(8) $\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \boxed{1}$

(9) $(\cos x - \sin x)(\cos x - \sin x)$

$\cos^2 x - 2\cos x \sin x + \sin^2 x$

$\cos^2 x + \sin^2 x - 2\cos x \sin x$

$\boxed{1 - 2\cos x \sin x}$

(10) $\frac{\cos x}{1} \div \frac{\sin x}{1} \Rightarrow \frac{\cos x}{1} \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x} = \boxed{\cot x}$

(11) $\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \cos^2 x - 1 = 0$

$\cos^2 x - 1 = \boxed{-\sin^2 x}$

(12) $\sin^2 x + \sin^2 x \cot^2 x$

$\sin^2 x (1 + \cot^2 x)$

$\sin^2 x (\csc^2 x)$

$\frac{\sin^2 x}{1} \cdot \frac{1}{\sin^2 x} = \boxed{1}$

(13) $\cot x (\sin x - \sec x)$

$\cot x \sin x - \cot x \sec x$

$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} - \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$\frac{\cos x}{1} - \frac{1}{\sin x} \Rightarrow \boxed{\cos x - \csc x}$

(14) $\sin x \sec x + \sin x \csc x - \cos x \sec x - \cos x \csc x$

$\frac{\sin x}{1} \cdot \frac{1}{\cos x} + \frac{\sin x}{1} \cdot \frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{1}{\sin x}$

$\frac{\sin x}{\cos x} + 1 - 1 - \frac{\cos x}{\sin x} \Rightarrow \boxed{\tan x - \cot x}$

(15) $1+\cos x-\cos x-\cos^2 x$

$1-\cos^2 x = \boxed{\sin^2 x}$

(16) $\frac{\cos\theta}{1}\frac{\sin\theta}{1}\left(\frac{1}{\cos\theta}-\frac{1}{\sin\theta}\right) \Rightarrow \frac{\cos\theta\sin\theta}{\cos\theta}-\frac{\cos\theta\sin\theta}{\sin\theta}$

$\Rightarrow \boxed{\sin\theta-\cos\theta}$

(17) $\frac{\cos x}{1}+\frac{\sin x}{1}\cdot\frac{\sin x}{\cos x} \Rightarrow \frac{\cos x}{1}+\frac{\sin^2 x}{\cos x} \Rightarrow \frac{\cos^2 x}{\cos x}+\frac{\sin^2 x}{\cos x}$

$\Rightarrow \frac{1}{\cos x} \Rightarrow \boxed{\sec x}$

(18) $\frac{\cot x}{\cos x} = \frac{\cos x}{\sin x}\div\frac{\cos x}{1} \Rightarrow \frac{\cos x}{\sin x}\cdot\frac{1}{\cos x} = \frac{1}{\sin x} \Rightarrow \csc x$

$\csc x-\csc x = \boxed{0}$

(19) $\csc\theta\cot\theta = \frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta} \Rightarrow \frac{\cos\theta}{\sin^2\theta}$

$\frac{\cos\theta}{\sin^2\theta}\div\frac{1}{\cos\theta} \Rightarrow \frac{\cos\theta}{\sin^2\theta}\cdot\frac{\cos\theta}{1} \Rightarrow \frac{\cos^2\theta}{\sin^2\theta} = \boxed{\cot^2\theta}$

(20) $\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta} = \frac{1}{\cos^2\theta} = \boxed{\sec^2\theta}$

(21) $\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\pm\sqrt{3}}{2} \Rightarrow \boxed{\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ}$

(22) $3x^2-x-4=0$

$(3x-4)(x+1)=0$

$(3\sin x-4)(\sin x+1)=0$

$3\sin x-4=0 \qquad \sin x+1=0$

$\sin x=\frac{4}{3} \qquad \sin x=-1$

$\boxed{x=270^\circ}$

↑ no solution, outside domain

(23) $2\cos^3\theta - \cos\theta = 0$

$\cos\theta(2\cos^2\theta - 1) = 0$

$\cos\theta = 0$ $\quad 2\cos^2\theta - 1 = 0$

$\theta = 90°, 270°$ $\quad \cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

$\theta = 45°, 135°, 225°, 315°$

(24) $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$(\sin x - 4)(\sin x + 3) = 0$

$\sin x - 4 = 0$ $\quad \sin x + 3 = 0$

$\sin x = 4$ $\quad \sin x = -3$

No solution Both are outside domain

(25) $x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$(\cos x + 1)(\cos x + 1) = 0$

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

(26) $2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$ $\quad 2\sin x - 1 = 0$

$x = 0, \pi$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

** (27) $\cos 2x = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4}$ $\quad 2x = \frac{7\pi}{4}$

$\boxed{x = \frac{\pi}{8}}$ $\quad \boxed{x = \frac{7\pi}{8}}$

(28) $\cos^2 x = 1$

$\cos x = \pm 1$

$\boxed{x = 0, \pi}$

(29) $\cos^2 x + 2\cos x + 1 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$(\cos x + 1)(\cos x + 1) = 0$

$\cos x + 1 = 0$

$\cos x = -1$

$\boxed{x = \pi}$

(30) $\cos(u+v) = \cos(105+45) = \cos(150) = \boxed{-\frac{\sqrt{3}}{2}}$

(31) $\sin(u+v) = \sin(15+105) = \sin(120) = \boxed{\frac{\sqrt{3}}{2}}$

(32) $\tan(u-v) = \tan(175-55) = \tan(120^\circ) = \boxed{-\sqrt{3}}$

(33) $\sin(u-v) = \sin\left(\frac{7\pi}{24} - \frac{\pi}{8}\right) = \sin\left(\frac{7\pi}{24} - \frac{3\pi}{24}\right) = \sin\left(\frac{4\pi}{24}\right) = \sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$

(34) $\tan(u+v) = \tan\left(\frac{\pi}{15} + \frac{4\pi}{15}\right) = \tan\left(\frac{5\pi}{15}\right) = \tan\left(\frac{\pi}{3}\right) = \boxed{\sqrt{3}}$

(35) $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{11\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{11\pi}{6}\right) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot -\frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

(36) $\dfrac{\tan\left(\frac{3\pi}{4}\right)-\tan\left(\frac{\pi}{6}\right)}{1+\tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)} = \dfrac{-1-\frac{\sqrt{3}}{3}}{1+(-1)\left(\frac{\sqrt{3}}{3}\right)} = \dfrac{-\frac{3}{3}-\frac{\sqrt{3}}{3}}{\frac{3}{3}-\frac{\sqrt{3}}{3}} = \dfrac{\frac{-3-\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \dfrac{-3-\sqrt{3}}{\cancel{3}} \cdot \dfrac{\cancel{3}}{3-\sqrt{3}} = \dfrac{-3-\sqrt{3}}{3-\sqrt{3}}\dfrac{(3+\sqrt{3})}{(3+\sqrt{3})} = \dfrac{-9-6\sqrt{3}-3}{9-3} = \dfrac{-12-6\sqrt{3}}{6}$

$= \boxed{-2-\sqrt{3}}$

(37) $\cos(60)\cos(45)+\sin(60)\sin(45)$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

(38) $\frac{\sin 11.2}{98.7} = \frac{\sin 37.2}{b} = \frac{\sin 131.6}{c}$

$\frac{\sin 11.2}{98.7} = \frac{\sin 131.6}{c}$

$98.7\sin 37.2 = b\sin 11.2$

$\boxed{b = 307.2}$

$98.7\sin 131.6 = c\sin 11.2$

$\boxed{c = 380}$

(39) $\frac{\sin A}{17.8} = \frac{\sin 114.4}{5.57} = \frac{\sin C}{c}$

$17.8\sin 114.4 = 5.57\sin A$

$2.9103 = \sin A$

↑

No solution; outside domain

(40) $\frac{\sin 30}{3.64} = \frac{\sin B}{7.28} = \frac{\sin C}{c} \Rightarrow \frac{\sin 30}{3.64} = \frac{\sin 90}{7.28} = \frac{\sin 60}{c}$ $\boxed{m\angle C = 60°}$

$7.28\sin 30 = 3.64\sin B$

$1 = \sin B$

$\boxed{m\angle B = 90°}$

$3.64\sin 60 = c\sin 30$

$\boxed{c = 6.3}$

(41) $\frac{\sin A}{12.73} = \frac{\sin 24.8}{10.68} = \frac{\sin C}{c} \Rightarrow \frac{\sin 30}{12.73} = \frac{\sin 125.2}{c}$

$12.73\sin 24.8 = 10.68\sin A$

$.4999 = \sin A$

$\boxed{m\angle A = 30°}$ $\boxed{m\angle C = 125.2°}$

$12.73\sin 125.2 = c\sin 30$

$\boxed{c = 20.8}$

OR

$m\angle A = 180 - 30° = 150° = m\angle A$

$\boxed{m\angle C = 5.2°}$

$\frac{\sin 150}{12.73} = \frac{\sin 24.8}{10.68} = \frac{\sin 5.2}{c}$

$10.68\sin 5.2 = c\sin 24.8$

$\boxed{c = 2.3}$

(42) $\frac{\sin A}{18.9} = \frac{\sin 16.6}{9.52} = \frac{\sin C}{c} \Rightarrow \frac{\sin 16.6}{9.52} = \frac{\sin 128.8}{c}$

$18.9 \sin 16.6 = 9.52 \sin A$

$9.52 \sin 128.8 = c \sin 16.6$

$.5672 = \sin A$

$c = 26$

$m\angle A = 34.6°$

$m\angle C = 128.8°$

OR

$m\angle A = 180 - 34.6 = 145.4 = m\angle A$

$m\angle C = 18°$

$\frac{\sin 145.4}{18.9} = \frac{\sin 16.6}{9.52} = \frac{\sin 18}{c}$

$9.52 \sin 18 = c \sin 16.6$

$c = 10.3$

(43) $\frac{\sin A}{7.1} = \frac{\sin B}{11.81} = \frac{\sin 105.6}{c}$ ⇒ use Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 7.1^2 + 11.81^2 - 2(7.1)(11.81)\cos 105.6$

$c^2 = 189.89 - 167.702 \cos 105.6$

$c^2 = 234.98 \Rightarrow c = 15.3$

$\frac{\sin A}{7.1} = \frac{\sin B}{11.81} = \frac{\sin 105.6}{15.3}$

$11.81 \sin 105.6 = 15.3 \sin B$

$.7435 = \sin B$

$m\angle B = 48.0°$

$m\angle A = 26.4°$

(44) $b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 12.2^2 + 7.8^2 - 2(12.2)(7.8)\cos 63.5$

$b^2 = 209.68 - 190.32 \cos 63.5$

$b^2 = 124.76$

$b = 11.2$

$\frac{\sin A}{12.2} = \frac{\sin 63.5}{11.2} = \frac{\sin C}{7.8}$

$12.2 \sin 63.5 = 11.2 \sin A$

$.9748 = \sin A$

$m\angle A = 77.1°$

$m\angle C = 39.4°$

(45) $c^2 = a^2 + b^2 - 2ab\cos C$

$15.1^2 = 6.4^2 + 13.6^2 - 2(6.4)(13.6)\cos C$

$228.01 = 225.92 - 174.08 \cos C$

$2.09 = -174.08 \cos C$

$-.0120 = \cos C$

$m\angle C = 90.7°$

$\frac{\sin A}{6.4} = \frac{\sin B}{13.6} = \frac{\sin 90.7}{15.1}$

$13.6 \sin 90.7 = 15.1 \sin B$

$.9006 = \sin B$

$m\angle B = 64.2°$

$m\angle C = 25.1°$

(46) $c^2 = a^2 + b^2 - 2ab\cos C$

$15.4^2 = 7.4^2 + 13.9^2 - 2(7.4)(13.9)\cos C$

$237.16 = 247.97 - 205.72\cos C$

$-10.81 = -205.72\cos C$

$.0525 = \cos C$

$\boxed{m\angle C = 87°}$

$\frac{\sin A}{7.4} = \frac{\sin B}{13.9} = \frac{\sin 87}{15.4}$

$13.9\sin 87 = 15.4\sin B$

$.9014 = \sin B$

$\boxed{m\angle B = 64.3°}$

$\boxed{m\angle C = 28.7°}$

(47) $21 = \frac{1}{2}(9)(6)\sin C$

$21 = 27\sin C$

$.7778 = \sin C$

$\boxed{m\angle C = 51.06°}$

(48) $A = \frac{1}{2}bc\sin A$

$= \frac{1}{2}(86)(12.7)\sin 48.3$

$\boxed{A = 407.7 m^2}$

* (49) $\frac{\sin 32.6}{a} = \frac{\sin 32.6}{b} = \frac{\sin 114.8}{6}$

$6\sin 32.6 = b\sin 114.8$

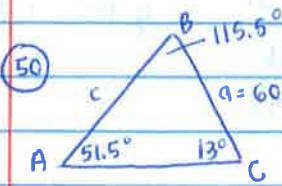
$b = 3.56$

$A = \frac{1}{2}bc\sin A$

$= \frac{1}{2}(3.56)(6)\sin 32.6$

$\boxed{A = 5.75 in^2}$

(50)



$\frac{\sin 51.5}{608} = \frac{\sin 115.5}{b} = \frac{\sin 13}{c}$

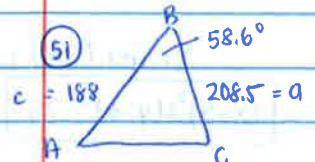
$608\sin 13 = c\sin 51.5$

$c = 174.7 \approx \boxed{175 m}$

$608\sin 115.5 = b\sin 51.5$

$b = 701.2$

(51)

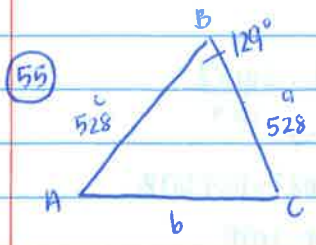


$A = \frac{1}{2}ac\sin B$

$= \frac{1}{2}(208.5)(188)\sin(58.6)$

$\boxed{= 16729 m^2}$

(55)



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = (528)^2 + (528)^2 - 2(528)(528)\cos 129$

$b^2 = 557568 + 350888.9118$

$b^2 = 206679.0882$

$b = 454.6 \approx$ **455 mi**

(74) $\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3} + \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3} = 1$

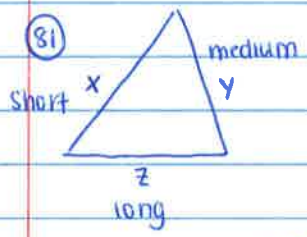
$2\sin x \cos\frac{\pi}{3} = 1$

$2\left(\frac{1}{2}\right)\sin x = 1$

$\sin x = 1$

$x = \pi/2$

(81)



① $x + y + z = 180$ ← first sentence

② $z = 2x - 9$ ← second sentence

③ $x + y = z + 30$ ← third sentence

sub ② into ①:

$x + y + 2x - 9 = 180$

$3x + y = 189$

sub ② into ③:

$x + y = 2x - 9 + 30$

$x + y = 2x + 21$

$-x + y = 21$

$3x + y = 189$

$-1(-x + y = 21)$ ⇒ $3x + y = 189$, $x - y = -21$

$2x = 168$

$x = 84$

$-84 + y = 21$

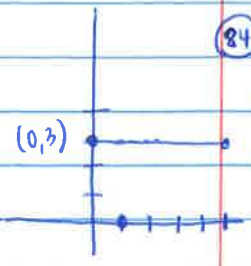
$y = 105$

$z = 2(84) - 9$

$z = 159$

Shortest: 84 ft, medium: 105 ft, long: 159 ft

(84)



center $(h, 3)$ tangent $(0,3)$ (h k, x y)

$(0-h)^2 + (3-3)^2 = r^2$

$(-h)^2 + (0)^2 = r^2$

$h^2 = r^2$

center $(h,3)$ x-int $(1,0)$ (h k, x y)

$(1-h)^2 + (0-3)^2 = h^2$

$(1-h)(1-h) + 9 = h^2$

$1 - 2h + h^2 + 9 = h^2$

$1 - 2h = -9$

$-2h = -10$

$h = 5$

center $(5,3)$ radius : 5

$(x-5)^2 + (y-3)^2 = 25$