Pre-calc Midterm Review : Chapter 1

① $j(0) = 3$

$j(1) = (1)^2 = 1$

② $f(4) = 3(4)+5$

$\boxed{f(4) = 17}$

③ $g(-3) = (-3)^2 - 6$

$= 9-6$

$\boxed{= 3}$

④ $fog = f(g(x)) = f(x^2-6)$

$= 3(x^2-6)+5$

$= 3x^2-18+5$

$\boxed{= 3x^2-13}$

⑤ $y = 3x+5$ → to find inverse, switch x's and y's and re-solve for y

$x = 3y+5$

$x-5 = 3y$

$y = \frac{x-5}{3} \Rightarrow f^{-1}(x) = \frac{x-5}{3} \Rightarrow$. $\boxed{(a)}$

⑥ $(f+g)(x) = 3x+5+x^2-6$

$= \boxed{x^2+3x-1}$

⑦ $(f-g)(x) = 3x+5-(x^2-6)$

$= 3x+5-x^2+6$

(9) $\left(\frac{f}{g}\right)(x) = \boxed{\frac{3x+5}{x^2-6}}$

(10) $(fog)(9) = f(g(9))$

$g(9) = -2(9)^2 - 8(9) + 3$

$= -2(81) - 72 + 3$

$= -162 - 72 + 3$

$= -231$

$\Rightarrow$

$f(g(9)) = f(-231)$

$= -2(-231) - 2$

$= 462 - 2$

$= 460 \Rightarrow \boxed{(c)}$

(11) $(gof)(4) = g(f(4))$

$f(4) = -4(4) - 9$

$= -16 - 9$

$= -25$

$\Rightarrow$

$g(-25) = 2(25)^2 - 3(25) - 6$

$= 2(625) + 75 - 6$

$= 1250 + 75 - 6$

$= \boxed{1319}$

(12) $f(g(x)) = f(3x^2+1)$

$= 2(3x^2+1)$

$= \boxed{6x^2+2}$

(13) $g(f(x)) = g(3x+2)$

$= 2(3x+2)^2 - 1$

$= 2(3x+2)(3x+2) - 1$

$= 2(9x^2+6x+6x+4) - 1$

$= 2(9x^2+12x+4) - 1$

$= 18x^2+24x+8-1$

$= \boxed{18x^2+24x+7}$

(14) $f(x-1) = 3(x-1)^2 + 2(x-1) - 7$

$= 3(x-1)(x-1) + 2x - 2 - 7$

$= 3(x^2-2x+1) + 2x - 9$

$= 3x^2 - 6x + 3 + 2x - 9$

$= \boxed{3x^2-4x-6}$

(15) $(f \circ g) = f(g(x)) = f\left(\frac{1}{x+2}\right)$

$= \frac{1}{x+2} + \frac{1}{1}\frac{(x+2)}{(x+2)}$

$= \frac{1+1(x+2)}{x+2}$

$= \frac{1+1x+14}{x+2}$

$= \frac{1x+21}{x+2}$ ← for domain, the denominator cannot equal zero

domain: $x+2 \neq 0 \Rightarrow (-\infty,-2) \cup (2,\infty) \Rightarrow$ **(b)**

$x \neq -2$

(16) domain: $x^2-9 \neq 0$

$(x+3)(x-3) \neq 0$

$x+3 \neq 0 \quad x-3 \neq 0 \Rightarrow (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

$x \neq -3, \quad x \neq 3$

(17) domain: $4-x \neq 0 \Rightarrow (-\infty,4) \cup (4,\infty)$

$4 \neq x$

(18) domain: $3x+2 \geq 0$ ← under the square root cant be negative, but could still be zero

$3x \geq -2 \Rightarrow [-2/3, \infty)$

$x \geq -2/3$

(19) domain: $x^2+2x-3 \neq 0$

$(x+3)(x-1) \neq 0$

$x+3 \neq 0 \quad x-1 \neq 0 \Rightarrow (-\infty,-3) \cup (-3,1) \cup (1,\infty)$

$x \neq -3 \quad x \neq 1$

(20) $g(x) = \frac{x^2-5x-6}{x^2-3x-4} \Rightarrow \frac{(x-6)(x+1)}{(x-4)(x+1)} = \frac{x-6}{x-4}$ ← holes are not included in domain ← denom. cant be zero

domain: $x+1 \neq 0, \quad x-4 \neq 0 \Rightarrow$ **(d)**

$x \neq -1 \quad x \neq 4$

(21) not one to one; doesnt pass the horizontal line test

(22) one to one; passes the horizontal line test

(23) $y = \frac{5}{x-7} \Rightarrow \frac{x}{1} = \frac{5}{y-7} \Rightarrow 5 = x(y-7)$

$5 = xy - 7x$

$5 + 7x = xy$

$y = \frac{5+7x}{x} \Rightarrow$ **(a)**

(24) $y = x^3 + 3 \Rightarrow x = y^3 + 3 \Rightarrow x - 3 = y^3$

$y = \sqrt[3]{x-3} \Rightarrow$ **(b)**

(25) $y = 4x + a \Rightarrow x = 4y + a \Rightarrow x - a = 4y$

$y = \frac{x-a}{4} \Rightarrow$ **$f^{-1}(x) = \frac{x-a}{4}$**

(26) $y = (x-4)^3 + 1 \Rightarrow x = (y-4)^3 + 1$

$x - 1 = (y-4)^3$

$\sqrt[3]{x-1} = y - 4$

$y = \sqrt[3]{x-1} + 4 \Rightarrow$ **$f^{-1}(x) = \sqrt[3]{x-1} + 4$**

(27) $f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2}$ ← same as original so **even**

(28) $f(-x) = \frac{-x}{(-x)^2+4} \Rightarrow \frac{-x}{x^2+4} \Rightarrow -\frac{x}{x^2+4}$ ← exactly opposite from original so **odd**

(29) $f(-x) = 3(-x)^3 - 5 = -3x^3 - 5$ ← not the same or opposite so **neither**

(30) $f(-x) = \sqrt{(-x)^2+1} = \sqrt{x^2+1}$ ← same as original so **even**