Pre-Calculus (A)
Law of Sines/Cosines Quiz Review

Name : Key
Date : _____

Determine whether the Law of Cosines or the Law of Sines is needed to solve each of the following triangles. It might help to sketch a triangle for each one.

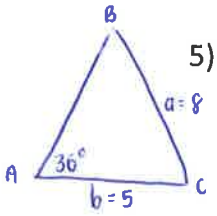
1) $m\angle A = 15°,\ m\angle B = 58°,\ c = 94$ AAS → Law of Sines

2) $a = 96,\ b = 43,\ m\angle A = 105°$ SSA → Law of Sines

3) $a = 24,\ b = 16,\ c = 29$ SSS → Law of Cosines

4) $a = 15,\ c = 42,\ m\angle B = 49°$ SAS → Law of Cosines

Solve the triangle(s) for questions #5-9.



5) $a = 8,\ b = 5,\ m\angle A = 36°$ (SSA → Law of Sines)

$\frac{\sin 36}{8} = \frac{\sin B}{5} = \frac{\sin C}{c}$

$\frac{5\sin 36}{8} = \frac{8\sin B}{8}$

$.3674 = \sin B$

$m\angle B = 21.6°$

$m\angle C = 122.4°$

$\frac{\sin 36}{8} = \frac{\sin 21.6}{5} = \frac{\sin 122.4}{c}$

$\frac{8\sin 122.4}{\sin 36} = \frac{c\sin 36}{\sin 36}$

$c = 11.5$

Second solution?

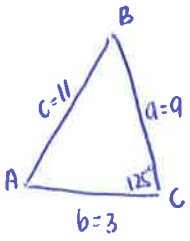
Possible $m\angle B = 180 - 21.6 = 158.4°$

$m\angle A = 36°$

$m\angle C = 180 - 158.4 - 36 = -14.4°$

X

No second solution; $m\angle C$ can't be negative



6) $a = 9,\ b = 3,\ c = 11$ (SSS → Law of Cosines)

* Solve for angle C first:

$c^2 = a^2 + b^2 - 2ab\cos C$

$11^2 = 9^2 + 3^2 - 2(9)(3)\cos C$

$121 = 90 - 54\cos C$

$\frac{31}{-54} = \frac{-54\cos C}{-54}$

$-0.5741 = \cos C$

$m\angle C = 125.0°$

$\frac{\sin A}{9} = \frac{\sin B}{3} = \frac{\sin 125°}{11}$

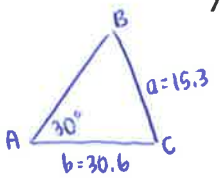
$\frac{3\sin 125°}{11} = \frac{9\sin B}{9}$

$.2234 = \sin B$

$m\angle B = 12.9°$

So $m\angle A = 42.1°$

7) $a = 15.3,\ b = 30.6,\ m\angle A = 30°$ (SSA - Law of Sines)



$\frac{\sin 30}{15.3} = \frac{\sin B}{30.6} = \frac{\sin C}{c}$

$\frac{30.6 \sin 30}{15.3} = \frac{15.3 \sin B}{15.3}$

$\sin B = 1$

$m\angle B = 90°$

so $m\angle C = 60°$

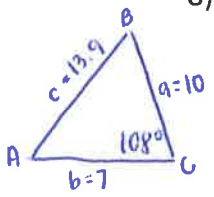
$\frac{\sin 30}{15.3} = \frac{\sin 90}{30.6} = \frac{\sin 60}{c}$

$\frac{30.6 \sin 60}{\sin 90} = \frac{c \sin 90}{\sin 90}$

$c = 26.5$

No possible second solution since the other possibility for $m\angle B = 180 - 90 = 90°$ which would produce the exact same Δ

8) $a = 10,\ b = 7,\ m\angle C = 108°$ (SAS - Law of Cosines)



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 10^2 + 7^2 - 2(10)(7)\cos 108°$

$c^2 = 149 - 140\cos 108°$

$c^2 = 192.26$

$c = 13.9$

$\frac{\sin A}{10} = \frac{\sin B}{7} = \frac{\sin 108}{13.9}$

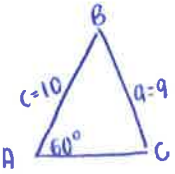
$\frac{7 \sin 108}{13.9} = \frac{13.9 \sin B}{13.9}$

$\sin B = .4789$

$m\angle B = 28.6°$

so $m\angle C = 43.4°$

9) $a = 9,\ c = 10,\ m\angle A = 60°$ (SSA - Law of Sines)



$\frac{\sin 60}{9} = \frac{\sin B}{b} = \frac{\sin C}{10}$

$\frac{10 \sin 60}{9} = \frac{9 \sin C}{9}$

$\sin C = .9623$

$m\angle C = 74.2°$

$m\angle B = 45.8°$

$\frac{\sin 60}{9} = \frac{\sin 45.8}{b} = \frac{\sin 74.2}{10}$

$\frac{9 \sin 45.8}{\sin 60} = \frac{b \sin 60}{\sin 60}$

$b = 7.5$

Second solution:

Possible $m\angle C = 180 - 74.2° = 105.8° = m\angle C$

$m\angle A = 60°$

$m\angle B = 14.2°$

$\frac{\sin 60}{9} = \frac{\sin 14.2}{b} = \frac{\sin 105.8}{10}$

$\frac{9 \sin 14.2}{\sin 60} = \frac{b \sin 60}{\sin 60}$

$b = 2.5$

Find the area of the triangles for questions #10-11.

10) $a=5,\ b=8,\ c=10$

*Find s first:

$s=\frac{a+b+c}{2}$

$s=\frac{5+8+10}{2}$

$s=\frac{23}{2}$

$s=11.5$

*Now find area:

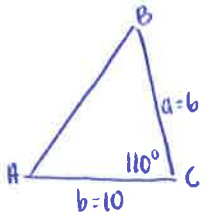
$A=\sqrt{s(s-a)(s-b)(s-c)}$

$A=\sqrt{11.5(11.5-5)(11.5-8)(11.5-10)}$

$A=\sqrt{11.5(6.5)(3.5)(1.5)}$

$A=\sqrt{392.4375} \approx$ **19.8 units²**

11) $a=6,\ b=10,\ m\angle C=110°$

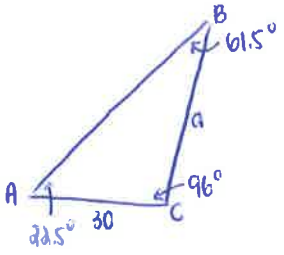


$A=\frac{1}{2}ab\sin C$

$A=\frac{1}{2}(6)(10)\sin(110°)$

A = 28.2 units²

12) Because of prevailing winds, a tree grew so that is was leaning 6 degrees from vertical away from you. You are at a point 30 meters from the tree, the angle of elevation to the top of the tree is 22.5 degrees. Find the height of the tree.

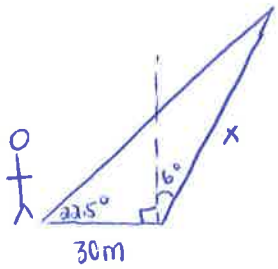


$\frac{\sin 22.5}{a}=\frac{\sin 61.5}{30}=\frac{\sin 96}{c}$

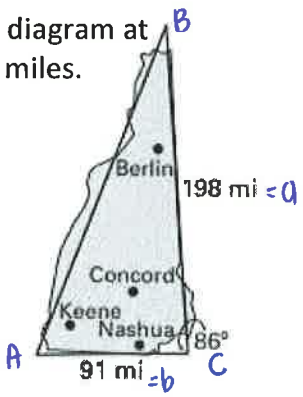
$\frac{30\sin 22.5}{\sin 61.5}=\frac{a\sin 61.5}{\sin 61.5}$

$a=13.1$

The tree is about 13.1 m tall



13) The state of New Hampshire is approximately triangular in shape. Use the diagram at the right to estimate the area of the state to the nearest thousand square miles.



$A=\frac{1}{2}ab\sin C$

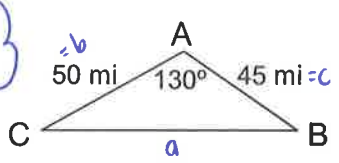
$A=\frac{1}{2}(198)(91)\sin(86°)$

$A=8987.1$

A = 8987 mi²

14) An emergency rescue helicopter has flown from its home base at point C to pick up an accident victim at point A and then from there to the hospital at point B. The pilot needs to know how far he is now from his home base so he can decide whether to refuel before returning. How far is the hospital from the helicopter's base?

SAS - Law of Cosines



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 50^2 + 45^2 - 2(50)(45)\cos 130°$

$a^2 = 4525 - 4500\cos 130°$

$a^2 = 7417.54$

$a = 86.1$

It is about 86.1 miles to the hospital

15) A plane leaves JFK International Airport and travels due west at 570 miles per hour. Another plane leaves 20 minutes later and travels N 22° W at the rate of 585 miles per hour. To the nearest mile, how far apart are they 40 minutes after the second plane leaves?

First plane: Travels 20 min + 40 min = 1 hr
So distance is 570 mi

Second plane: Travels 40 min

585 mph ÷ 60 min = 9.75 mi/min

So distance = 9.75 × 40 = 390 mi

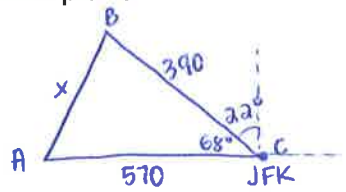
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 390^2 + 570^2 - 2(390)(570)\cos 68°$

$c^2 = 477000 - 444600\cos 68$

$c^2 = 310449.9086$

$c = 557.2$



The planes are about 557 mi apart

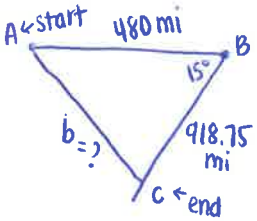
16) A plane flies in a straight line at 400 mph for 1 hour and 12 minutes. The plane then makes a 15 degree turn and flies at 375 mph for 2 hours and 27 minutes. How far is the plane from the starting point?

$\frac{12}{60} = .2$ hr

$\frac{27}{60} = .45$ hr

400 mph × 1.2 hrs = 480 miles traveled on the 1st leg

375 mph × 2.45 hrs = 918.75 miles traveled on the 2nd leg



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = (918.75)^2 + (480)^2 - 2(918.75)(480)\cos 15°$

$b^2 = 1074501.563 - 882000\cos 15°$

$b^2 = 222554.9842$

$b = 471.8$

The plane is about 471.8 miles from its starting point