Pre-Calculus A
Section 5.1 Mixed Practice

Name : Key
Date : _____ Period : ___

Verify the following trigonometric expressions.

1. $(\sec x - \tan x)(\csc x + 1) = \cot x$ (FOIL)

LHS: $\sec x\csc x + \sec x - \tan x\csc x - \tan x$

$\Rightarrow \frac{1}{\cos x}\cdot\frac{1}{\sin x} + \frac{1}{\cos x} - \frac{\sin x}{\cos x}\cdot\frac{1}{\sin x} - \frac{\sin x}{\cos x}$

$\Rightarrow \frac{1}{\cos x\sin x} + \frac{1}{\cos x} - \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$\Rightarrow \frac{1}{\cos x\sin x} - \frac{\sin x}{\cos x}$ ← LCD: $\cos x\sin x$

$\Rightarrow \frac{1}{\cos x\sin x} - \frac{\sin^2 x}{\cos x\sin x}$

$\Rightarrow \frac{1-\sin^2 x}{\cos x\sin x}$

$\Rightarrow \frac{\cos^2 x}{\cos x\sin x}$

$\Rightarrow \frac{\cos x}{\sin x}$

$\Rightarrow \cot x$

$\cot x = \cot x$

LHS = RHS

2. Verify $(\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$

LHS: $\csc^2\theta + \csc\theta\cot\theta - \cot\theta\csc\theta - \cot^2\theta$

$\Rightarrow \csc^2\theta - \cot^2\theta$ ← Pythag Identity

$\Rightarrow 1$

$1 = 1$

LHS = RHS

3. $\tan x\csc^2 x - \tan x = \cot x$ (GCF = $\tan x$)

LHS: $\tan x(\csc^2 x - 1)$

$\Rightarrow \tan x(\cot^2 x)$

$\Rightarrow \frac{\sin x}{\cos x}\cdot\frac{\cos^2 x}{\sin^2 x}$

$\Rightarrow \frac{\cos x}{\sin x}$

$\Rightarrow \cot x$

$\cot x = \cot x$

LHS = RHS

4. $\sin\theta - \sin\theta\cos^2\theta = \sin^3\theta$ (GCF: $\sin\theta$)

LHS: $\sin\theta(1 - \cos^2\theta)$

$\Rightarrow \sin\theta(\sin^2\theta)$

$\Rightarrow \sin^3\theta$

$\sin^3\theta = \sin^3\theta$

LHS = RHS

5. $\frac{1+\csc\theta}{\sec\theta} - \cot\theta = \cos\theta$ (LCD: $\sin\theta$)

LHS: $\frac{1 + \frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \frac{\frac{\sin\theta}{\sin\theta} + \frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \frac{\frac{\sin\theta+1}{\sin\theta}}{\frac{1}{\cos\theta}} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \frac{\sin\theta+1}{\sin\theta}\cdot\frac{\cos\theta}{1} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \frac{\cos\theta(\sin\theta+1) - \cos\theta}{\sin\theta}$

$\Rightarrow \frac{\cos\theta[(\sin\theta+1) - 1]}{\sin\theta}$ (the 1's cancel)

$\Rightarrow \frac{\cos\theta(\sin\theta)}{\sin\theta}$

$\Rightarrow \cos\theta$

$\cos\theta = \cos\theta$

LHS = RHS

6. $\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta} = 1$

LHS: $\frac{\frac{\cos\theta}{1}}{\frac{1}{\cos\theta}} + \frac{\frac{\sin\theta}{1}}{\frac{1}{\sin\theta}}$

$\Rightarrow \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{1} + \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{1}$

$\Rightarrow \cos^2\theta + \sin^2\theta$

$\Rightarrow 1$

$1 = 1$

LHS = RHS

7. $\frac{1+\sec\theta}{\tan\theta+\sin\theta} = \csc\theta$

LCD: $\cos\theta$

LHS: $\frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta}{1}}$ ← LCD: $\cos\theta$

$\Rightarrow \frac{\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta\cos\theta}{\cos\theta}}$

$\Rightarrow \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}$

$\Rightarrow \frac{\cos\theta+1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta+\sin\theta\cos\theta}$

$\Rightarrow \frac{\cos\theta+1}{\sin\theta+\sin\theta\cos\theta}$

↑ GCF: $\sin\theta$

$\Rightarrow \frac{\cos\theta+1}{\sin\theta(1+\cos\theta)}$

$\Rightarrow \frac{1+\cos\theta}{\sin\theta(1+\cos\theta)}$

$\Rightarrow \frac{1}{\sin\theta}$

$\Rightarrow \csc\theta$

$\csc\theta = \csc\theta$

LHS = RHS

8. $\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta} = 2\sec^2\theta$

LCD: $(1-\sin\theta)(1+\sin\theta)$

LHS: $\frac{(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}1 + \frac{1}{(1+\sin\theta)(1-\sin\theta)}(1-\sin\theta)$

$\Rightarrow \frac{1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} + \frac{1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$\Rightarrow \frac{1+\sin\theta+1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$\Rightarrow \frac{2}{1-\sin\theta+\sin\theta-\sin^2\theta}$

$\Rightarrow \frac{2}{1-\sin^2\theta}$

$\Rightarrow \frac{2}{\cos^2\theta}$

$\Rightarrow 2\sec^2\theta$

$2\sec^2\theta = 2\sec^2\theta$

LHS = RHS

LCD: $(\sin\theta)(\cos\theta)$

9. $\frac{\sin\theta+\cos\theta}{\cos\theta} - \frac{\sin\theta-\cos\theta}{\sin\theta} = \sec\theta\csc\theta$

LHS: $\frac{(\sin\theta+\cos\theta)\sin\theta}{\cos\theta\cdot\sin\theta} - \frac{(\sin\theta-\cos\theta)\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$

$\Rightarrow \frac{\sin^2\theta+\cos\theta\sin\theta}{\cos\theta\sin\theta} - \frac{\cos\theta\sin\theta-\cos^2\theta}{\cos\theta\sin\theta}$

$\Rightarrow \frac{\sin^2\theta+\cos\theta\sin\theta-(\cos\theta\sin\theta-\cos^2\theta)}{\cos\theta\sin\theta}$

$\Rightarrow \frac{\sin^2\theta+\cos\theta\sin\theta-\cos\theta\sin\theta+\cos^2\theta}{\cos\theta\sin\theta}$

$\Rightarrow \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$

$\Rightarrow \frac{1}{\cos\theta\sin\theta}$

$\Rightarrow \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$\Rightarrow \sec\theta\csc\theta$

$\sec\theta\csc\theta = \sec\theta\csc\theta$

LHS = RHS