

Solve in radians. Please solve everywhere if no interval is noted.

1. $4 \sin^2 x - 3 = 0$ on $[0, 2\pi)$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \sqrt{\frac{3}{4}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

3. $2 \cos^2 x - \cos x - 1 = 0$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$2 \cos x + 1 = 0 \quad \cos x - 1 = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$x = \frac{2\pi}{3} + 2\pi n$$

$$x = \frac{4\pi}{3} + 2\pi n$$

$$x = 0 + 2\pi n$$

5. $\sin x + 1 = \cos^2 x$

$$\sin x + 1 = 1 - \sin^2 x$$

$$\sin^2 x + \sin x + 1 = 1$$

$$\sin^2 x + \sin x = 0$$

$$\sin x (\sin x + 1) = 0$$

$$\sin x = 0 \quad \sin x + 1 = 0$$

$$x = 0 + 2\pi n$$

$$x = \pi + 2\pi n$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2} + 2\pi n$$

7. $\cos^2 x + \sin x = 1$

$$(1 - \sin^2 x) + \sin x = 1$$

$$1 - \sin^2 x + \sin x - 1 = 0$$

$$-\sin^2 x + \sin x = 0$$

$$\sin x (-\sin x + 1) = 0$$

$$\sin x = 0 \quad -\sin x + 1 = 0$$

$$x = 0 + 2\pi n$$

$$x = \pi + 2\pi n$$

$$-\sin x = -1$$

$$\sin x = 1$$

$$x = \frac{\pi}{2} + 2\pi n$$

2. $\sin^2 x = 2 \sin x$

$$\sin^2 x - 2 \sin x = 0$$

$$\sin x (\sin x - 2) = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$\sin x - 2 = 0$$

$$\sin x = 2$$

↑
no solution

4. $\sqrt{3} \sec x - 2 = 0$ on $[0, 2\pi)$

$$\sec x = \frac{2}{\sqrt{3}} \leftarrow \text{take reciprocal of both sides}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, x = \frac{11\pi}{6}$$

6. $4 \cos x = 1 + 2 \cos x$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} + 2\pi n$$

$$x = \frac{5\pi}{3} + 2\pi n$$

8. $\sec^2 x - 2 \tan x = 4$

$$\tan^2 x + 1 - 2 \tan x = 4$$

$$\tan^2 x - 2 \tan x + 1 - 4 = 0$$

$$\tan^2 x - 2 \tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

$$\tan x = 3$$

↑
no solution
for now

$$\tan x = -1$$

$$x = \frac{3\pi}{4} + 2\pi n$$

$$x = \frac{7\pi}{4} + 2\pi n$$

9. $\tan x \sin^2 x = 2 \tan x$ on $[0, 2\pi)$

$$\tan x \sin^2 x - 2 \tan x = 0$$

$$\tan x (\sin^2 x - 2) = 0$$

$$\tan x = 0 \quad \sin^2 x - 2 = 0$$

$$\boxed{x = 0, \pi}$$

$$\sin^2 x = 2$$

$$\sin x = \pm \sqrt{2}$$

↑
no solution

10. $\sin x - \sqrt{2} = -\sin x$

$$2 \sin x - \sqrt{2} = 0$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{\pi}{4} + 2\pi n}$$

$$\boxed{x = \frac{3\pi}{4} + 2\pi n}$$

11. $\cos x (\cos x - 1) = 0$ on $[0, 2\pi)$

$$\cos x = 0 \quad \cos x - 1 = 0$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$

$$\cos x = 1$$

$$\boxed{x = 0}$$

12. $3 \tan^3 x = \tan x$ on $[0, 2\pi)$

$$3 \tan^3 x - \tan x = 0$$

$$\tan x (3 \tan^2 x - 1) = 0$$

$$\tan x = 0 \quad 3 \tan^2 x - 1 = 0$$

$$\boxed{x = 0, \pi}$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

Answer Key:

1) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ 2) $0 + 2\pi n, \pi + 2\pi n$ 3) $\frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, 0 + 2\pi n$ 4) $\frac{\pi}{6}, \frac{11\pi}{6}$

5) $0 + 2\pi n, \pi + 2\pi n, \frac{3\pi}{2} + 2\pi n$ 6) $\frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$ 7) $0 + 2\pi n, \pi + 2\pi n, \frac{\pi}{2} + 2\pi n$

8) $\frac{3\pi}{4} + 2\pi n, \frac{7\pi}{4} + 2\pi n$ 9) $0, \pi$ 10) $\frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n$ 11) $0, \frac{\pi}{2}, \frac{3\pi}{2}$

12) $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$