Pre-Calculus A
Final Exam Review

Name : Key
Date : ______________ Period : ______

DISCLAIMER! This review guide is intended to help you review for the final exam. Refer to your course notes, test and quiz reviews, homework worksheets, and quizzes and tests as well when preparing for the final exam.

Basic Unit Circle Review: Find the exact value of the following expression without using a calculator.

1. $\sin\left(\frac{3\pi}{4}\right)$

a. $\frac{1}{2}$ b. $\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{2}}{2}$ d. $-\frac{1}{2}$

2. $\cos(30°)$

a. $\sqrt{3}$ b. $\frac{1}{2}$ c. $\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{2}}{2}$

3. $\cos\left(-\frac{5\pi}{4}\right)$ $-\frac{5\pi}{4}+\frac{2\pi}{1} = -\frac{5\pi}{4}+\frac{8\pi}{4} = \frac{3\pi}{4} \Rightarrow \cos\left(\frac{3\pi}{4}\right)$

a. $\frac{1}{2}$ b. $-\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}}{2}$

Find the exact value of each real number. Do not use a calculator.

4. $y=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ The angle whose sin is $\frac{\sqrt{2}}{2}$ (in Q1 or Q4)

a. $\frac{7\pi}{4}$ b. $\frac{\pi}{4}$ c. $\frac{3\pi}{4}$ d. $\frac{\pi}{4}, \frac{3\pi}{4}$

5. $y=arc\csc(2)$ $\Rightarrow \arcsin\left(\frac{1}{2}\right)$ The angle whose sin is $\frac{1}{2}$ in Q1 or Q4

a. $\frac{\pi}{6}$ b. $\frac{5\pi}{6}, \frac{\pi}{6}$ c. $\frac{5\pi}{6}$ d. No solution

6. $y=\tan^{-1}(\sqrt{3})$ The angle whose tan is $\sqrt{3}$ in Q1 or Q4

a. $\frac{\pi}{6}, \frac{7\pi}{6}$ b. $\frac{\pi}{3}, \frac{4\pi}{3}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{3}$

7. $y=\text{arccot}(\sqrt{3})$ $\arctan\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \arctan\left(\frac{\sqrt{3}}{3}\right)$ The angle whose tan is $\frac{\sqrt{3}}{3}$ in Q1 or Q4

a. $\frac{\pi}{6}, \frac{7\pi}{6}$ b. $\frac{\pi}{3}, \frac{4\pi}{3}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{3}$

Use the fundamental identities to simplify the expression. #8-51 on separate paper

8. $\cot\theta \sec\theta \sin\theta$

a. 1 b. $\csc^2\theta$ c. $\sec^2\theta$ d. $\tan^2\theta$

9. $(\cos x - \sin x)^2$

a. $1 - 2\sin x \cos x$ b. $\cos^2 x + 2\sin^2 x$ c. 1 d. $\cos^2 x + 2\sin x - \sin^2 x$

10. $\dfrac{\cos x}{\sin x}$

a. $\cos x$ b. $\sin x$ c. $\cot x$ d. $\tan x$

11. $\cos^2 x - 1$

a. $\sin^2 x$ b. $\sec^2 x$ c. $\csc^2 x$ d. $-\sin^2 x$

12. $\sin^2 x + \sin^2 x \cot^2 x$

a. $\sin^2 x + 1$ b. $\cot^2 x + 1$ c. $\cot^2 x - 1$ d. 1

13. $\cot x(\sin x - \sec x)$

a. $\cot x + \csc x$ b. $\cos x - \csc x$ c. $\cos x - \cot x$ d. $\tan x - \sin x$

14. $(\sin x - \cos x)(\sec x + \csc x)$

a. $\tan x - \cot x$ b. $\cos x - \cot x$ c. $\tan x - \sin x$ d. $\cot x + \csc x$

15. $(1 - \cos x)(1 + \cos x)$

a. $\sin^2 x$ b. $1 + 2\sin^2 x$ c. $\sin x + 2\csc x$ d. $\cos^2 x - 1$

16. $\cos\theta\sin\theta(\sec\theta-\csc\theta)$

a. $\dfrac{\cos^2\theta-1}{\sin\theta-2\csc\theta}$ b. $\sin\theta-\cos\theta$ c. 1 d.

17. $\cos x+\sin x\tan x$

a. $\csc x$ b. $\tan x-1$ c. $\sec x$ d. $\cot x-1$

18. $\dfrac{\cot x}{\cos x}-\csc x$

a. 1 b. 0 c. $\tan x$ d. $\sec x$

19. $\dfrac{\csc\theta\cot\theta}{\sec\theta}$

a. $\cot^2\theta$ b. $\csc^2\theta$ c. $\sec^2\theta$ d. 1

20. $\tan^2\theta\csc^2\theta$

a. $\cos^3\theta$ b. $\sin\theta$ c. $\sec^2\theta$ d. $\tan^2\theta$

Solve the equation in the interval $[0°,360°)$.

21. $4\sin^2\theta=3$

a. 240°, 300° b. 60°, 120° c. No Solution d. 60°, 120°, 240°, 300°

22. $3\sin^2\theta-\sin\theta-4=0$

a. 270° b. 0° c. 180° d. 90°

23. $2\cos^3\theta = \cos\theta$

a. No Solution

b. 45°, 90°, 135°, 225°, 270°, 315°

c. 45°, 135°, 225°, 315°

d. 90, 270

24. $\sin^2\theta - \sin\theta - 12 = 0$

a. 45°, 135°

b. 45°, 315°

c. 45°

d. No Solution

Solve the equation in the interval $[0, 2\pi)$.

25. $\cos^2 x + 2\cos x + 1 = 0$

a. $\frac{\pi}{4}, \frac{7\pi}{4}$

b. $\frac{\pi}{2}, \frac{3\pi}{2}$

c. 2π

d. π

26. $2\sin^2 x = \sin x$

a. $\frac{\pi}{6}, \frac{5\pi}{6}$

b. $\frac{\pi}{3}, \frac{2\pi}{3}$

c. $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

d. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$

27. $\sqrt{2}\cos 2x = 1$

a. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

b. $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

c. No Solution

d. $x = \frac{\pi}{8}, \frac{9\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}$

28. $\cos^2 x - 1 = 0$

a. $x = 0, 2\pi$

b. $x = 0,\ \pi,\ 2\pi$

c. $x = 0, \pi$

d. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

29. $\cos^2 x + 2\cos x = -1$

a. $x = 0, 2\pi$

b. $x = 0,\ \pi,\ 2\pi$

c. $x = 0, \pi$

d. $x = \pi$

Use trigonometric identities to find the exact value.

30. $\cos(105°)\cos(45°) - \sin(105°)\sin(45°)$

a. $\frac{7}{4}$ b. $\frac{1}{2}$ c. $\frac{\sqrt{3}}{3}$ d. $-\frac{\sqrt{3}}{2}$

31. $\sin(15°)\cos(105°) + \cos(15°)\sin(105°)$

a. $-\frac{1}{2}$ b. $\frac{\sqrt{3}}{2}$ c. $-\frac{\sqrt{3}}{2}$ d. $\frac{1}{4}$

32. $\dfrac{\tan(175°) - \tan(55°)}{1 + \tan(175°)\tan(55°)}$

a. $-\frac{\sqrt{3}}{3}$ b. $-\sqrt{3}$ c. -2 d. $-\frac{1}{2}$

Use an appropriate identity to simplify the expression.

33. $\sin\left(\frac{7\pi}{24}\right)\cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{7\pi}{24}\right)\sin\left(\frac{\pi}{8}\right)$

a. $\frac{1}{2}$ b. 1 c. $\frac{\sqrt{2}}{2}$ d. $\frac{\sqrt{3}}{2}$

34. $\dfrac{\tan\left(\frac{\pi}{15}\right) + \tan\left(\frac{4\pi}{15}\right)}{1 - \tan\left(\frac{\pi}{15}\right)\tan\left(\frac{4\pi}{15}\right)}$

a. $\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{3}}{3}$ c. $\frac{1}{2}$ d. $\sqrt{3}$

Use an identity to find the exact value of the expression.

35. $\sin\left(\overset{u}{\frac{\pi}{4}}-\overset{v}{\frac{11\pi}{6}}\right)$

a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ b. $-\frac{\sqrt{6}+\sqrt{2}}{4}$ c. $\frac{\sqrt{6}+\sqrt{2}}{4}$ d. $\frac{\sqrt{2}-\sqrt{6}}{4}$

36. $\tan\left(\overset{u}{\frac{3\pi}{4}}-\overset{v}{\frac{\pi}{6}}\right)$

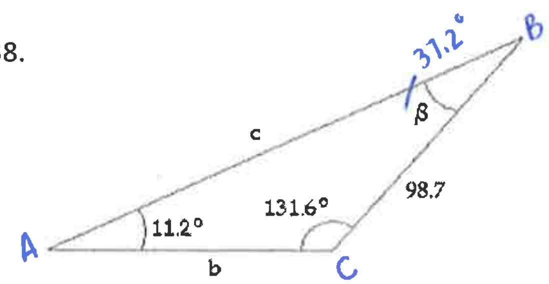
a. $2+\sqrt{3}$ b. $-2-\sqrt{3}$ c. $\sqrt{3}-2$ d. $2-\sqrt{3}$

37. $\cos(\overset{u}{60^\circ}-\overset{v}{45^\circ})$

a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ b. $\frac{\sqrt{2}-\sqrt{6}}{4}$ c. $-\frac{\sqrt{6}+\sqrt{2}}{4}$ d. $\frac{\sqrt{6}+\sqrt{2}}{4}$

Solve the triangle. If there is more than one triangle with the given parts, give both solutions.

38.



a. $\beta=37.2^\circ,\ b=380,\ c=307.3$ c. $\beta=37.2^\circ,\ b=307,\ c=380$

b. $\beta=36.8^\circ,\ b=304,\ c=380$ d. $\beta=37.2^\circ,\ b=31.7,\ c=25.8$

39. $\beta=114.4^\circ,\ b=5.57,\ a=17.8$

a. $\alpha=58.2^\circ,\ C=8.4^\circ,\ c=27.37$ c. $\alpha=57.2^\circ,\ C=8.4^\circ,\ c=23.37$

b. $\alpha=56.2^\circ,\ C=8.4^\circ,\ c=25.37$ d. No solution

40. $\alpha = 30°$, $a = 3.64$, $b = 7.28$

a. $\beta = 60°$, $C = 90°$, $c = 6.3$

b. No solution

c. $\beta = 60°$, $C = 60°$, $c = 6.3$

d. $\beta = 90°$, $C = 60°$, $c = 6.3$

41. $\beta = 24.8°$, $b = 10.68$, $a = 12.73$

a. No solution

b. $\alpha = 150°$, $C = 5.2°$, $c = 2.31$

c. $\alpha = 30°$, $C = 125.2°$, $c = 20.8$

d. $\alpha = 30°$, $C = 125.2°$, $c = 20.8$
$\alpha' = 150°$, $C' = 5.2°$, $c' = 2.31$

42. $\beta = 16.6°$, $b = 9.52$, $a = 18.9$

a. $\alpha = 9.3°$, $C = 155.1°$, $c = 31.4$

b. $\alpha = 8.3°$, $C = 155.1°$, $c = 28.4$

c. $\alpha = 7.3°$, $C = 155.1°$, $c = 30.4$

d. $\alpha = 34.6°$, $C = 128.8°$, $c = 26.0$
$\alpha' = 145.4°$, $C' = 18.0°$, $c' = 10.3$

43. $C = 105.6°$, $a = 7.1$, $b = 11.81$

a. $c = 18.2$, $\alpha = 28.5°$, $\beta = 45.9°$

b. $c = 15.3$, $\alpha = 26.5°$, $\beta = 47.9°$

c. $c = 21.1$, $\alpha = 24.5°$, $\beta = 49.9°$

d. No solution

44. $\beta = 63.5°$, $a = 12.20$, $c = 7.80$

a. $b = 13.2$, $\alpha = 73.1°$, $C = 37.4°$

b. $b = 11.2$, $\alpha = 77.8°$, $C = 38.7°$

c. $b = 12.2$, $\alpha = 75.1°$, $C = 41.4°$

d. No solution

45. $a = 6.4$, $b = 13.6$, $c = 15.1$ ← solve for largest angle 1st ($\angle C$)

a. $\alpha = 25.1°$, $\beta = 64.3°$, $C = 90.7°$

b. $\alpha = 23.1°$, $\beta = 64.3°$, $C = 92.7°$

c. No solution

d. $\alpha = 27.1°$, $\beta = 62.3°$, $C = 90.7°$

46. $a = 7.4$, $b = 13.9$, $c = 15.4$ ← solve for largest angle 1st ($\angle C$)

a. $\alpha = 30.7°$, $\beta = 62.3°$, $C = 87.0°$

b. $\alpha = 26.7°$, $\beta = 64.3°$, $C = 89.0°$

c. No triangle satisfies the given conditions

d. $\alpha = 28.7°$, $\beta = 64.3°$, $C = 87.0°$

Solve the problem.

47. The area of a triangle is given by $A = \frac{1}{2}ab\sin C,$ where a and b are the lengths of two of the sides and C is in the included angle. If A = 21 in^2, a = 9 in, b = 6 in, and C is an acute angle, what must C be?

a. 128.94° b. 25.53° c. 51.06° d. No such triangle exists

48. Find the area of the triangle ABC for which $\alpha = 48.3°$, b = 86 meters, and c = 12.7 meters.

a. 403.5 m^2 b. 417 m^2 c. 407.7 m^2 d. 422.8 m^2

* 49. Find the area of triangle ABC for which $\alpha = 32.6°$, $\beta = 32.6°$, and c = 6.0 inches.

a. 14 in^2 b. 16 in^2 c. 38 in^2 d. 36 in^2 e. 5.75 in^2

50. To find the distance AB across a river, a distance BC of 608 m is laid off on one side of the river. It is found that B = 115.5° and C = 13.0°. Find AB.

a. 175 m b. 178 m c. 137 m d. 140 m

51. A triangular-shaped field has sides of 188.0 meters and 208.5 meters, and the angle between them measures 58.60°. Find the area of the field.

a. 10,211m^2 b. 20,422 m^2 c. 33,458 m^2 d. 16,729 m^2

52. Find the area of the triangle given a = 240 feet, b = 121 feet, and c = 180 feet.

a. 6,149 ft^2 b. 10,565 ft^2 c. 6,135 ft^2 d. 6,129 ft^2

$S = \frac{240+121+180}{2}$ Area: $\sqrt{270.5(270.5-240)(270.5-121)(270.5-180)}$

$S = 270.5$

53. Find the area of the triangle given a = 59.7 inches, b = 65.4 inches, and c = 73.7 inches.

a. 2,324 in^2 b. 1,857 in^2 c. 2,303 in^2 d. 2,310 in^2

$S = \frac{59.7+65.4+73.7}{2}$ Area = $\sqrt{(99.4)(99.4-59.7)(99.4-65.4)(99.4-73.7)}$

$S = \frac{198.8}{2} = 99.4$

$s = \frac{16.9+13.3+9.1}{2}$ $\quad$ $\text{Area} = \sqrt{19.65(19.65-16.9)(19.65-13.3)(19.65-9.1)}$

↑ $s = 19.65$ $\quad$ $= 60.17$

54. A room in the shape of a triangle has sides of length 16.9 yards, 13.3 yards, and 9.1 yards. If carpeting costs $21.82 per square yard, padding costs $2.10 per square yard, and there is no charge for installation, how much will it cost to carpet the room?

a. $2,045 b. $60 c. $1,439 d. $325

Carpet: $21.82 \times 60.17 = 1312.91$

Padding: $2.10 \times 60.17 = 126.36$

$\rangle$ 1439

55. Two ships leave a harbor together traveling on courses that have an angle of 129° between them. If they each travel 528 miles, how far apart are they?

a. 953 mi b. 455 mi c. 1,906 mi d. 41 mi

work on separate paper

56. A painter needs to cover a triangular region 64 meters by 69 meters by 75 meters. A can of paint covers 70 square meters. How many cans will be needed?

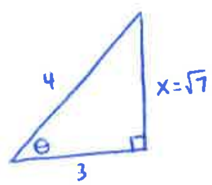
a. 3 cans b. 15 cans c. 30 cans d. 337 cans

$s = \frac{64+69+75}{2}$ $\quad$ $\text{Area} = \sqrt{104(104-64)(104-69)(104-75)}$

$s = 104$ $\quad$ $= 2054.85 \div 70 = 29.4$ cans ≈ 30

Give the exact value of each expression without using a calculator.

57. $\tan\left(\arccos\left(\frac{3}{4}\right)\right)$



$3^2 + x^2 = 4^2$

$9 + x^2 = 16$

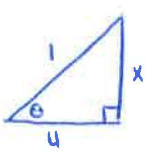
$x^2 = 7$

$x = \sqrt{7}$

$\tan(\theta) = \boxed{\frac{\sqrt{7}}{3}}$

58. $\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) \Rightarrow \cos^{-1}(-1) = \boxed{\pi}$

59. $\sin(\arccos(u))$



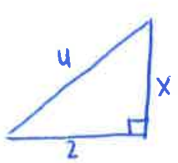
$x^2 + u^2 = 1^2$

$x^2 = 1 - u^2$

$x = \sqrt{1-u^2}$

$\sin(\theta) = \frac{\sqrt{1-u^2}}{1} = \boxed{\sqrt{1-u^2}}$

60. $\sin\left(\sec^{-1}\left(\frac{u}{2}\right)\right)$



$x^2 + 2^2 = u^2$

$x^2 + 4 = u^2$

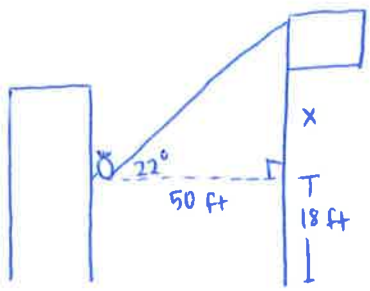
$x^2 = u^2 - 4$

$x = \sqrt{u^2-4}$

$\boxed{\sin(\theta) = \frac{\sqrt{u^2-4}}{u}}$

Applications

61. Noah is looking out of Mrs. D'Emanuele's window and sees the top of the school flagpole at an angle of elevation of 22°. Noah is 18 feet above the ground and 50 feet from the flagpole. Find the height of the flagpole. Round your answer to the nearest tenth of a foot.



$\tan 22 = \frac{X}{50}$

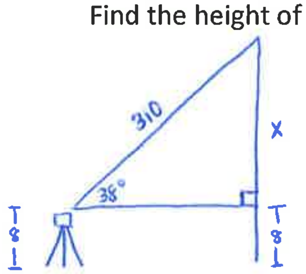
$X = 50\tan 22$

$X = 20.2$

height $= 20.2 + 18$

$= \boxed{38.2 \text{ ft}}$

62. A surveyor is taking measurements from a transit that is 8 feet tall. The angle of elevation to the top of a cell tower is 38°. The distance from the transit to the top of the cell tower is 310 feet. Find the height of the tower. Round your answer to two decimal places.



$\sin 38 = \frac{X}{310}$

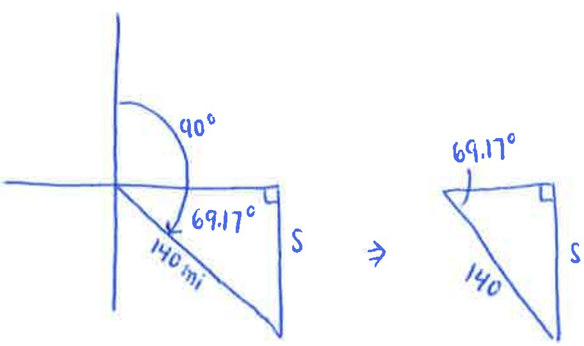
$X = 310\sin 38$

$X = 190.9$

height $= 190.86 + 8$

$= \boxed{198.86 \text{ ft}}$

63. A boat sails for 4 hours at 35 miles per hour in a direction 159° 10'. How far south has it sailed? Round your answer to the nearest mile. $4 \times 35 = 140$ miles ↳ 159.17°



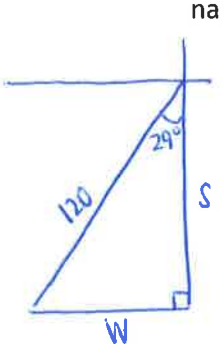
$\sin 69.17 = \frac{S}{140}$

$S = 140\sin 69.17$

$\boxed{S = 130.8 \text{ miles}}$

↳ 29° west of south

64. A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots. How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?



20 knots × 6 hrs = 120 mi

$\cos 129 = \frac{S}{120}$

$S = 120\cos 129$

$\boxed{S = 105 \text{ mi}}$

$\sin 29 = \frac{W}{120}$

$W = 120\sin 29$

$\boxed{W = 58.2 \text{ mi}}$

Solve the following equations. $3x^2-x-1=0 \Rightarrow b^2-4ac \Rightarrow (-1)^2-4(3)(-1) = 1+12=13$

not factorable

65. $3\sin^2 x - \sin x - 1 = 0$ on the interval $[0°, 360°)$.

$\sin x = \frac{1 \pm \sqrt{13}}{2(3)} = \frac{1 \pm \sqrt{13}}{6} < \begin{matrix} .7676 \\ -.4343 \end{matrix}$

$\sin x = .7676$

$\boxed{x = 50.13°}$

Ref angle in Q2:

$x = 180 - 50.13$

$\boxed{x = 129.87°}$

$\sin x = -.4343$

$x = -25.7°$

$+360$

$\boxed{334.3°}$

Ref angle in Q3:

$x = 180 + 25.7$

$\boxed{x = 205.7°}$

66. $\cos^2 x - \cos x - 1 = 0$ on the interval $[0°, 360°)$.

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x - 2 = 0$ $\quad$ $\cos x + 1 = 0$

$\cos x = 2$ $\quad$ $\cos x = -1$

no solution $\quad$ $\boxed{x = \pi}$

67. $2\cos 3x - 1 = 0$ on the interval $[0, 2\pi)$.

$2\cos 3x = 1$

$\cos 3x = \frac{1}{2}$

$3x = \frac{\pi}{3}$ $\quad$ $3x = \frac{5\pi}{3}$

$\boxed{x = \frac{\pi}{9}}$ $\quad$ $\boxed{x = \frac{5\pi}{9}}$

68. $\sin 2x = -\frac{\sqrt{3}}{2}$ on the interval $[0, 2\pi)$.

$2x = \frac{4\pi}{3}$ $\quad$ $2x = \frac{5\pi}{3}$

$x = \frac{4\pi}{6}$ $\quad$ $\boxed{x = \frac{5\pi}{6}}$

$\boxed{x = \frac{2\pi}{3}}$

Use an appropriate identity to find the exact value of the expression.

69. $\sin\left(\frac{11\pi}{12}\right) \Rightarrow \sin(165°) = \sin(120+45) = \sin 120 \cos 45 + \cos 120 \sin 45$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

70. $\cos\left(\frac{\pi}{12}\right) \Rightarrow \cos(15^\circ) = \cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \Rightarrow \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

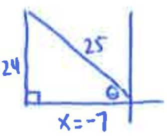
71. $\tan\left(\frac{7\pi}{12}\right) \Rightarrow \tan(105) = \tan(60+45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3}+1}{1-(\sqrt{3})(1)} = \frac{\sqrt{3}+1}{1-\sqrt{3}}\frac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$$= \frac{1+2\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{-2}$$

$$= \boxed{-2-\sqrt{3}}$$

72. Find $\sin(u-v)$ where $\sin u = \frac{24}{25}$ with $\frac{\pi}{2} < u < \pi$ and $\sin v = \frac{5}{13}$ with $0 < v < \frac{\pi}{2}$.

Δ for u:

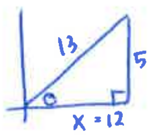


$\sin u = 24/25$

$\cos u = -7/25$

$\tan u = -24/7$

Δ for v:



$\sin v = 5/13$

$\cos v = 12/13$

$\tan v = 5/12$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$$= \left(\frac{24}{25}\right)\left(\frac{12}{13}\right) - \left(\frac{-7}{25}\right)\left(\frac{5}{13}\right)$$

$$= \frac{288}{325} + \frac{35}{325}$$

$$= \boxed{\frac{323}{325}}$$

73. Solve $\cos(x+\pi) - \cos x - 1 = 0$ on the interval $[0, 2\pi)$.

$\cos x \cos \pi - \sin x \sin \pi - \cos x - 1 = 0$

$-\cos x - (0)\cancel{\sin x} - \cos x - 1 = 0$

$-2\cos x - 1 = 0$

$-2\cos x = 1$

$\cos x = -\frac{1}{2} \Rightarrow \boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

74. Solve $\sin\left(x+\frac{\pi}{3}\right) + \sin\left(x-\frac{\pi}{3}\right) = 1$ on the interval $[0, 2\pi)$.

on separate page

Solve each system of equations using either substitution or elimination.

75. $\begin{cases} x - y = 6 \\ -2x + 2y = 1 \end{cases}$

$\Rightarrow x = 6 + y$

$\Rightarrow -2(6+y) + 2y = 1$

$-12 - \cancel{2y} + \cancel{2y} = 1$

$-12 = 1 \leftarrow$ $\boxed{\text{no solution}}$

76. $\begin{cases} 4(5x + 3y = 9) \\ 3(2x - 4y = 14) \end{cases}$

$\Rightarrow$

$20x + \cancel{12y} = 36$

$6x - \cancel{12y} = 42$

$26x = 78$

$x = 3$

$5(3) + 3y = 9$

$15 + 3y = 9$

$3y = -6$

$y = -2$

$\boxed{(3,-2)}$

77. $\begin{cases} 2x + y = 1 \\ 3x + 4y = 14 \end{cases}$

$\Rightarrow y = 1 - 2x$

$\Rightarrow 3x + 4(1-2x) = 14$

$3x + 4 - 8x = 14$

$-5x + 4 = 14$

$-5x = 10$

$x = -2$

$y = 1 - 2(-2)$

$y = 1 + 4$

$y = 5$

$\boxed{(-2,5)}$

78. $\begin{cases} 5(2x + 5y = -1) \\ -10x - 25y = 5 \end{cases}$

$\Rightarrow$

$\cancel{10x} + \cancel{25y} = \cancel{-5}$

$\cancel{-10x} - \cancel{25y} = \cancel{5}$

$0 = 0$

$\boxed{\text{Infinitely many solutions}}$

*Eliminate y

79. $\begin{cases} ① \; 3x + 2y + 4z = 11 \\ ② \; 2x - y + 3z = 4 \\ ③ \; 5x - 3y + 5z = -1 \end{cases}$

① and ②

$3x + 2y + 4z = 11$

$2(2x - y + 3z = 4)$

$\Rightarrow$

$3x + \cancel{2y} + 4z = 11$

$4x - \cancel{2y} + 6z = 8$

$7x + 10z = 19$

② and ③

$-3(2x - y + 3z = 4)$

$5x - 3y + 5z = -1$

$\Rightarrow$

$-6x + \cancel{3y} - 9z = -12$

$5x - \cancel{3y} + 5z = -1$

$-x - 4z = -13$

$7x + 10z = 19$

$(-x - 4z = -13)7$

$\Rightarrow$

$\cancel{7x} + 10z = 19$

$- \cancel{7x} - 28z = -91$

$-18z = -72$

$z = 4$

$7x + 10(4) = 19$

$7x + 40 = 19$

$7x = -21$

$x = -3$

$2(-3) - y + 3(4) = 4$

$-6 - y + 12 = 4$

$-y + 6 = 4$

$y = -2$

$y = 2$

$\boxed{(-3,2,4)}$

$x=-8+6y+2z$

80. $\begin{cases} ① \; x-6y-2z=-8 \\ ② \; -x+5y+3z=2 \\ ③ \; 3x-2y-4z=18 \end{cases}$

Sub into ②:
$-(-8+6y+2z)+5y+3z=2$
$8-6y-2z+5y+3z=2$
$-y+z=-6$

Sub into ③:
$3(-8+6y+2z)-2y-4z=18$
$-24+18y+6z-2y-4z=18$
$16y+2z=42$

$(-y+z=-6)\times 16$
$16y+2z=42$

$\Rightarrow$ $-\cancel{16y}+16z=-96$
$\cancel{16y}+2z=42$
$18z=-54$
$z=-3$

$-y-3=-6$
$-y=-3$
$y=3$

$x=-8+6(3)+2(-3)$
$x=-8+18-6$
$x=4$

$\boxed{(4,3,-3)}$

81. The perimeter of a triangle is 180 feet. The longest side of the triangle is 9 feet shorter than twice the shortest side. The sum of the lengths of the two shorter sides is 30 feet more than the length of the longest side. Find the lengths of the sides of the triangle.

on separate paper

Write the standard form of the equation of the circle with the given information for #82-84.

82. Center $(\overset{h}{4}, \overset{k}{-7})$, radius 13

$(x-4)^2+(y--7)^2=13^2$

$\boxed{(x-4)^2+(y+7)^2=169}$

83. The center is $(\overset{h}{2}, \overset{k}{4})$, and a point on the circle is $(\overset{x}{-3}, \overset{y}{16})$.

$(x-h)^2+(y-k)^2=r^2$
$(-3-2)^2+(16-4)^2=r^2$
$(-5)^2+(12)^2=r^2$
$25+144=r^2$
$169=r^2$
$r=13$

center $(2,4)$ radius 13

$\boxed{(x-2)^2+(y-4)^2=169}$

84. A circle is tangent to the y-axis at $y=3$ and has an x-intercept at x = 1.

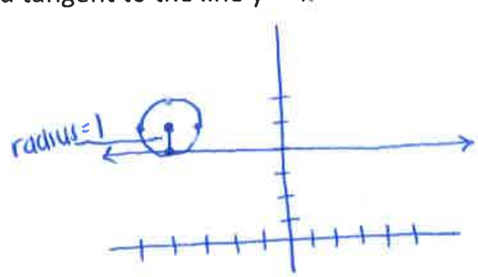
on separate paper

85. Write the standard equation of the circle with center at (-4, 5) and tangent to the line y = 4.

center: $(-4,5)$ radius: 1

$(x--4)^2+(y-5)^2=1^2$

$\boxed{(x+4)^2+(y-5)^2=1}$



86. Find the center and foci of the ellipse.

$$\frac{(x-\overset{h}{1})^2}{\underset{\uparrow \atop b^2}{28}}+\frac{(y-\overset{k}{5})^2}{\underset{\uparrow \atop a^2}{64}}=1$$

center: (1,5)

$c^2=a^2-b^2$

$c^2=64-28$

$c^2=36$

$c=6$

A) center: (1, 5) foci: (–7, –5), (5, –5)

~~B)~~ center: (–1, –5) foci: (–1, –11), (–1, 1)

~~C)~~ center: (–1, –5) foci: (–7, 5), (5, 5)

D) center: (1, 5) foci: (1, –1), (1, 11)

E) center: (1, 5) foci: (–5, 5), (7, 5)

87. Find the standard form of the equation of the ellipse with the following characteristics.

foci: (±8,0) ; major axis of length: 20 ⇒ $2a=20$ ⇒ $a=10$

↳ horizontal major axis

$c=8$

$c^2=a^2-b^2$

$64=100-b^2$

$-36=-b^2$

$36=b^2$

$b=6$

~~A)~~ $\frac{x^2}{400}+\frac{y^2}{64}=1$

~~B)~~ $\frac{x^2}{400}+\frac{y^2}{336}=1$

C) $\frac{x^2}{100}+\frac{y^2}{64}=1$

~~D)~~ $\frac{x^2}{64}+\frac{y^2}{100}=1$

E) $\frac{x^2}{100}+\frac{y^2}{36}=1$