Pre-Calculus A
Section 5.3 Quiz Review

Name : Key
Date : _____ Period: _____

Solve the following equations. Solve everywhere if no interval is noted.

1) $7\csc x - 1 = 9$ on $[0°, 360°)$

$7\csc x = 10$

$\csc x = \frac{10}{7}$ ← take reciprocal of both sides

$\sin x = \frac{7}{10}$

$x = \sin^{-1}\left(\frac{7}{10}\right)$

$x = 44.4°$

Sine is also positive in Quad II:

$x = 180 - 44.4$

$x = 135.6°$

2) $\tan\theta = -0.23$ on $[0°, 360°)$

$\theta = \tan^{-1}(-0.23)$

$\theta = -12.95°$

↑ angle must be positive so find coterminal

$\theta = -12.95 + 360$

$\theta = 347.05°$

Tan is also negative in quad II:

$\theta = 180 - 12.95$

$\theta = 167.05°$

3) $\sin x + 1 = \cos x$ on $[0, 2\pi)$

$(\sin x + 1)^2 = (\cos x)^2$

$(\sin x + 1)(\sin x + 1) = \cos^2 x$

$\sin^2 x + 2\sin x + 1 = \cos^2 x$

$\sin^2 x + 2\sin x + 1 = 1 - \sin^2 x$

$2\sin^2 x + 2\sin x = 0$

$2\sin x(\sin x + 1) = 0$

$2\sin x = 0$ $\quad$ $\sin x + 1 = 0$

$\sin x = 0$ $\quad$ $\sin x = -1$

$x = 0, \pi$ $\quad$ $x = \frac{3\pi}{2}$

↑ possible solutions ↑

Check:

$x = 0$: $\sin(0) + 1 = \cos(0)$

$0 + 1 = 1$ ✓

$x = \pi$: $\sin(\pi) + 1 = \cos(\pi)$

$0 + 1 = -1$ ✗

$x = \frac{3\pi}{2}$: $\sin\left(\frac{3\pi}{2}\right) + 1 = \cos\left(\frac{3\pi}{2}\right)$

$-1 + 1 = 0$ ✓

So solutions are:

$x = 0, \ x = \frac{3\pi}{2}$

4) $\cos^2 x = 1 - \sin x$ NO INTERVAL

↑ Rewrite as sin using Pythag identity

$1 - \sin^2 x = 1 - \sin x$

$-\sin^2 x + \sin x = 0$

$\sin x(-\sin x + 1) = 0$

$\sin x = 0$ $\quad$ $-\sin x + 1 = 0$

$-\sin x = -1$

$\sin x = 1$

$x = 0 + 2\pi n$

$x = \pi + 2\pi n$

$x = \frac{\pi}{2} + 2\pi n$

5) $4\sin^2 x - 3 = 0$ on $[0, 2\pi)$

$4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

6) $2\cos^2 x - \cos x - 1 = 0$ NO INTERVAL

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ $\quad$ $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = 1$

$x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$

$x = 0 + 2\pi n$

7) $6\sin^2 x - 7\sin x + 2 = 0$ on $[0°, 360°)$

$(3\sin x - 2)(2\sin x - 1) = 0$

$3\sin x - 2 = 0$ $\quad$ $2\sin x - 1 = 0$

$\sin x = \frac{2}{3}$ $\quad$ $\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{2}{3}\right)$ $\quad$ $x = 30°, 150°$

$x = 41.8°$

Sin is positive in Quad 2:

$x = 180 - 41.8$

$x = 138.2°$

8) $9\sin^2 x + 6\sin x = 2$ on $[0°, 360°)$

$b^2 - 4ac = (6)^2 - 4(9)(-2) = 36 + 72 = 108$ ← not factorable use Quad. form.

$9\sin^2 x + 6\sin x - 2 = 0$

$\frac{-6 \pm \sqrt{108}}{18} \begin{matrix} \nearrow 0.2440 \\ \searrow -0.9107 \end{matrix}$

$\sin x = 0.2440$ $\quad$ $\sin x = -0.9107$

$x = \sin^{-1}(.2440)$ $\quad$ $x = \sin^{-1}(-.9107)$

$x = 14.1°$ $\quad$ $x = -65.6°$ ← Find pos. coterminal

Sin pos. in Quad 2: $\quad$ $x = -65.6 + 360$

$x = 180 - 14.1$ $\quad$ $x = 294.4°$

$x = 165.9°$ $\quad$ Sin neg in Q3:

$x = 180 + 65.6$

$x = 245.6°$

9) $\tan^2 x - 2\tan x - 5 = 0$ on $[0°, 360°)$

$b^2 - 4ac = (-2)^2 - 4(1)(-5) = 4 + 20 = 24$

$\frac{2 \pm \sqrt{24}}{2} \begin{matrix} \nearrow 3.4495 \\ \searrow -1.4495 \end{matrix}$

$\tan x = 3.4495$ $\quad$ $\tan x = -1.4495$

$x = \tan^{-1}(3.4495)$ $\quad$ $x = \tan^{-1}(-1.4495)$

$x = 73.8°$ $\quad$ $x = -55.4°$ ← Find pos. coterminal

Tan is pos in Quad 3: $\quad$ $x = -55.4 + 360$

$x = 180 + 73.8$ $\quad$ $x = 304.6°$

$x = 253.8°$ $\quad$ Tan is neg in Quad 2:

$x = 180 - 55.4$

$x = 124.6°$

10) $2\cos^2 \theta - 4\cos\theta + 1 = 0$ on $[0°, 360°)$

$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$

$\frac{4 \pm \sqrt{8}}{4} \begin{matrix} \nearrow 1.7071 \\ \searrow 0.2929 \end{matrix}$

$\cos x = 1.7071$ $\quad$ $\cos x = 0.2929$

$x = \cos^{-1}(1.7071)$ $\quad$ $x = \cos^{-1}(0.2929)$

↑ $\quad$ $x = 72.9°$

no solution; outside domain of cosine $\quad$ Cosine is pos in Quad 4:

$x = 360 - 72.9$

$x = 287.1°$

11) $\cos x \sin x = -\sin x$ NO INTERVAL

get all terms to one side

$\cos x \sin x + \sin x = 0$ ← GCF of $\sin x$

$\sin x (\cos x + 1) = 0$

$\sin x = 0$ $\quad$ $\cos x + 1 = 0$

$\cos x = -1$

$x = 0 + 2\pi n$

$x = \pi + 2\pi n$ ← $x = \pi + 2\pi n$ already a solution from 1st equation; no need to list it twice