

Find the exact value of each expression.

$$1. \sin\left(\frac{2\pi}{3} + \frac{5\pi}{6}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{5\pi}{6}\right)$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right) \cdot \frac{1}{2}$$

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \boxed{1}$$

$$2. \cos\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right) = \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{9\pi}{4}\right) + \sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{9\pi}{4}\right) = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}}$$

$$3. \sin\left(\frac{13\pi}{12}\right) = \sin(195^\circ) = \sin(150 + 45) = \sin(150)\cos(45) + \cos(150)\sin(45)$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

$$4. \tan(105^\circ) = \tan(60 + 45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} = \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = \boxed{-2 - \sqrt{3}}$$

5. Write the expression as the sine, cosine, or tangent of an angle:

$$\sin\left(\frac{4\pi}{9}\right)\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{4\pi}{9}\right)\sin\left(\frac{\pi}{8}\right)$$

$$\sin(u+v)$$

$$\sin\left(\frac{4\pi}{9} + \frac{\pi}{8}\right)$$

$$\sin\left(\frac{32\pi}{72} + \frac{9\pi}{72}\right)$$

$$\boxed{\sin\left(\frac{41\pi}{72}\right)}$$

6. Simplify and evaluate:

a. $\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{7\pi}{12}\right) + \cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{7\pi}{12}\right)$
 $\sin\left(\frac{5\pi}{12} + \frac{7\pi}{12}\right) = \sin\left(\frac{12\pi}{12}\right) = \sin(\pi) = \boxed{0}$

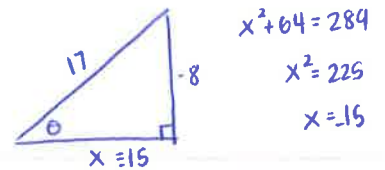
b. $\cos(105^\circ)\cos(15^\circ) + \sin(105^\circ)\sin(15^\circ)$
 $\cos(105-15) = \cos(90) = \boxed{0}$

Find the exact value of the trigonometric function given that $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{4}{5}$ when both u and v are in **quadrant III**. x and y are negative

7. $\cos(u+v) = \cos u \cos v - \sin u \sin v$

$= \left(-\frac{15}{17}\right)\left(-\frac{4}{5}\right) - \left(-\frac{8}{17}\right)\left(-\frac{3}{5}\right)$
 $\frac{60}{85} - \frac{24}{85} = \boxed{\frac{36}{85}}$

Δ for u :

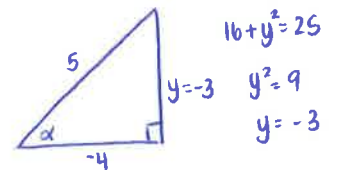


$\cos u = -\frac{15}{17}$ $\tan u = \frac{8}{15}$

8. $\sin(u-v) = \sin u \cos v - \cos u \sin v$

$= \left(-\frac{8}{17}\right)\left(-\frac{4}{5}\right) - \left(-\frac{15}{17}\right)\left(-\frac{3}{5}\right)$
 $\frac{32}{85} - \frac{45}{85} = \boxed{-\frac{13}{85}}$

Δ for v :



$\sin v = -\frac{3}{5}$ $\tan v = \frac{3}{4}$

9. $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

num: $\frac{8}{15} + \frac{3}{4} = \frac{32}{60} + \frac{45}{60} = \frac{77}{60}$

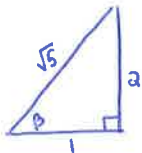
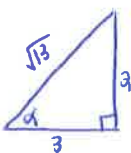
$\Rightarrow \frac{\frac{77}{60}}{\frac{36}{60}} = \frac{77}{60} \cdot \frac{60}{36} = \boxed{\frac{77}{36}}$

denom: $\frac{1}{1} - \left(\frac{8}{15}\right)\left(\frac{3}{4}\right) = \frac{1}{1} - \frac{24}{60} = \frac{60}{60} - \frac{24}{60} = \frac{36}{60}$

10. Suppose $\cot \alpha = \frac{3}{2}$ and $\sec \beta = \sqrt{5}$, where $0 < \alpha, \beta < \frac{\pi}{2}$. Find $\tan(\alpha - \beta)$. = $\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

Δ for α :

Δ for β :



$\frac{-\frac{4}{3}}{\frac{1}{3}} = \frac{-4}{3} \cdot \frac{3}{1} = \boxed{-\frac{4}{1}}$

Num = $\frac{2}{3} - \frac{2}{1} = \frac{2}{3} - \frac{6}{3} = -\frac{4}{3}$

Denom = $\frac{1}{1} + \left(\frac{2}{3}\right)\left(\frac{2}{1}\right)$
 $= \frac{1}{1} + \frac{4}{3}$
 $= \frac{3}{3} + \frac{4}{3}$
 $= \frac{7}{3}$

$2^2 + 3^2 = x^2$
 $13 = x^2$
 $x = \sqrt{13}$
 $\tan \alpha = \frac{2}{3}$

$1^2 + x^2 = (\sqrt{5})^2$
 $1 + x^2 = 5$
 $x^2 = 4$
 $x = 2$
 $\tan \beta = 2$

Answer Key:

1) -1

2) $\frac{-\sqrt{2} + \sqrt{6}}{4}$

3) $\frac{\sqrt{2} - \sqrt{6}}{4}$

4) $-2 - \sqrt{3}$

5) $\sin\left(\frac{41\pi}{72}\right)$

6) a. 0 b. 0

7) $\frac{36}{85}$

8) $-\frac{13}{85}$

9) $\frac{77}{36}$

10) $-\frac{4}{7}$