Name : __key__

Date : _____ Period :____

Please solve the following equations on the interval $[0, 2\pi]$.

1) $\sin x + 1 = \cos x$

$(\sin x + 1)^2 = (\cos x)^2$

$(\sin x + 1)(\sin x + 1) = \cos^2 x$

$\sin^2 x + 2\sin x + 1 = \cos^2 x$

$\sin^2 x + 2\sin x + 1 = 1 - \sin^2 x$

$2\sin^2 x + 2\sin x = 0$

$2\sin x(\sin x + 1) = 0$

$2\sin x = 0 \qquad \sin x + 1 = 0$

$\sin x = 0 \qquad \sin x = -1$

$x = 0, x = \pi \qquad x = \frac{3\pi}{2}$

check:

$x = 0: \sin(0) + 1 = \cos(0)$
$\qquad 0 + 1 = 1$ ✓

$x = \pi: \sin(\pi) + 1 = \cos(\pi)$
$\qquad 0 + 1 = -1$ ✗

$x = \frac{3\pi}{2}: \sin\left(\frac{3\pi}{2}\right) + 1 = \cos\left(\frac{3\pi}{2}\right)$
$\qquad -1 + 1 = 0$ ✓

Solutions: $x = 0, x = \frac{3\pi}{2}$

2) $8 = 9\cos x + 2$

$6 = 9\cos x$

$\cos x = \frac{6}{9}$

$\cos x = \frac{2}{3}$

$x = \cos^{-1}\left(\frac{2}{3}\right)$

$x = 48.19°$

cosine is also + in Q4 :
$x = 360 - 48.19$
$x = 311.81°$

* interval is in radians so convert degrees :

$x = 48.19° \cdot \frac{\pi}{180} = \boxed{0.84}$

$x = 311.81° \cdot \frac{\pi}{180} = \boxed{5.44}$

3) $4\cos^2 x + 3\cos x + 1 = 0$

Check discriminant: $b^2 - 4ac$

$= (3)^2 - 4(4)(1)$

$= 9 - 16$

$= -7$

Since discriminant is negative,

$\boxed{\text{no real solutions}}$

4) $8\tan^2 x - 3\sec x = -3$ ← use Pythag identity

$8(\sec^2 x - 1) - 3\sec x = -3$

$8\sec^2 x - 8 - 3\sec x + 3 = 0$

$8\sec^2 x - 3\sec x - 5 = 0$

$8\sec^2 x - 8\sec x + 5\sec x - 5 = 0$

$8\sec x(\sec x - 1) + 5(\sec x - 1) = 0$

$(8\sec x + 5)(\sec x - 1) = 0$

$8\sec x + 5 = 0 \qquad \sec x - 1 = 0$

$\sec x = -\frac{5}{8} \qquad \sec x = 1$

$\cos x = -\frac{8}{5} \qquad \cos x = 1$

↑ $\qquad\qquad \boxed{x = 0}$

no solution;

$-\frac{8}{5}$ is outside the domain restrictions

Please solve the following equations on the interval $[0°, 360°)$.

5) $2\tan^2 x = 3\tan x - 1$

$2\tan^2 x - 3\tan x + 1 = 0$

$(2\tan x - 1)(\tan x - 1) = 0$

$2\tan x - 1 = 0 \qquad \tan x - 1 = 0$

$\tan x = \frac{1}{2} \qquad \tan x = 1$

$x = \tan^{-1}\left(\frac{1}{2}\right) \qquad \boxed{x = 45°, 225°}$

$\boxed{x = 26.56°}$

tan is pos in Q3:

$x = 180 + 26.56$

$\boxed{x = 206.56°}$

6) $\sin x = -\dfrac{7}{16}$

$x = \sin^{-1}\left(-\frac{7}{16}\right)$

$x = -25.94°$ ← needs to be + so find coterminal:

$x = -25.94 + 360 = \boxed{334.06°}$

sine is also neg. in Q3 so Ref angle:

$x = 180 + 25.94$

$\boxed{x = 205.94°}$

7) $12\sin^2 x - 13\sin x + 3 = 0$

$(4\sin x - 3)(3\sin x - 1) = 0$

$4\sin x - 3 = 0 \qquad 3\sin x - 1 = 0$

$\sin x = \frac{3}{4} \qquad \sin x = \frac{1}{3}$

$x = \sin^{-1}\left(\frac{3}{4}\right) \qquad x = \sin^{-1}\left(\frac{1}{3}\right)$

$\boxed{x = 48.59°} \qquad \boxed{x = 19.47°}$

Sine is also positive in Q2:

$x = 180 - 48.59 \qquad x = 180 - 19.47$

$\boxed{x = 131.41°} \qquad \boxed{x = 160.53°}$

Answer Key:

1) $0, \dfrac{3\pi}{2}$ 2) 0.84, 5.44   3) No Solutions   4) 0

5) 45°, 225°, 26.57°, 206.57°   6) 205.94°, 334.06°   7) 19.47°, 48.59°, 131.41°, 160.53°