Pre-Calculus A
Chapter 4 (Part II) Test Review

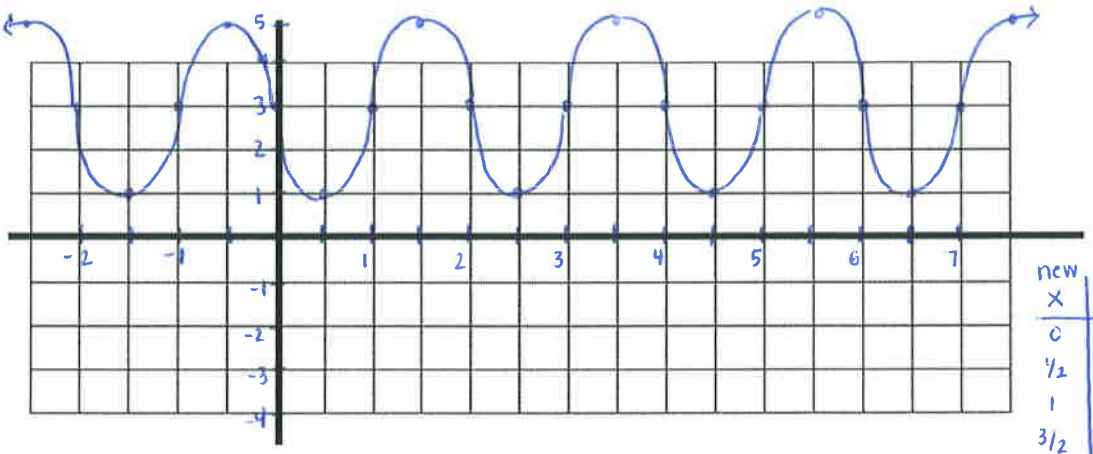
Name : Key
Date : _____ Period : ____

Graphing Practice

Graph each of the following trigonometric functions. Be sure to include at least two full periods and identify all of the key elements (amplitude, period, vertical shifts, phase shifts, asymptotes, x-intercepts, etc.)

$y = -2\sin(\pi x) + 3$

1. $y = 3 - 2\sin(\pi x)$



$\pi x = 0 \Rightarrow x = 0$

$\pi x = \frac{\pi}{2} \Rightarrow x = 1/2$

$\pi x = \pi \Rightarrow x = 1$

$\pi x = \frac{3\pi}{2} \Rightarrow x = 3/2$

$\pi x = 2\pi \Rightarrow x = 2$

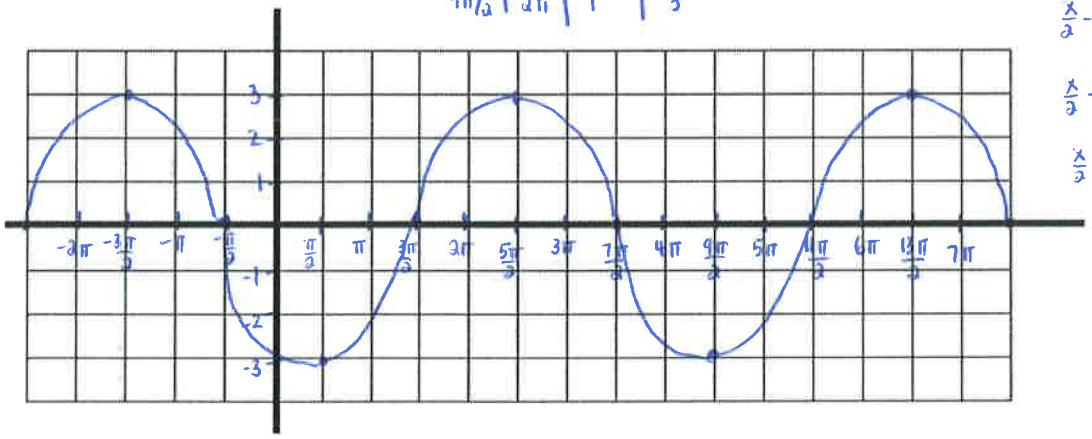
new x	x	y	-2y	+3
0	0	0	0	3
1/2	π/2	1	-2	1
1	π	0	0	3
3/2	3π/2	-1	2	5
2	2π	0	0	3

amplitude = 2 vertical shift: up 3 units domain: $(-\infty, \infty)$

period: $\frac{2\pi}{\pi} = 2$ no phase shift range: $[1, 5]$

2. $y = -3\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

new x	x	y	-3y
π/2	0	1	-3
3π/2	π/2	0	0
5π/2	π	-1	3
7π/2	3π/2	0	0
9π/2	2π	1	-3



$\frac{x}{2} - \frac{\pi}{4} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{2}$

$\frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \frac{x}{2} = \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{2}$

$\frac{x}{2} - \frac{\pi}{4} = \pi \Rightarrow \frac{x}{2} = \frac{5\pi}{4} \Rightarrow x = \frac{5\pi}{2}$

$\frac{x}{2} - \frac{\pi}{4} = \frac{3\pi}{2} \Rightarrow \frac{x}{2} = \frac{7\pi}{4} \Rightarrow x = \frac{7\pi}{2}$

$\frac{x}{2} - \frac{\pi}{4} = 2\pi \Rightarrow \frac{x}{2} = \frac{9\pi}{4} \Rightarrow x = \frac{9\pi}{2}$

amplitude = 3 no vertical shift domain: $(-\infty, \infty)$

period: $\frac{2\pi}{b} = \frac{2\pi}{1/2} = \frac{2\pi}{1} \div \frac{1}{2} = \frac{2\pi}{1} \cdot \frac{1}{2} = \pi$ phase shift: $\frac{c}{b} = \frac{\frac{\pi}{4}}{1/2} = \frac{\pi}{4} \div \frac{1}{2} = \frac{\pi}{4} \cdot \frac{2}{1} = \frac{2\pi}{4} = \frac{\pi}{2}$ Right range: $[-3, 3]$

3. $y = \frac{1}{4}\tan\left(x - \frac{\pi}{2}\right)$ — increasing

Consec Asymptotes:

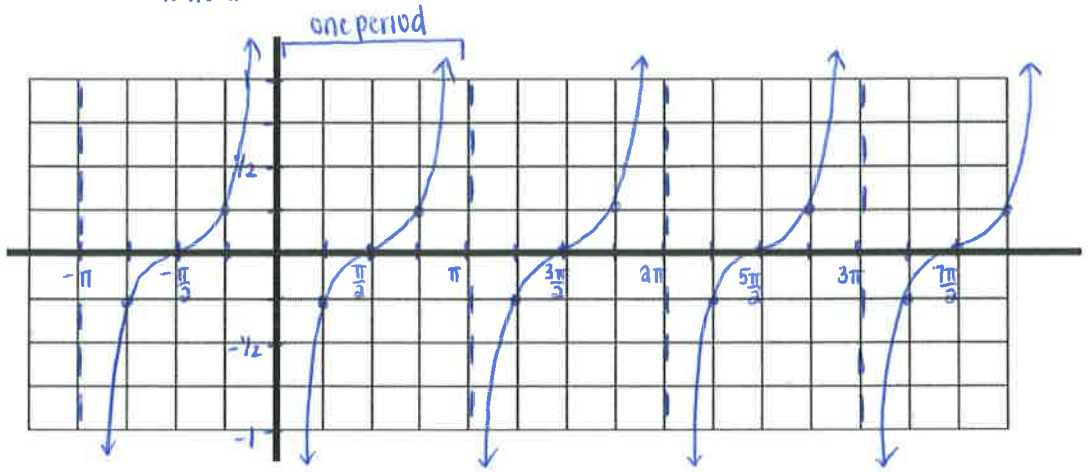
$x - \frac{\pi}{2} = -\frac{\pi}{2}$ $\quad$ $x - \frac{\pi}{2} = \frac{\pi}{2}$

$x = 0$ $\quad$ $x = \pi$

1st Q point: $-1 \times \frac{1}{4} = -\frac{1}{4}$

2nd Q point: $1 \times \frac{1}{4} = \frac{1}{4}$

* midpoint is at zero since there is no d-value



period = $\pi - 0 = \pi$

Range: $(-\infty, \infty)$

Asymptotes: $0, \pi, 2\pi, 3\pi$, etc.

4. $y = 4\cot(\pi x)$ — decreasing

Consec Asymptotes:

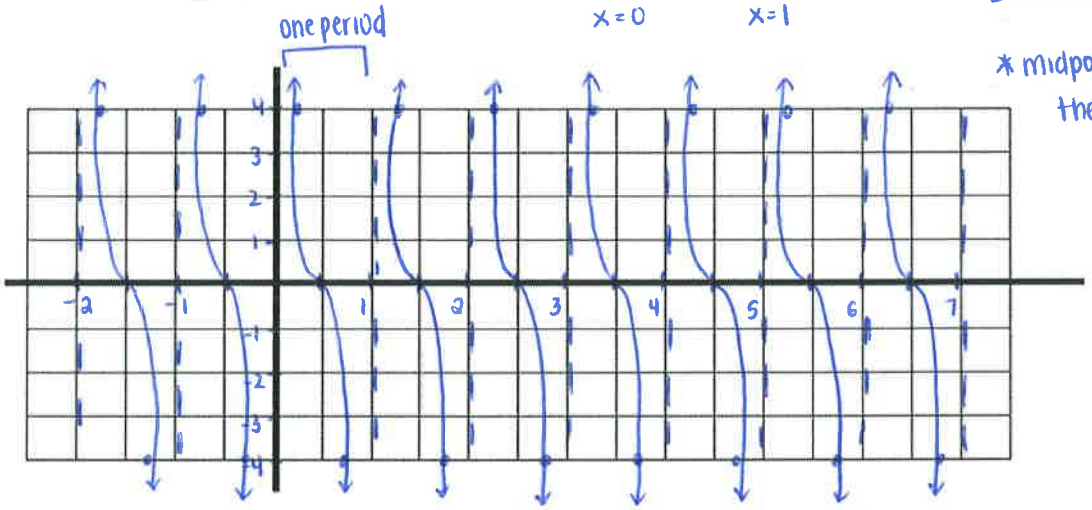
$\pi x = 0$ $\quad$ $\pi x = \pi$

$x = 0$ $\quad$ $x = 1$

1st Q point: $1 \times 4 = 4$

2nd Q point: $-1 \times 4 = -4$

* midpoint is at zero since there is no d-value



period = $1 - 0 = 1$

Range: $(-\infty, \infty)$

Asymptotes: $0, 1, 2, 3, 4$, etc.

5. $y = 2\csc\left(x - \frac{\pi}{4}\right)$ ↱ sine

new x	x	y	2y
asymp. → $\pi/4$	0	0	0
$3\pi/4$	$\pi/2$	1	2
asymp. → $5\pi/4$	π	0	0
$7\pi/4$	$3\pi/2$	-1	-2
asymp. → $9\pi/4$	2π	0	0

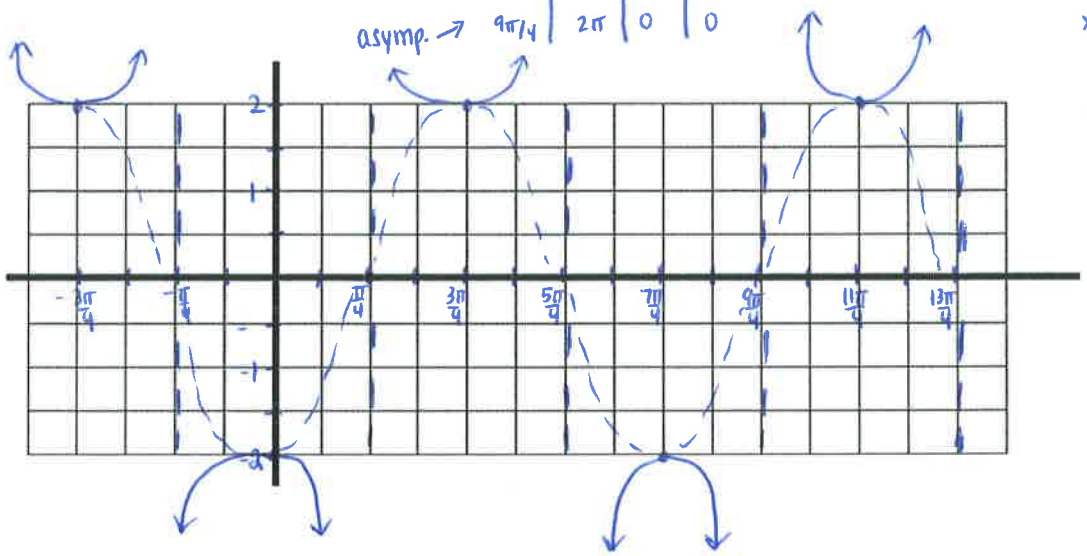
$x - \frac{\pi}{4} = 0 \Rightarrow x = \frac{\pi}{4}$

$x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{4}$

$x - \frac{\pi}{4} = \pi \Rightarrow x = \frac{5\pi}{4}$

$x - \frac{\pi}{4} = \frac{3\pi}{2} \Rightarrow x = \frac{7\pi}{4}$

$x - \frac{\pi}{4} = 2\pi \Rightarrow x = \frac{9\pi}{4}$



period = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

6. $y = -\frac{1}{2}\sec(x - \pi)$ ↱ cosine

$x - \pi = 0$

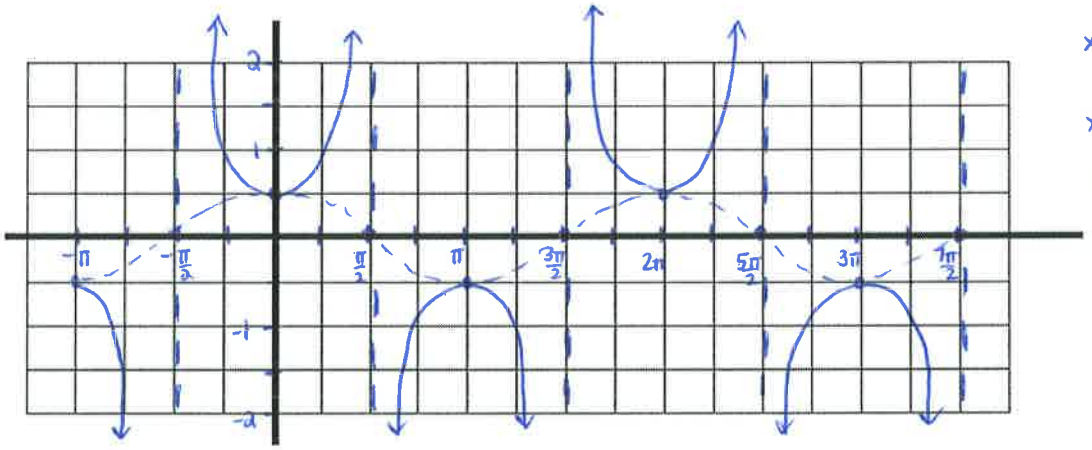
$x = \pi$

$x - \pi = \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{2}$

$x - \pi = \pi \Rightarrow x = 2\pi$

$x - \pi = \frac{3\pi}{2} \Rightarrow x = \frac{5\pi}{2}$

$x - \pi = 2\pi \Rightarrow x = 3\pi$



period = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

new x	x	y	$-\frac{1}{2}y$
π	0	1	$-1/2$
asymp. → $3\pi/2$	$\pi/2$	0	0
2π	π	-1	$1/2$
asymp. → $5\pi/2$	$3\pi/2$	0	0
3π	2π	1	$-1/2$

Inverse Trigonometric Functions Practice

Find the exact value of each expression in radians.

7. $\arcsin(-1)$ = $\boxed{-\frac{\pi}{2}}$
8. $\arccos(4)$ ⇒ no solution; 4 is outside the domain of cosine
9. $\arcsin\left(-\frac{1}{2}\right)$ = $\boxed{-\frac{\pi}{6}}$
10. $\csc^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$ ⇒ $\sin^{-1}\left(-\frac{3}{2\sqrt{3}}\right) = \sin^{-1}\left(-\frac{3\sqrt{3}}{6}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$
11. $\sec^{-1}\left(\sqrt{2}\right)$ ⇒ $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ ⇒ $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$
12. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ = $\boxed{\frac{5\pi}{6}}$
13. $\tan^{-1}\left(-\sqrt{3}\right)$ = $\boxed{-\frac{\pi}{3}}$
14. Find the exact value of $\sin(\arccos 1)$

$\sin(0) = \boxed{0}$

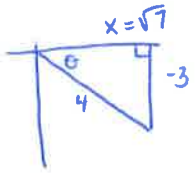
15. Find the exact value of $\arccos\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$\arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

16. Find the exact value of $\sin(\arctan(-1))$

$\sin\left(-\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

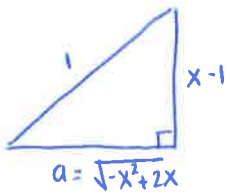
17. Find the exact value of $\tan\left(\arcsin\left(-\frac{3}{4}\right)\right)$ ← Quad IV



$x^2+(-3)^2=4^2$
$x^2+9=16$
$x^2=7$
$x=\sqrt{7}$

$\tan(\Delta) = \frac{opp}{adj} = \frac{-3}{\sqrt{7}} = \boxed{\frac{-3\sqrt{7}}{7}}$

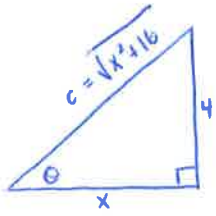
18. Find the exact value of $\sec\left(\arcsin(x-1)\right)$ opp $x-1$, hyp 1



$a^2+(x-1)^2=1^2$
$a^2+x^2-2x+1=1$
$a^2=1-x^2+2x-1$
$a^2=-x^2+2x$
$a=\sqrt{-x^2+2x}$

$\sec(\Delta) = \frac{hyp}{adj} = \frac{1}{\sqrt{-x^2+2x}} = \boxed{\frac{\sqrt{-x^2+2x}}{-x^2+2x}}$

19. Find the exact value of $\sec\left(\cot^{-1}\left(\frac{x}{4}\right)\right)$ adj x, opp 4

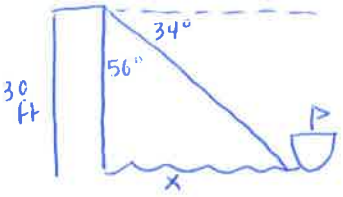


$x^2+4^2=c^2$
$x^2+16=c^2$
$c=\sqrt{x^2+16}$

$\sec(\Delta) = \frac{hyp}{adj} = \boxed{\frac{\sqrt{x^2+16}}{x}}$

Applications and Models Practice

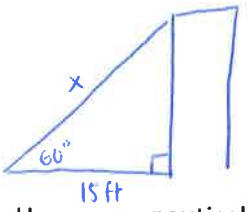
20. You are standing on the top of a cliff that is 30 feet above the ocean. You see a sailboat down in the water below. The angle of depression that you spot the sailboat at is 34°. How far from the base of the cliff is the boat?



$\frac{\tan 56}{1} = \frac{x}{30}$

$x = 30 \tan 56 \Rightarrow$ **$x \approx 44.5$ ft away**

21. A damsel is in distress and is being held captive in a tower. Her knight in shining armor is on the ground below with a ladder. When the knight stands 15 feet from the base of the tower and looks up at his precious damsel, the angle of elevation to her window is 60°. How long does the ladder have to be in order for the knight to rescue her?



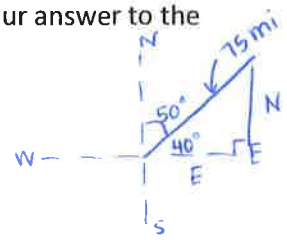
$\frac{\cos 60}{1} = \frac{15}{x}$

$15 = x \cos 60$

$x = \frac{15}{\cos 60}$

$x = 30$ ft

22. A ship travels on a $N\,50°\,E$ course. The ship travels for 3 hours at 25 knots. How many nautical miles north and how many nautical miles east has the ship traveled? Round your answer to the nearest tenth of a mile.



$3 \text{ hrs} \times 25 = 75 \text{ miles}$

$\frac{\sin 40}{1} = \frac{N}{75}$

$N = 75 \sin 40$

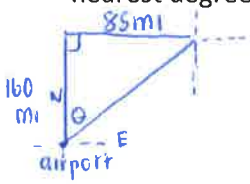
$N = 48.2$ mi

$\frac{\cos 40}{1} = \frac{E}{75}$

$E = 75 \cos 40$

$E = 57.5$ mi

23. A plane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What is the bearing from the airport to the plane's current position? Round to the nearest degree.

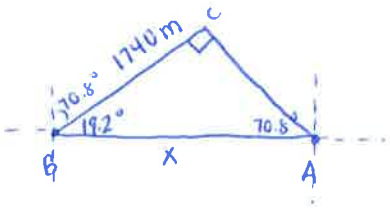


$\tan\theta = \frac{85}{160}$

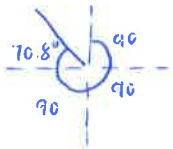
$\theta = \tan^{-1}\left(\frac{85}{160}\right) = 28°$

N 28° E

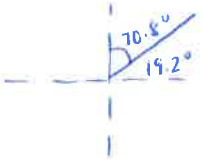
24. Two lighthouses are located on an east-west line; lighthouse A is east of lighthouse B. From lighthouse A, the bearing of a ship is 340.8°. From lighthouse B, the bearing of a ship 1,740 meters away is $N\,70°\,48'\,E$. Find the distance between the lighthouses. Round to the nearest tenth of a meter.

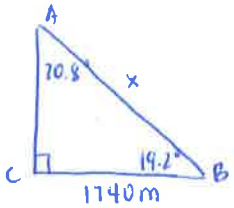


Bearing A:



Bearing B: N 70.8° E





$\frac{\cos 19.2}{1} = \frac{1740}{x}$

$\frac{1740}{\cos 19.2} = \frac{x \cos 19.2}{\cos 19.2}$

$x = \frac{1740}{\cos 19.2} \approx$

1842.5m

25. Write a sinusoidal model for each of the following graphs:

amp: $\frac{1}{2}(\max - \min) = \frac{1}{2}(9 - -3) = \frac{1}{2}(12) = 6$

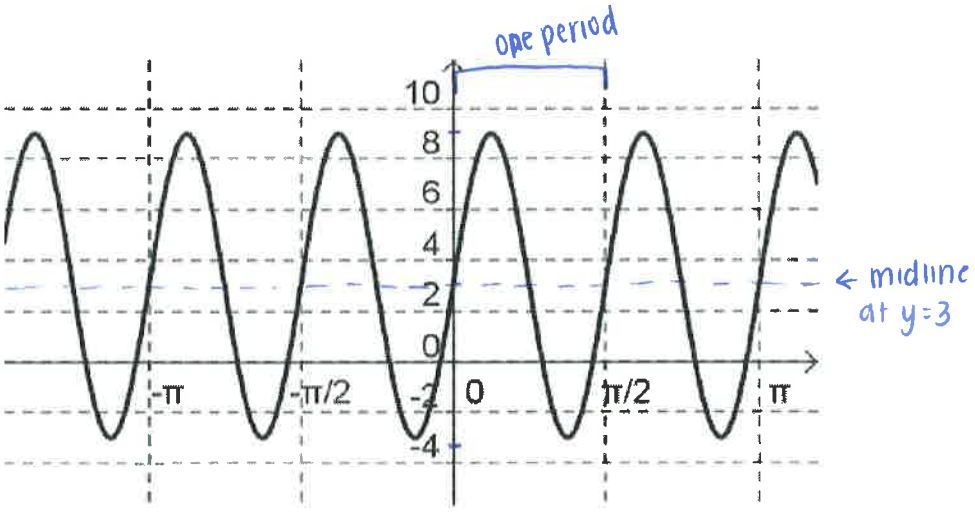
d = 3

period = $\frac{\pi}{2}$

$\frac{2\pi}{b} = \frac{\pi}{2} \Rightarrow \frac{4\pi}{\pi} = \frac{\pi b}{\pi} \Rightarrow b = 4$

no phase shift

$y = 6\sin(4x) + 3$



amp: $\frac{1}{2}(4 - -4) = \frac{1}{2}(8) = 4$

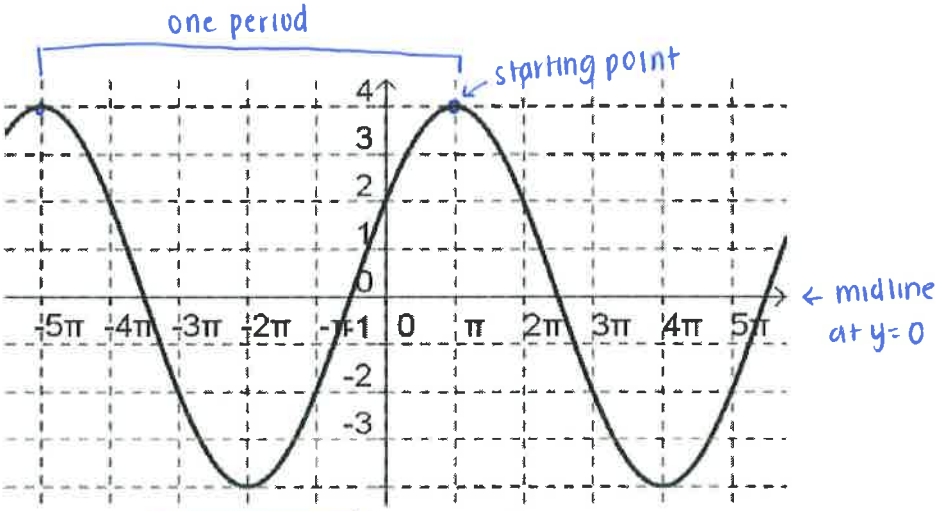
d = 0

period = $\pi - -5\pi = \pi + 5\pi = 6\pi$

$\frac{2\pi}{b} = \frac{6\pi}{1} \Rightarrow \frac{2\pi}{6\pi} = \frac{6\pi b}{6\pi} \Rightarrow b = \frac{2}{6} \Rightarrow b = \frac{1}{3}$

P.S: π units right

$\frac{c}{b} = \frac{\pi}{1} \Rightarrow \frac{c}{\frac{1}{3}} = \frac{\pi}{1} \Rightarrow c = \frac{1}{3}\pi$ or $\frac{\pi}{3}$



$y = 4\cos\left(\frac{1}{3}x - \frac{\pi}{3}\right)$

Answers:

#1-6 : See following page 7. $-\frac{\pi}{2}$ 8. No Solution 9. $-\frac{\pi}{6}$ 10. $-\frac{\pi}{3}$ 11. $\frac{\pi}{4}$ 12. $\frac{5\pi}{6}$

13. $-\frac{\pi}{3}$ 14. 0 15. $\frac{2\pi}{3}$ 16. $-\frac{\sqrt{2}}{2}$ 17. $-\frac{3\sqrt{7}}{7}$ 18. $\frac{\sqrt{-x^2+2x}}{-x^2+2x}$ 19. $\frac{\sqrt{x^2+16}}{x}$

20. 44.5 ft 21. 30 ft 22. N: 48.2 mi, E: 57.5 mi 23. N 28° E 24. 1842.5 meters

25. $y = 6\sin(4x) + 3$

$y = 4\cos\frac{1}{3}(x - \pi)$