 2} \end{array}$$

$$\begin{array}{r} 15(35r + 10t = -25) \\ -10(20r + 15t = -5) \\ \hline 525r + 150t = -375 \\ -200r - 150t = 50 \\ \hline 325r = -325 \\ \hline r = -1 \end{array}$$

Sub into new 1:

$$\begin{array}{r} 35(-1) + 10t = -25 \\ -35 + 10t = -25 \\ 10t = 10 \\ \hline t = 1 \end{array}$$

Sub into orig eqn 1:

$$\begin{array}{r} 6(-1) - s + 3(1) = -9 \\ -6 - s + 3 = -9 \\ -3 - s = -9 \\ -s = -6 \\ \hline s = 6 \end{array}$$

Solution: (r, s, t)
(-1, 6, 1)

$$\begin{cases} \textcircled{1} & -x - 5y + z = 17 \Rightarrow z = 17 + x + 5y \\ \textcircled{2} & -5x - 5y + 5z = 5 \\ \textcircled{3} & 2x + 5y - 3z = -10 \end{cases}$$

Sub into Eqn 2:

$$-5x - 5y + 5(17 + x + 5y) = 5$$

$$-5x - 5y + 85 + 5x + 25y = 5$$

$$20y + 85 = 5$$

$$20y = -80$$

$$y = -4$$

Sub into eqn 3:

$$2x + 5y - 3(17 + x + 5y) = -10$$

$$2x + 5y - 51 - 3x - 15y = -10$$

$$-x - 10y - 51 = -10$$

$$-x - 10y = 41$$

$$-x - 10(-4) = 41$$

$$-x + 40 = 41$$

$$-x = 1$$

$$x = -1$$

Sub $x = -1$ and $y = -4$ into orig eqn 1:

$$z = 17 + (-1) + 5(-4)$$

$$z = 17 - 1 - 20$$

$$z = -4$$

$$\text{Solution: } (-1, -4, -4)$$

4. An arcade in Lynchburg, Tennessee uses 3 different colored tokens for their game machines. For \$20, you can purchase any of the following mixtures of tokens: 14 gold, 20 silver, and 24 bronze; OR 20 gold, 15 silver, and 19 bronze; OR 30 gold, 5 silver, and 13 bronze. What is the monetary value of each token?

x : Gold, y : Silver, z : Bronze

Mixture 1: $14x + 20y + 24z = 20$

Mixture 2: $20x + 15y + 19z = 20$

Mixture 3: $30x + 5y + 13z = 20$

*Eliminate y with coefficient of 60

① and ②

$$\begin{aligned} 3(14x + 20y + 24z = 20) & \Rightarrow 42x + 60y + 72z = 60 \\ -4(20x + 15y + 19z = 20) & \Rightarrow -80x - 60y - 76z = -80 \end{aligned}$$

$$-38x - 4z = -20 \text{ new 1}$$

① and ③ *Eliminate y with coefficient of 20

$$\begin{aligned} 14x + 20y + 24z = 20 \\ -4(30x + 5y + 13z = 20) & \Rightarrow -120x - 20y - 52z = -80 \end{aligned}$$

$$-106x - 28z = -60 \text{ new 2}$$

$$\begin{aligned} \text{Solution: Gold: } \$0.50 \\ \text{Silver: } \$0.35 \\ \text{Bronze: } \$0.25 \end{aligned}$$

$$\begin{aligned} -7(-38x - 4z = -20) & \Rightarrow 266x + 28z = 140 \\ -106x - 28z = -60 & \Rightarrow -106x - 28z = -60 \end{aligned}$$

$$160x = 80$$

$$x = .50$$

sub into new 1:

$$-38(.50) - 4z = -20$$

$$-19 - 4z = -20$$

$$-4z = -1$$

$$z = 0.25$$

sub into orig. 1:

$$14(.50) + 20y + 24(.25) = 20$$

$$7 + 20y + 6 = 20$$

$$20y = 7$$

$$y = .35$$

smallest: x , middle: y , largest: z

5. A triangle has three angles (duh, Ms. R). The middle angle measures 5° more than twice the smallest angle, and the largest angle measures 11° less than 3 times the measure of the smallest angle. Find the measures of the three angles.

Equation 1 (Δ sum): $x+y+z=180$

Equation 2: $y=5+2x$

Equation 3: $z=3x-11$

$$\begin{cases} \textcircled{1} & x+y+z=180 \\ \Rightarrow \textcircled{2} & \begin{cases} 2x-y+0z=-5 & \Rightarrow y=5+2x \\ \textcircled{3} & \begin{cases} 3x+0y-z=11 & \Rightarrow z=3x-11 \end{cases} \end{cases} \end{cases}$$

Sub $y=5+2x$ into eqn 1:

$$x+5+2x+z=180$$

$$3x+z=175 \text{ new 1}$$

Sub $z=3x-11$ into new 1:

$$3x+3x-11=175$$

$$6x=186$$

$$x=31$$

Sub $x=31$ into new 1:

$$3(31)+z=175$$

$$93+z=175$$

$$z=82$$

Sub into orig eqn 1:

$$31+y+82=180$$

$$y+113=180$$

$$y=67$$

Solution: smallest = 31°
middle = 67°
largest = 82°

Answer Key:

1. (4, 2, 0)

2. (-1, 6, 1)

3. (-1, -4, -4)

4. Gold: \$0.50, Silver: \$0.35, Bronze: \$0.25

5. Small: 31° , Middle: 67° , Large: 82°