

Simplify the following expressions.

$$\begin{aligned} 1. \cos\left(\theta - \frac{3\pi}{2}\right) &= \cos\theta \cos \frac{3\pi}{2} + \sin\theta \sin \frac{3\pi}{2} \\ &= \cos\theta (0) + \sin\theta (-1) \\ &= 0 - \sin\theta \\ &= \boxed{-\sin\theta} \end{aligned}$$

$$\begin{aligned} 2. \sin\left(\frac{3\pi}{2} + \theta\right) &= \sin \frac{3\pi}{2} \cos\theta + \cos \frac{3\pi}{2} \sin\theta \\ &= (-1) \cos\theta + (0) \sin\theta \\ &= \boxed{-\cos\theta} \end{aligned}$$

$$3. \tan(\pi + \theta) = \frac{\tan\pi + \tan\theta}{1 - \tan\pi \tan\theta} = \frac{0 + \tan\theta}{1 - (0)\tan\theta} = \frac{\tan\theta}{1} = \boxed{\tan\theta}$$

Prove the following identities.

$$4. \sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\text{LHS: } \sin \frac{\pi}{2} \cos x + \cos \frac{\pi}{2} \sin x$$

$$(1) \cos x + (0) \sin x$$

$$\cos x$$

$$\begin{aligned} \text{LHS} &= \text{RHS} \\ \cos x &= \cos x \quad \checkmark \end{aligned}$$

$$5. \cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$$

$$\text{LHS: } \cos \pi \cos \theta + \sin \pi \sin \theta + \sin \frac{\pi}{2} \cos \theta + \cos \frac{\pi}{2} \sin \theta$$

$$\Rightarrow (-1) \cos \theta + (0) \sin \theta + (1) \cos \theta + (0) \sin \theta$$

$$\Rightarrow -\cos \theta + \cos \theta$$

$$\Rightarrow 0$$

$$\text{LHS} = \text{RHS} \quad \checkmark$$

$$0 = 0$$

Find the solution(s) of the equation in the interval $[0, 2\pi)$.

$$6. \sin(x + \pi) - \sin x + 1 = 0$$

$$\sin x \cos \pi + \cos x \sin \pi - \sin x + 1 = 0$$

$$\sin x (-1) + \cos x (0) - \sin x + 1 = 0$$

$$-\sin x - \sin x + 1 = 0$$

$$-2\sin x = -1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$7. \cos\left(x + \frac{\pi}{4}\right) - 1 = \cos\left(x - \frac{\pi}{4}\right)$$

$$\cancel{\cos x \cos \frac{\pi}{4}} - \cancel{\sin x \sin \frac{\pi}{4}} - 1 = \cancel{\cos x \cos \frac{\pi}{4}} + \cancel{\sin x \sin \frac{\pi}{4}}$$

$$- \cancel{\sin x \sin \frac{\pi}{4}} - 1 = \cancel{\sin x \sin \frac{\pi}{4}}$$

$$+ \cancel{\sin x \sin \frac{\pi}{4}} + \cancel{\sin x \sin \frac{\pi}{4}}$$

$$-1 = 2 \sin x \sin \frac{\pi}{4}$$

$$-1 = 2 \left(\frac{\sqrt{2}}{2}\right) \sin x$$

$$-1 = \sqrt{2} \sin x$$

$$\frac{-1}{\sqrt{2}} = \sin x$$

$$\frac{-1}{\sqrt{2}} = \sin x$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$8. \sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$$\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} - \left(\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cancel{\sin x \cos \frac{\pi}{6}} + \cos x \sin \frac{\pi}{6} - \cancel{\sin x \cos \frac{\pi}{6}} + \cos x \sin \frac{\pi}{6} = \frac{1}{2}$$

$$2 \cos x \sin \frac{\pi}{6} = \frac{1}{2}$$

$$2\left(\frac{1}{2}\right) \cos x = \frac{1}{2}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$9. \sin\left(x + \frac{\pi}{2}\right) - \cos^2 x = 0$$

$$\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2} - \cos^2 x = 0$$

$$\cancel{\sin x (0)} + \cos x (1) - \cos^2 x = 0$$

$$\cos x - \cos^2 x = 0$$

$$\cos x (1 - \cos x) = 0$$

$$\cos x = 0$$

$$1 - \cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$1 = \cos x$$

$$x = 0$$