

Solve the following systems using substitution.

$$1. \begin{cases} 5x + 2y = 4 \\ -2x + y = 11 \Rightarrow y = 11 + 2x \end{cases}$$
$$5x + 2(11 + 2x) = 4$$
$$5x + 22 + 4x = 4$$
$$9x + 22 = 4$$
$$9x = -18$$
$$x = -2$$

$$y = 11 + 2(-2)$$
$$y = 11 - 4$$
$$y = 7$$

Solution:  $(-2, 7)$

$$2. \begin{cases} x - 2y = -4 \Rightarrow x = -4 + 2y \\ -3x + 6y = 0 \end{cases}$$
$$-3(-4 + 2y) + 6y = 0$$
$$12 - 6y + 6y = 0$$
$$12 = 0$$

No solution;  $12$  is never equal to  $0$

Solve the following systems using elimination.

$$3. \begin{cases} 8x + 7y = 38 \\ 3x - 5y = -1 \end{cases} \Rightarrow \begin{array}{r} 40x + 35y = 190 \\ 21x - 35y = -7 \\ \hline 61x = 183 \\ x = 3 \end{array}$$

$$8(3) + 7y = 38$$
$$24 + 7y = 38$$
$$7y = 14$$
$$y = 2$$

Solution:  $(3, 2)$

$$4. \begin{cases} 7x + 2y = -1 \\ 5x - 3y = -14 \end{cases} \Rightarrow \begin{array}{r} 21x + 6y = -3 \\ 10x - 6y = -28 \\ \hline 31x = -31 \\ x = -1 \end{array}$$

$$7(-1) + 2y = -1$$
$$-7 + 2y = -1$$
$$2y = 6$$
$$y = 3$$

Solution:  $(-1, 3)$

Solve the following systems using whichever method you choose. Check your solution using matrices.

$$\begin{cases} \textcircled{1} & 2x + 4y - z = 7 & * \text{Eliminate } y \\ 5. \textcircled{2} & 2x - 4y + 2z = -6 \\ \textcircled{3} & x + 4y + z = 0 \end{cases}$$

Eqns  $\textcircled{1} : \textcircled{2}$

$$\begin{array}{r} 2x + 4y - z = 7 \\ 2x - 4y + 2z = -6 \\ \hline \end{array}$$

$$4x + z = 1 \text{ new 1}$$

Eqns  $\textcircled{2} : \textcircled{3}$

$$\begin{array}{r} 2x - 4y + 2z = -6 \\ x + 4y + z = 0 \\ \hline \end{array}$$

$$3x + 3z = -6 \text{ new 2}$$

$$\begin{array}{r} -3(4x + z = 1) \\ 3x + 3z = -6 \\ \hline \end{array} \Rightarrow \begin{array}{r} -12x - 3z = -3 \\ 3x + 3z = -6 \\ \hline \end{array}$$

$$-9x = -9$$

$$x = 1$$

Sub  $x=1$  back into new 1:

$$4(1) + z = 1$$

$$4 + z = 1$$

$$z = -3$$

Sub  $x=1$  and  $z=-3$  into orig eqn:

$$2(1) + 4y - (-3) = 7$$

$$2 + 4y + 3 = 7$$

$$4y = 2$$

$$y = \frac{1}{2}$$

$$\text{Solution: } (1, \frac{1}{2}, -3)$$

$$\begin{cases} \textcircled{1} & x + y + z = 3 \Rightarrow x = 3 - y - z \\ 6. \textcircled{2} & 2x - y + 3z = 16 \\ \textcircled{3} & x - 2y - z = 1 \end{cases}$$

Sub into Eqn 2:

$$2(3 - y - z) - y + 3z = 16$$

$$6 - 2y - 2z - y + 3z = 16$$

$$-3y + z = 10 \text{ new 1}$$

Sub into Eqn 3:

$$3 - y - z - 2y - z = 1$$

$$-3y - 2z = -2 \text{ new 2}$$

$$\begin{array}{r} 2(-3y + z = 10) \\ -3y - 2z = -2 \\ \hline \end{array} \Rightarrow \begin{array}{r} -6y + 2z = 20 \\ -3y - 2z = -2 \\ \hline \end{array}$$

$$-9y = 18$$

$$y = -2$$

Sub  $y=-2$  and  $z=4$  into orig eqn:

$$x - 2 + 4 = 3$$

$$x + 2 = 3$$

$$x = 1$$

Sub  $y=-2$  into new 1:

$$-3(-2) + z = 10$$

$$6 + z = 10$$

$$z = 4$$

$$\text{Solution: } (1, -2, 4)$$

$$\begin{array}{l}
 \textcircled{1} \left\{ \begin{array}{l} x - 6y + 4z = -12 \\ x + y - 4z = 12 \Rightarrow x = 12 - y + 4z \\ 2x + 2y + 5z = -15 \end{array} \right.
 \end{array}$$

Sub into Eqn ①:

$$12 - y + 4z - 6y + 4z = -12$$

$$-7y + 8z = -24 \text{ new 1}$$

Sub into Eqn ③:

$$2(12 - y + 4z) + 2y + 5z = -15$$

$$24 - 2y + 8z + 2y + 5z = -15$$

$$13z = -39$$

$$z = -3$$

Sub  $z = -3$  into new 1:

$$-7y + 8(-3) = -24$$

$$-7y - 24 = -24$$

$$7y = 0$$

$$y = 0$$

Sub  $y = 0$  and  $z = -3$  into orig eqn:

$$x = 12 - 0 + 4(-3)$$

$$x = 12 - 0 - 12$$

$$x = 0$$

$$\text{Solution: } (0, 0, -3)$$

$$\begin{array}{l}
 \textcircled{1} \left\{ \begin{array}{l} -6x + 5y + 2z = -11 \quad * \text{Eliminate } y \\ -2x + y + 4z = -9 \\ 4x - 5y + 5z = -4 \end{array} \right.
 \end{array}$$

Eqn ① : ②

$$-6x + 5y + 2z = -11$$

$$-5(-2x + y + 4z = -9) \Rightarrow$$

$$-6x + 5y + 2z = -11$$

$$10x - 5y - 20z = 45$$

$$4x - 18z = 34 \text{ new 1}$$

Eqn ① : ③

$$-6x + 5y + 2z = -11$$

$$4x - 5y + 5z = -4$$

$$-2x + 7z = -15 \text{ new 2}$$

$$4x - 18z = 34 \Rightarrow$$

$$2(-2x + 7z = -15) \Rightarrow$$

$$-4x + 14z = -30$$

$$-4z = 4$$

$$z = -1$$

$$\text{Solution: } (4, 3, -1)$$

Sub into new 2:

$$-2x + 7(-1) = -15$$

$$-2x - 7 = -15$$

$$-2x = -8$$

$$x = 4$$

Sub  $x = 4$  and  $z = -1$  into orig eqn:

$$-2(4) + y + 4(-1) = -9$$

$$-8 + y - 4 = -9$$

$$-12 + y = -9$$

$$y = 3$$

x: smallest #, y: middle #, z: largest #

9. The sum of three numbers is 24. Twice the smallest number is 2 less than the largest number, and the largest number is equal to the sum of the other two. What are the three numbers?

3rd Eqn  $\leftarrow$

$$\begin{cases} \text{Sum: } x+y+z=24 & \textcircled{1} \\ \text{2nd Eqn: } 2x=z-2 & \textcircled{2} \\ z=x+y & \textcircled{3} \end{cases} \Rightarrow \begin{cases} x+y+z=24 \\ 2x-z=-2 \\ x+y-z=0 \end{cases}$$

Solution: smallest: 5  
Middle: 7  
Largest: 12

Sub Eqn ③ into ①:

$x+y+x+y=24$   
 $2x+2y=24$  new 1

Sub Eqn ③ into ②:

$2x = x+y-2$   
 $x-y=-2$  new 2

$2x+2y=24$   
 $2(x-y)=-2$   
 $\Rightarrow \frac{2x+2y=24}{2x-2y=-2}$   
 $4y=26$   
 $y=6.5$

Sub  $x=5$  into new 2:

$5-y=-2$   
 $-y=-7$   
 $y=7$

Sub into orig #3:

$z=5+7$   
 $z=12$

10. A theater has tickets at \$6 for adults, \$3.50 for students, and \$2.50 for children under 12 years old. A total of 278 tickets were sold for one showing with a total revenue of \$1,300. If the number of adult tickets sold was 10 less than twice the number of student tickets, how many of each type of ticket were sold for the showing? x = adults, y = students, z = children

Revenue:  $6x+3.50y+2.50z=1300$  ①  
Tickets sold:  $x+y+z=278$  ②  
 $x=2y-10$  ③

new system:  
 $-2.5(3y+z=288)$   
 $15.50y+2.50z=1360$   
 $\Rightarrow \frac{-7.50y-2.50z=-720}{15.50y+2.50z=1360}$

Sub ③ into ①

$6(2y-10)+3.50y+2.50z=1300$   
 $12y-60+3.50y+2.50z=1300$   
 $15.50y+2.50z=1360$  new 1

Sub ③ into ②:

$2y-10+y+z=278$   
 $3y+z=288$  new 2

Sub into new 2:

$3(80)+z=288$   
 $240+z=288$   
 $z=48$

$8y=640$   
 $y=80$

Sub into orig eqn #3:

$x=2(80)-10$   
 $x=150$

Solution: 150 adult  
80 student  
48 child

Answer Key:

- 1. (-2, 7)
- 2. No Solution
- 3. (3, 2)
- 4. (-1, 3)
- 5.  $(1, \frac{1}{2}, -3)$
- 6. (1, -2, 4)
- 7. (0, 0, -3)
- 8. (4, 3, -1)
- 9. Smallest: 5, Middle: 7, Largest: 12
- 10. Adult: 150, Student: 80, Child: 48