Solve each equation. If there is no interval specified, please find all solutions.

1) $2\cos(2x) = \sqrt{2}$ on $[0, 2\pi)$

$\cos(2x) = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4}$ $2x = \frac{7\pi}{4}$

$\boxed{x = \frac{\pi}{8}}$ $\boxed{x = \frac{7\pi}{8}}$

2) $\cancel{4} + \sin 3\theta = 3$
   $_{-4} _{-4}$

$\sin 3\theta = -1$

$3\theta = \frac{3\pi}{2} + 2\pi n$

$\boxed{\theta = \frac{\pi}{2} + \frac{2\pi n}{3}}$

3) $\cancel{1} + 3\tan^2 3x = 10$ on $[0, 2\pi)$
   $_{-1} _{-1}$

$\frac{3\tan^2 3x}{3} = \frac{9}{3}$

$\sqrt{\tan^2 3x} = \sqrt{3}$

$\tan 3x = \pm\sqrt{3}$

$3x = \frac{\pi}{3}$ $3x = \frac{2\pi}{3}$ $3x = \frac{4\pi}{3}$ $3x = \frac{5\pi}{3}$

$\boxed{x = \frac{\pi}{9}}$ $\boxed{x = \frac{2\pi}{9}}$ $\boxed{x = \frac{4\pi}{9}}$ $\boxed{x = \frac{5\pi}{9}}$

4) $\frac{1}{5}\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{10}$

$\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{10} \cdot \frac{5}{1}$

$\cos\left(\frac{x}{2}\right) = \frac{5\sqrt{2}}{10}$

$\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2}$

$\frac{x}{2} = \frac{\pi}{4} + 2\pi n$ $\frac{x}{2} = \frac{7\pi}{4} + 2\pi n$

$x = \frac{2\pi}{4} + 4\pi n$ $x = \frac{14\pi}{4} + 4\pi n$

$\boxed{x = \frac{\pi}{2} + 4\pi n}$ $\boxed{x = \frac{7\pi}{2} + 4\pi n}$

For questions #5 – 8, simplify and evaluate.

5) $\sin(-15°) = \sin(\overset{u}{30°} - \overset{v}{45°}) = \sin(30)\cos(45) - \cos(30)\sin(45)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$ OR $\boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

6) $\cos\left(\frac{7\pi}{12}\right) = \cos(105°) = \cos(60°+45°) = \cos(60)\cos(45) - \sin(60)\sin(45)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$ OR $\boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

7) $\cos 250° \cos 40° + \sin 250° \sin 40°$ (u over 250, v over 40)

$$\cos(u-v) = \cos(250-40)$$
$$= \cos(210°)$$
$$= \boxed{-\frac{\sqrt{3}}{2}}$$

8) $\sin\left(\dfrac{\pi}{6}\right)\cos\left(\dfrac{5\pi}{6}\right) + \sin\left(\dfrac{5\pi}{6}\right)\cos\left(\dfrac{\pi}{6}\right)$ ← same as $\sin\left(\dfrac{\pi}{6}\right)\cos\left(\dfrac{5\pi}{6}\right) + \cos\left(\dfrac{\pi}{6}\right)\sin\left(\dfrac{5\pi}{6}\right)$

$$\sin(u+v) = \sin\left(\dfrac{\pi}{6} + \dfrac{5\pi}{6}\right)$$
$$= \sin\left(\dfrac{6\pi}{6}\right)$$
$$= \sin(\pi) = \boxed{0}$$

9) Simplify $\tan(x - 2\pi)$. Do not evaluate. (u over x, v over 2π)

$$\frac{\tan u - \tan v}{1 + \tan u \tan v} = \frac{\tan x - \tan 2\pi}{1 + \tan x \tan 2\pi} = \frac{\tan x - 0}{1 + \tan x (0)} = \frac{\tan x}{1 + 0} = \frac{\tan x}{1} = \boxed{\tan x}$$

at 2π: $(1, 0)$

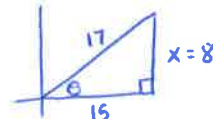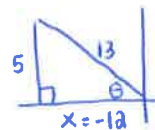$\tan = \dfrac{\sin}{\cos} = \dfrac{0}{1} = 0$

10) If $\sin A = \dfrac{5}{13}$, $\cos B = \dfrac{15}{17}$, and $0 < B < \dfrac{\pi}{2} < A < \pi$, please evaluate:

B in Quad 1   A in Quad 2

a. $\sin(A+B)$

$= \sin A \cos B + \cos A \sin B$

$= \left(\dfrac{5}{13}\right)\left(\dfrac{15}{17}\right) + \left(-\dfrac{12}{13}\right)\left(\dfrac{8}{17}\right)$

$= \dfrac{75}{221} - \dfrac{96}{221} = \boxed{\dfrac{-21}{221}}$

b. $\tan(A-B)$

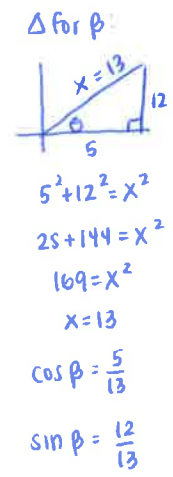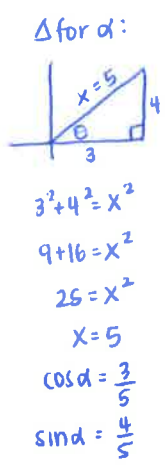$= \dfrac{\tan A - \tan B}{1 + \tan A \tan B} = \dfrac{-\dfrac{5}{12} - \dfrac{8}{15}}{\dfrac{1}{1} + \left(-\dfrac{5}{12}\right)\left(\dfrac{8}{15}\right)} = \dfrac{-\dfrac{75}{180} - \dfrac{96}{180}}{\dfrac{1}{1} - \dfrac{40}{180}}$

$\dfrac{-\dfrac{171}{180}}{\dfrac{180}{180} - \dfrac{40}{180}} = \dfrac{-\dfrac{171}{180}}{\dfrac{140}{180}} = -\dfrac{171}{180} \cdot \dfrac{180}{140} = \boxed{-\dfrac{171}{140}}$

Δ for A:

$x = -12$

$x^2 + 5^2 = 13^2$
$x^2 + 25 = 169$
$x^2 = 144$
$x = 12$
$x = -12$ in QII

$\sin A = \dfrac{5}{13}$

$\cos A = -\dfrac{12}{13}$

$\tan A = -\dfrac{5}{12}$

Δ for B:

$x = 8$

$x^2 + 15^2 = 17^2$
$x^2 + 225 = 289$
$x^2 = 64$
$x = 8$

$\sin B = \dfrac{8}{17}$

$\cos B = \dfrac{15}{17}$

$\tan B = \dfrac{8}{15}$

11) If $\tan\alpha = \dfrac{4}{3}$, $\tan\beta = \dfrac{12}{5}$, and $0 < \alpha < \beta < \dfrac{\pi}{2}$, please find $\cos(\alpha - \beta)$.

$\cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\dfrac{3}{5}\right)\left(\dfrac{5}{13}\right) + \left(\dfrac{4}{5}\right)\left(\dfrac{12}{13}\right)$

$= \dfrac{15}{65} + \dfrac{48}{65}$

$= \boxed{\dfrac{63}{65}}$

Δ for α:

$x = 5$, 4, 3

$3^2 + 4^2 = x^2$
$9 + 16 = x^2$
$25 = x^2$
$x = 5$
$\cos\alpha = \dfrac{3}{5}$
$\sin\alpha = \dfrac{4}{5}$

Δ for β:

$x = 13$, 12, 5

$5^2 + 12^2 = x^2$
$25 + 144 = x^2$
$169 = x^2$
$x = 13$
$\cos\beta = \dfrac{5}{13}$
$\sin\beta = \dfrac{12}{13}$

12) If $\tan\alpha = 2$ and $\tan\beta = \dfrac{1}{4}$, please evaluate $\tan(\alpha - \beta)$.

$\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \dfrac{2 - \frac{1}{4}}{1 + (2)\left(\frac{1}{4}\right)} = \dfrac{\frac{2}{1} - \frac{1}{4}}{\frac{1}{1} + \frac{2}{4}} = \dfrac{\frac{8}{4} - \frac{1}{4}}{\frac{4}{4} + \frac{2}{4}} = \dfrac{\frac{7}{4}}{\frac{6}{4}} = \dfrac{7}{4} \cdot \dfrac{4}{6} = \boxed{\dfrac{7}{6}}$

For questions #~~10-12~~ 13-15, solve the trigonometric equations on the interval $[0, 2\pi)$.

13) $\cos\left(\overset{u}{x} - \overset{v}{\dfrac{\pi}{6}}\right) = 1 + \cos\left(\overset{u}{x} + \overset{v}{\dfrac{\pi}{6}}\right)$

$\cos x \cos\left(\frac{\pi}{6}\right) + \sin x \sin\left(\frac{\pi}{6}\right) = 1 + \cos x \cos\left(\frac{\pi}{6}\right) - \sin x \sin\left(\frac{\pi}{6}\right)$

$- \cos x \cos\left(\frac{\pi}{6}\right)$ $- \cos x \cos\left(\frac{\pi}{6}\right)$

_____

$\sin x \sin\left(\frac{\pi}{6}\right) = 1 - \sin x \sin\left(\frac{\pi}{6}\right)$
$+ \sin x \sin\left(\frac{\pi}{6}\right)$ $+ \sin x \sin\left(\frac{\pi}{6}\right)$

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$2\sin x \sin\left(\frac{\pi}{6}\right) = 1$

$\cancel{2} \cdot \dfrac{1}{\cancel{2}}\sin x = 1$

$\sin x = 1$

$\boxed{x = \dfrac{\pi}{2}}$

14) $\sin\left(\overset{u}{x} + \overset{v}{\dfrac{3\pi}{4}}\right) + \sin\left(\overset{u}{x} - \overset{v}{\dfrac{3\pi}{4}}\right) = 1$

$\sin x \cos\left(\frac{3\pi}{4}\right) + \cos x \sin\left(\frac{3\pi}{4}\right) + \sin x \cos\left(\frac{3\pi}{4}\right) - \cos x \sin\left(\frac{3\pi}{4}\right) = 1$

$2\sin x \cos\left(\frac{3\pi}{4}\right) = 1$

$\dfrac{2}{1}\left(-\dfrac{\sqrt{2}}{2}\right)\sin x = 1$

$-\sqrt{2}\sin x = 1$

$\sin x = \dfrac{1}{-\sqrt{2}}$

$\sin x = -\dfrac{\sqrt{2}}{2}$

$\boxed{x = \dfrac{5\pi}{4}, \dfrac{7\pi}{4}}$

15) $-\cos x = 1 + 2\cos(x - \pi)$

$-\cos x = 1 + 2(\cos x \cos \pi + \sin x \sin \pi)$

$-\cos x = 1 + 2(\cos x (-1) + \sin x (0))$

$-\cos x = 1 + 2(-\cos x + 0)$

$-\cos x = 1 + 2(-\cos x)$

$-\cos x = 1 - 2\cos x$

$+2\cos x \qquad +2\cos x$

$\cos x = 1 \longrightarrow \boxed{x = 0}$

16) Simplify $\dfrac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{\cos \alpha \cos \beta}$

$= \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \dfrac{2\sin\alpha\cos\beta}{\cos\alpha\cos\beta}$

$= 2\left(\dfrac{\sin\alpha}{\cos\alpha}\right)$

$= 2(\tan\alpha)$

$= \boxed{2\tan\alpha}$

17) Simplify $(\sec x \sec y)(\sin(x + y))$

$\left(\dfrac{1}{\cos x} \cdot \dfrac{1}{\cos y}\right)\left(\dfrac{\sin x\cos y + \cos x\sin y}{1}\right) = \dfrac{\sin x\cos y + \cos x\sin y}{\cos x\cos y} = \dfrac{\sin x\cos y}{\cos x\cos y} + \dfrac{\cos x\sin y}{\cos x\cos y}$

$= \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}$

$= \boxed{\tan x + \tan y}$

Answer Key:

1) $\dfrac{\pi}{8} + \pi n, \dfrac{7\pi}{8} + \pi n$ ✓

2) $\dfrac{\pi}{2} + \dfrac{2\pi n}{3}$ ✓

3) $\dfrac{\pi}{9}, \dfrac{2\pi}{9}, \dfrac{4\pi}{9}, \dfrac{5\pi}{9}$ ✓

4) $\dfrac{\pi}{2} + 4\pi n, \dfrac{7\pi}{2} + 4\pi n$ ✓

5) $\dfrac{\sqrt{2} - \sqrt{6}}{4}$ ✓

6) $\dfrac{\sqrt{2} - \sqrt{6}}{4}$ ✓

7) $-\dfrac{\sqrt{3}}{2}$ ✓

8) 0 ✓

9) $\tan x$ ✓

10) a. $-\dfrac{21}{221}$ ✓ b. $-\dfrac{171}{140}$ ✓

11) $\dfrac{63}{65}$ ✓

12) $\dfrac{7}{6}$ ✓

13) $\dfrac{\pi}{2}$ ✓

14) $\dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ ✓

15) 0 ✓

16) $2\tan\alpha$ ✓

17) $\tan x + \tan y$ ✓