Pre-Calculus (A) Name : _____
Midterm Review 2016-2017 Date : _____ Period : _____

Please do all work on a separate sheet of paper!

Chapter 1

1) $j(t)=\begin{cases}2x+9 & \text{if } x<0\\ 3 & \text{if } 0\le x<1\\ x^2 & \text{if } x\ge 1\end{cases}$ $j(0)=$ _____

$j(1)=$ _____

For #2 – 9, let f(x) = 3x + 5 **and** g(x) = x^2-6**.**

2) Find f(4).

a.) 12 b.) 17 c.) 10 d.) 60

3) Find g(-3).

a.) -15 b.) 15 c.) 0 d.) 3

4) Find $f \circ g$.

a) $3x^2-13$ b) $9x^2+30x+19$ c) $3x^2-18$ d) $3x-1$

5) Find f^{-1}.

a) $\frac{x-5}{3}$ b) $3x-5$ c) $\frac{x+5}{3}$ d) $\frac{1}{3x+5}$

6) Find (f + g)(x)

7) Find (f – g)(x)

8) Find (fg)(x)

9) Find $\left(\frac{f}{g}\right)$(x)

10) Find $(f \circ g)(9)$ when $f(x)=-2x-2$ and $g(x)=-2x^2-8x+3$

a) -637 b) 172 c) 460 d) 203

11) Find $(g \circ f)(4)$ when $f(x)=-4x-9$ and $g(x)=2x^2-3x-6$

12) Find $f(g(x))$ when $f(x)=2x$ and $g(x)=3x^2+1$

13) Find $g(f(x))$ when $f(x)=3x+2$ and $g(x)=2x^2-1$

14) Find f(x – 1) when $f(x)=3x^2+2x-7$.

a) $3x^2-4x-6$ b) $-4x^2+3x-6$ c) $3x^2-19x-2$ d) $3x^2-4x-2$

15) Find the **domain** of the composite function $(f \circ g)$ when $f(x)=x+7, g(x)=\frac{7}{x+2}$.

a) $(-\infty,-9)\cup(-9,\infty)$ b) $(-\infty,-2)\cup(-2,\infty)$

c) $(-\infty,-2)\cup(-2,-7)\cup(-7,\infty)$ d) $(-\infty,\infty)$

Find the domain of each function using <u>interval notation</u>.

16) $f(x)=\frac{x}{x^2-9}$

17) $f(x)=\frac{15}{4-x}$

18) $f(x)=\sqrt{3x+2}$

19) $f(x)=\frac{x}{x^2+2x-3}$

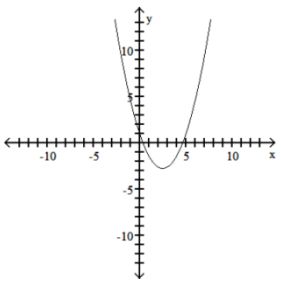
20) What are the domain restrictions for g(x) = $\frac{x^2-5x-6}{x^2-3x-4}$?

a) x ≠ -6, 1 b) x ≠ 6, -1

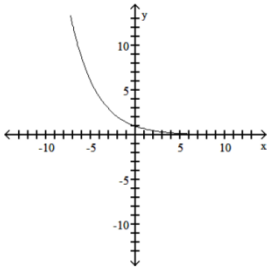
c) x ≠ -4, 1 d) x ≠ 4, -1

For #21 – 22, please use the graphs to determine whether the function is one-to-one.

21)



22)



For #23 – 24, find the inverse of the function.

23) $f(x)=\dfrac{5}{x-7}$

a) $f^{-1}(x)=\dfrac{7x+5}{x}$ b) $f^{-1}(x)=\dfrac{-7+5x}{x}$

c) $f^{-1}(x)=\dfrac{x}{-7+5x}$ d) Not invertible

24) $f(x)=x^3+3$

a) $f^{-1}(x)=\sqrt[3]{x}-3$ b) $f^{-1}(x)=\sqrt[3]{x-3}$

c) $f^{-1}(x)=\sqrt[3]{x+3}$ d) Not invertible

Determine the inverse of the following functions.

25) $f(x)=4x+2$
26) $f(x)=(x-4)^3+1$

Determine algebraically whether the function is even, odd or neither.

27) $f(x)=\frac{1}{x^2}$
28) $f(x)=\frac{x}{x^2+4}$
29) $f(x)=3x^3-5$
30) $f(x)=\sqrt{x^2+1}$

<u>Chapter 2</u>

31) a. Use synthetic division to show that $x=-4$ is a solution of $2x^3+9x^2+3x-4=0$.

b. Use the result from part (a) to factor the polynomial **completely**.

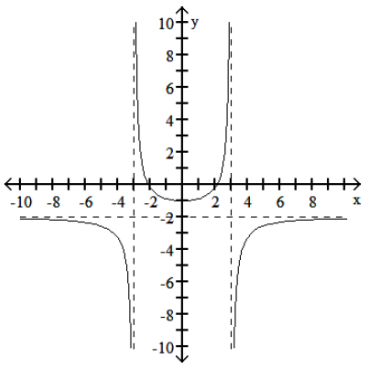
c. List **all** real solutions of the equation.

32) Write $\dfrac{4+2i}{3-i}$ in standard form.

33) Find all (real and/or imaginary) zeros of the equation $x^2+6x+10=0$

34) Find all (real and/or imaginary) zeros of the equation $x^3 - 3x^2 + 9x + 13 = 0$.

35) Determine the domain and the equations of the asymptotes for the graph of the rational function.



a. $(-\infty,-2)\cup(-2,\infty),\ y=-2,\ x=\pm 3$

b. $(-\infty,-2)\cup(-2,\infty),\ y=\pm 3,\ x=-2$

c. $(-\infty,-3)\cup(-3,3)\cup(3,\infty),\ y=\pm 3,\ x=-2$

d. $(-\infty,-3)\cup(-3,3)\cup(3,\infty),\ y=-2,\ x=\pm 3$

For the given function, find all asymptotes of the type indicated (if there are any).

36) $f(x)=\dfrac{(x-2)(x+1)}{x^2-4}$, Vertical

a) $x=2,\ x=-1$ b) $x=-2,\ x=1$ c) None d) $x=-2$

37) $f(x)=\dfrac{x^2+4x-3}{x-9}$, Slant (Oblique)

a) $y=x-5$ b) $y=x+13$ c) None d) $x=y+13$

38) $f(x)=\dfrac{5x^2-6x-4}{6x^2-7x+9}$, Horizontal

a) $y=\dfrac{6}{7}$ b) $y=\dfrac{5}{6}$ c) None d) $y=0$

Find the horizontal, vertical and slant asymptotes (if any) of the functions below.

39) $f(x)=\frac{3x+x^2-8x+2}{x^2-1}$

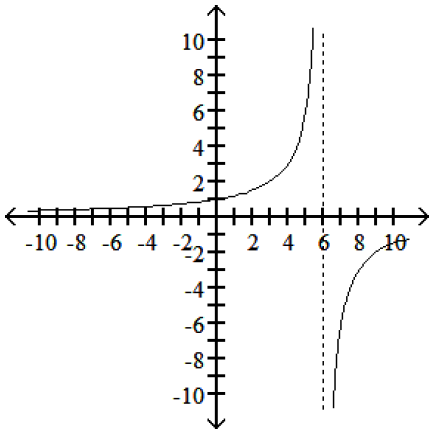
40) $f(x)=\frac{3x^2-5}{x+5}$

41) $f(x)=\frac{3x+8}{x-9}$

42) $g(x)=\frac{x+11}{x^2-25x}$

43) $h(x)=\frac{x^2+2x+7}{x+9}$

44) Match the graph of the rational function with its equation.

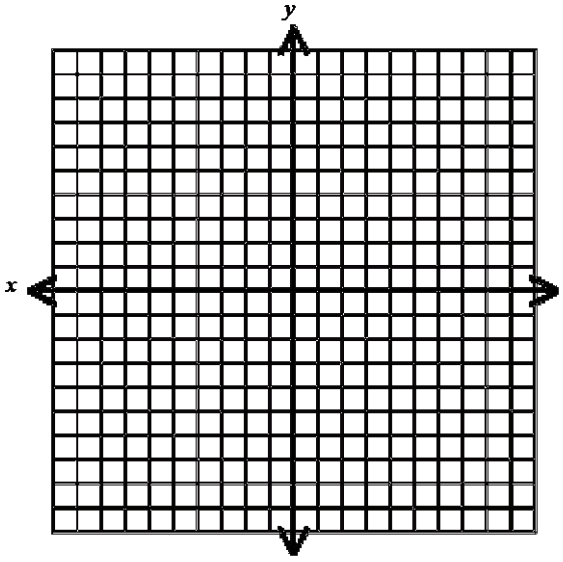


a) $f(x)=\dfrac{-6}{x+6}$ b) $f(x)=\dfrac{-6}{x-6}$

c) $f(x)=\dfrac{6}{x+6}$ d) $f(x)=\dfrac{6}{x-6}$

45) Sketch a graph of the rational function

$$k(x)=\frac{x-3}{x^2+x-12}$$



<u>Chapter 3</u>

Rewrite the following expressions as logarithmic expressions.

46) $6^a = b$

47) $2^5 = 32$

Rewrite the following expressions as exponential expressions.

48) $\log_7 a = b$

49) $\ln x = 3$

Find the value of the logarithmic function.

50) $\log_2 4$

a) 2 b) -2 c) $2\ln(2)$ d) $\dfrac{1}{2}$

51) $\log_6\left(\frac{1}{6}\right)$

a) 0 b) 6 c) -1 d) 1

52) $\log 100$

a) -3 b) 2 c) -1 d) -2

53) $\ln 1$

a) 1 b) 0 c) e d) -1

54) Write the equation $\ln(x) = -7$ as an equivalent exponential equation.

a) $10^{-7} = x$ b) $\ln(-7) = x$ c) $e^{-7} = x$ d) $e^{7} = x$

55) Write the equation $6^2 = 36$ as an equivalent logarithmic equation.

a) $\log_{36} 6 = 2$ b) $\log_2 36 = 6$ c) $\log_6 36 = 2$ d) $\log_6 2 = 36$

Evaluate the function using the change of base formula.

56) $\log_4 9$
57) $\log_3 0.28$

Use the properties of logarithms to expand each expression.

58) $\log_2 9x$
59) $\log_6 x^6$
60) $\ln\left(\frac{xy^3}{3e^x}\right)$
61) $\log_7 5x^3y\sqrt{z}$

Use the properties of logarithms to condense each expression.

62) $2\ln x + \ln 5$
63) $4\ln(x-4) - 2\ln x$
64) $5\log_4 2 + 7\log_4 x + 4\log_4 y$
65) $7\ln x - (3\ln y + 8\ln z)$
66) $\log_2 10 - \log_2 a$
67) $\log_6 15 - \log_6 24$

68) The projected population of California for the years 2015 to 2030 can be modeled by the function $P = 34.706e^{0.0097t}$, where P is the population (in millions) and t is the time (in years), with t=15 corresponding to 2015. Determine the year when the population of California will exceed 50 million.

69) The population of a certain rodent at a dump site is growing according to the function $R(x) = 97(3)^x$, where x is the number of years since 2008. What was the population in 2013?

70) You have deposited $500 in an account that pays 6.75% interest, compounded continuously. How long will it take for your money to double?

Solve the following equations.

71) $2^{2x+1} = 4$
72) $9^{-x} = \frac{1}{3}$
73) $4^x = 8$
74) $\ln(3x + 1) = 3$
75) $e^x = 13$
76) $8^x - 4 = 3$
77) $e^{2x} - 6 = 23$
78) $10\ln(x + 2) = 50$
79) $5 + 3\log_5 x = 8$
80) $10 + \log(x + 14) = 13$
81) $\log_{20}(x^2 - x) = 1$
82) $\log_6(x) = 2 - \log_6(x + 5)$

Chapter 4

Convert the following angle measures to degrees.

83) $\frac{5\pi}{3}$
84) 5

Convert the following angle measures to radians. Leave as a multiple of π.

85) 120°
86) -36°

Find the measures of two angles, one positive and one negative that are coterminal with the given angle.

87) -28°

88) $\frac{7\pi}{5}$

89) Convert $66°9'45''$ to decimal degrees and round to the nearest hundredth of a degree.

90) Convert $-92.23°$ to degrees, minutes and seconds.

91) Find the length of the arc intercepted by a central angle of 162° in a circle of radius $r = 9.5$ cm. Round to two decimal places.

92) Find the radius of a circle in which the given central angle, $\frac{\pi}{6}$ intercepts an arc of the length 7.9 feet.

93) Pittsburgh, Pennsylania and Miami, Florida, lie approximately on the same longitude. Pittsburgh has a latitude of 40.325° N and Miami has a latitude of 25.025° N. Find the distance between these two cities. (The radius of the earth is 3960 miles).

94) Assuming that Earth is a sphere of radius 4,000 miles, what is the difference in the latitudes of Buffalo, New York and Durham, North Carolina, where Buffalo is about 688 miles due north of Durham?

Find the exact value of the following trig functions.

95) $\tan\left(-\frac{2\pi}{3}\right)$

96) $\csc\left(\frac{4\pi}{3}\right)$

97) $\tan(1080°)$

98) $\cot\left(\frac{\pi}{3}\right)$

99) $\sec\left(\frac{\pi}{4}\right)$

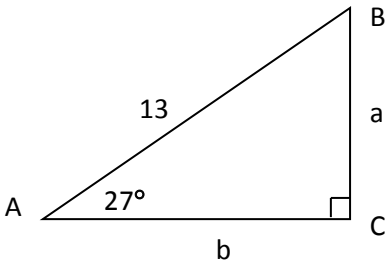
100) $\csc\left(-\frac{17\pi}{6}\right)$

101) $\sin\left(\frac{121\pi}{2}\right)$

102) $\cos\left(-\frac{5\pi}{4}\right)$

Complete the following.

103) Use the diagram on the right to solve the triangle.



104) θ is an angle in standard position whose terminal side contains $(-1,-3)$. Identify the **EXACT** value of $\csc\theta$.

105) From a boat on the lake, the angle of elevation to the top of a cliff is 12° 22'. If the base of the cliff is 146 feet from the boat, how high is the cliff (to the nearest foot)?

106) When sitting atop a tree and looking down at his pal Joey, the angle of depression of Mack's line of sight is 53° 49'. If Joey is known to be standing 32 feet from the base of the tree, how tall is the tree (to the nearest foot)?

107) Find the **EXACT** value of $\cos\theta$, if θ is in quadrant III and $\sin\theta = \frac{-5}{13}$.

108) Determine the **EXACT** value of $\sec\theta$ if $\frac{\pi}{2} \le \theta \le \pi$ and $\sin\theta = \frac{\sqrt{3}}{2}$.

109) Find the **EXACT** value of $\sin\theta$ for an angle θ in standard position which passes through the point $(8, -15)$.

110) In which quadrant is $\sec\theta < 0$ and $\tan\theta > 0$?

111) Find the reference angle for $-\frac{17\pi}{6}$ radians.

112) Find the reference angle for -440°.

113) Find $\sin(\alpha)$, given that $\cos(\alpha) = \frac{4}{9}$ and α is in quadrant IV.

a. $-\frac{\sqrt{65}}{9}$ b. $-\frac{9}{4}$ c. $-\frac{\sqrt{65}}{4}$ d. $-\sqrt{65}$

114) Find $\sin(\alpha)$, given that $\cos(\alpha) = \frac{4}{7}$ and $\sin(\alpha) > 0$.

a. $-\frac{\sqrt{33}}{7}$ b. $\frac{3}{7}$ c. $\frac{\sqrt{33}}{7}$ d. $-\frac{3}{7}$

115) Find $\cos(\alpha)$, given that $\sin(\alpha)=\frac{6}{7}$ and $\cos(\alpha)<0$.

a. $\frac{\sqrt{13}}{7}$ b. $-\frac{\sqrt{13}}{7}$ c. $-\frac{1}{7}$ d. $\frac{1}{7}$

116) Find $\csc(\alpha)$ if $\sin(\alpha)=-\frac{2}{3}$ and α is in quadrant IV.

a. $\frac{5}{4}$ b. $\frac{3\sqrt{7}}{7}$ c. $-\frac{\sqrt{7}}{9}$ d. $-\frac{3}{2}$

117) Find $\tan(\alpha)$ if $\sin(\alpha)=\frac{3}{4}$ and α is in quadrant II.

a. $-\frac{3\sqrt{7}}{7}$ b. $-\frac{\sqrt{7}}{9}$ c. $-\frac{3}{2}$ d. $\frac{5}{4}$

118) Find $\sec(\alpha)$ if $\tan(\alpha)=\frac{3}{4}$ and α is in quadrant I.

a. $-\frac{3}{2}$ b. $\frac{3\sqrt{7}}{7}$ c. $\frac{5}{4}$ d. $-\frac{\sqrt{7}}{9}$

Find the amplitude or period as requested.

119) Find the amplitude of $y=5\cos\left(2x+\frac{\pi}{3}\right)$.

a. $\frac{\pi}{2}$ b. 2 c. 5 d. -5

120) Find the amplitude of $y=-5\sin\left(3x+\frac{\pi}{3}\right)$.

a. 3 b. $\frac{\pi}{3}$ c. -5 d. 5

121) Find the period of $y=-2\sin\left(4x+\frac{\pi}{2}\right)$.

a. 2 b. $\frac{\pi}{2}$ c. π d. 4

122) Find the period of $y = -5\cos\left(3x + \frac{\pi}{2}\right)$

a. π b. 5 c. $\frac{\pi}{2}$ d. $\frac{2\pi}{3}$

123) Find the period of $y = -3\sin\left(\frac{1}{4}x - \frac{\pi}{2}\right)$

a. 4π b. 3π c. 8π d. $\frac{\pi}{2}$

Find the phase shift of the function.

124) $y = 2\sin\left(x - \frac{\pi}{4}\right)$

a. $\frac{\pi}{4}$ right b. 2 right c. 2 left d. $\frac{\pi}{4}$ left

125) $y = 2\sin\left(2x - \frac{\pi}{2}\right)$

a. 2π left b. $\frac{\pi}{4}$ right c. 2π right d. $\frac{\pi}{2}$ right

126) $y = 4\cos(6x + \pi)$

a. $\frac{\pi}{6}$ left b. 6π right c. $\frac{\pi}{4}$ right d. 4π right

127) $y = 5\cos\left(\frac{1}{4}x + \frac{\pi}{4}\right)$

a. $\frac{\pi}{4}$ left b. π left c. 5π right d. $\frac{\pi}{16}$ right

128) Given the function $y = -3\cos\left(\frac{1}{2}x + \frac{\pi}{3}\right) + 4$, identify the fundamental period, phase shift, vertical shift, amplitude, domain and range. Then graph the function and be sure to include a table of values.