Pre-Calculus A
Chapter 3 Review

Name : Key
Date : ______________ Period : __________

Convert each exponential equation to logarithmic form.

1. $4^5 = 1024$

$\log_4 1024 = 5$

2. $e^{2.708} = 15$

$\ln 15 = 2.708$

Convert each logarithmic equation to exponential form.

3. $\log_7 \frac{1}{49} = -2$

$7^{-2} = \frac{1}{49}$

4. $\log 0.0001 = -4$

$10^{-4} = 0.0001$

5. $\ln 5 = 1.6094$

$e^{1.6094} = 5$

Find the value of each logarithmic expression.

6. $\log 10 = x$

$10^x = 10 \Rightarrow \boxed{x=1}$

7. $\log_6 \sqrt{6} = x$

$6^x = \sqrt{6} \Rightarrow \boxed{x = 1/2}$

8. $\log_4 \left(\frac{1}{16}\right) = x$

$4^x = \frac{1}{16} \Rightarrow \boxed{x=-2}$

9. $\log_{64} 2 = x$

$64^x = 2 \Rightarrow \boxed{x = 1/6}$

$(2^6)^x = 2^1$

$6x = 1$

$x = 1/6$

10. $\ln(1) = x$

$e^x = 1 \Rightarrow \boxed{x=0}$

Approximate the value of each logarithm using the Change of Base Formula.

11. $\log_7 156$

$\frac{\log 156}{\log 7} \approx \boxed{2.5951}$

12. $\log_4 0.0376$

$\frac{\log 0.0376}{\log 4} \approx \boxed{-2.3666}$

Use $\log_b 2 \approx 0.3868$, $\log_b 3 \approx 0.6131$ and $\log_b 7 \approx 1.086$ to approximate the value of the expression.

13. $\log_b 18 = \log_b(3\cdot 3\cdot 2)$

$= \log_b 3 + \log_b 3 + \log_b 2$

$= 0.6131 + 0.6131 + 0.3868$

$= \boxed{1.613}$

14. $\log_b 4\sqrt{7} = \log_b(2\cdot 2\cdot 7^{1/2})$

$= \log_b 2 + \log_b 2 + \log_b 7^{1/2}$

$= \log_b 2 + \log_b 2 + \frac{1}{2}\log_b 7$

$= 0.3868 + 0.3868 + \frac{1}{2}(1.086)$

$= \boxed{1.3166}$

Expand each logarithmic expression.

15. $\ln\left(\frac{8}{6x}\right)$

$= \ln 8 - (\ln 6x)$

$= \ln 8 - (\ln 6 + \ln x)$

$= \boxed{\ln 8 - \ln 6 - \ln x}$

16. $\log_7\left(\frac{6x^2y}{t^3z}\right)$

$= \log_7 6x^2y - (\log_7 t^3z)$

$= \log_7 6 + \log_7 x^2 + \log_7 y - (\log_7 t^3 + \log_7 z)$

$= \boxed{\log_7 6 + 2\log_7 x + \log_7 y - 3\log_7 t - \log_7 z}$

17. $\log_3\left(\frac{3\sqrt{x}}{y^2}\right)$

$= \log_3 3\sqrt{x} - \log_3 y^2$

$= \log_3 3 + \log_3 \sqrt{x} - \log_3 y^2$

$= 1 + \log_3 x^{1/2} - \log_3 y^2$

$= \boxed{1 + \frac{1}{2}\log_3 x - 2\log_3 y}$

18. $\ln(16x^2\sqrt[5]{y^2z^3})$

$= \ln 16 + \ln x^2 + \ln \sqrt[5]{y^2z^3}$

$= \ln 16 + 2\ln x + \ln (y^2z^3)^{1/5}$

$= \ln 16 + 2\ln x + \ln y^{2/5} + \ln z^{3/5}$

$= \boxed{\ln 16 + 2\ln x + \frac{2}{5}\ln y + \frac{3}{5}\ln z}$

Condense each logarithmic expression.

19. $\log_3 5 + \log_3 7 + \frac{1}{2}\log_3 36$

$= \log_3 5 + \log_3 7 + \log_3 36^{1/2}$

$= \log_3 5 + \log_3 7 + \log_3 \sqrt{36}$

$= \log_3 5 + \log_3 7 + \log_3 6$

$= \log_3(5 \cdot 7 \cdot 6) = \boxed{\log_3 210}$

20. $3(\log_2 24 - \log_2 8)$

$= 3(\log_2 \frac{24}{8})$

$= 3(\log_2 3)$

$= \log_2 3^3$

$= \boxed{\log_2 27}$

21. $3\ln x + 0.5(\ln 36 - 2\ln 2)$

$= \ln x^3 + 0.5(\ln 36 - \ln 2^2)$

$= \ln x^2 + 0.5(\ln 36 - \ln 4)$

$= \ln x^2 + 0.5(\ln \frac{36}{4})$

$= \ln x^2 + 0.5(\ln 9)$

$= \ln x^2 + \ln 9^{1/2}$

$= \ln x^2 + \ln 3 = \boxed{\ln 3x^2}$

22. $\frac{1}{3}\log_4 x - \left(2\log_4 y + \frac{1}{2}\log_4 z\right)$

$= \log_4 x^{1/3} - (\log_4 y^2 + \log_4 z^{1/2})$

$= \log_4 \sqrt[3]{x} - (\log_4 y^2 + \log_4 \sqrt{z})$

$= \log_4 \sqrt[3]{x} - (\log_4 y^2\sqrt{z})$

$= \boxed{\log_4 \frac{\sqrt[3]{x}}{y^2\sqrt{z}}}$

Solve each equation. Round solutions to 4 decimal places. Be sure to check for extraneous solutions.

23. $16^{4x-2} = \left(\frac{1}{64}\right)^{2x-1}$

$(4^2)^{4x-2} = (4^{-3})^{2x-1}$

$4^{8x-4} = 4^{-6x+3}$

$8x-4 = -6x+3 \Rightarrow 14x-4=3$

$14x = 7 \Rightarrow \boxed{x = 1/2}$

24. $3(4^{3x+5}) - 2 = 17$

$3(4^{3x+5}) = 19$

$4^{3x+5} = \frac{19}{3}$

$\cancel{\log_4 4}^{3x+5} = \log_4 \frac{19}{3}$

$3x+5 = \log_4 \frac{19}{3}$

$3x = \log_4 \frac{19}{3} - 5$

$x = \frac{\log_4 19/3 - 5}{3}$

$\boxed{x \approx -1.223}$

25. $4^{x+2} - 5 = 3$

$4^{x+2} = 8$

$(2^2)^{x+2} = 2^3$

$2^{2x+4} = 2^3$

$2x+4 = 3 \Rightarrow 2x = -1 \Rightarrow \boxed{x = -1/2}$

26. $\frac{10}{1+e^{-x}} = \frac{2}{1}$

$10 = 2(1+e^{-x})$

$5 = 1+e^{-x}$

$4 = e^{-x}$

$\ln 4 = \cancel{\ln e}^{-x}$

$\ln 4 = -x$

$x = -\ln 4$

$\boxed{x = -1.3863}$

27. $e^{x-6} = 14$

$\cancel{\ln e}^{x-6} = \ln 14$

$x-6 = \ln 14$

$x = \ln 14 + 6$

$\boxed{x = 8.6391}$

28. $\log(x) = 1 - \log(x-3)$

$\log x + \log(x-3) = 1$

$\log x(x-3) = 1$

$\log x^2 - 3x = 1$

$10^1 = x^2 - 3x$

$10 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$\boxed{x=5}$ $\cancel{x=-2}$

29. $2 + 3\log_2(5x-1) = 20$

$3\log_2(5x-1) = 18$

$\log_2(5x-1) = 6$

$2^6 = 5x-1$

$64 = 5x-1 \Rightarrow 65 = 5x \Rightarrow \boxed{x = 13}$

30. $2\ln(3x+2) = 14$

$\ln(3x+2) = 7$

$e^7 = 3x+2$

$e^7 - 2 = 3x$

$x = \frac{e^7-2}{3} \Rightarrow \boxed{x \approx 364.8777}$

31. $3(\log_6(4x+1)) + 2 = 11$

$3(\log_6(4x+1)) = 9$

$\log_6(4x+1) = 3$

$6^3 = 4x+1$

$216 = 4x+1$

$215 = 4x \Rightarrow \boxed{x = 53.75}$

32. $\log_6 3 + \log_6 x = 2$

$\log_6 3x = 2$

$6^2 = 3x$

$36 = 3x$

$\boxed{x = 12}$

33. $\log_4(2x+1) = \log_4(x+2) - \log_4 3$

$\log_4(2x+1) = \log_4 \frac{x+2}{3}$

$\frac{2x+1}{1} = \frac{x+2}{3}$

$x+2 = 3(2x+1)$

$x+2 = 6x+3$

$2 = 5x+3 \Rightarrow -1 = 5x \Rightarrow \boxed{x = -1/5}$

34. $e^{-2x^2} = e^{-x^2+4x-12}$

$-2x^2 = -x^2+4x-12$

$0 = x^2+4x-12$

$0 = (x+6)(x-2)$

$\boxed{x=-6,\ x=2}$

35. Find the amount of time (in years) that it would take for a deposit of $1,000 to grow to $1 million at 14% compounded continuously. Round to the nearest tenth of a year.

$1{,}000{,}000 = 1000e^{.14t}$

$1000 = e^{.14t}$

$\ln(1000) = .14t$

$t = \frac{\ln(1000)}{.14} \Rightarrow \boxed{t = 49.3 \text{ years}}$

36. At what interest rate would a deposit of $30,000 grow to $2,540,689 in 40 years compounded continuously? Round to the nearest tenth.

$2540689 = 30000e^{40r}$

$\frac{2540689}{30000} = e^{40r}$

$\ln\left(\frac{2540689}{30000}\right) = 40r$

$r = \frac{\ln\left(\frac{2540689}{30000}\right)}{40} \Rightarrow r \approx .1109 \Rightarrow \boxed{11.1\%}$

37. The projected population of California for the years 2015 to 2030 can be modeled by the function $P = 34.706e^{0.0097t}$, where P is the population (in millions) and t is the time (in years), with t = 15 corresponding to 2015. Determine in which year the population of California will exceed 50 million.

$50 = 34.706e^{.0097t}$

$\frac{50}{34.706} = e^{.0097t}$

$\ln\left(\frac{50}{34.706}\right) = .0097t$

$t = \frac{\ln\left(\frac{50}{34.706}\right)}{.0097}$

$t = 37.6$

$t = 0: 2000$

$t = 15: 2015$

$t = 37: \boxed{2037}$

38. The speed of the wind S (in miles per hour) near the center of a tornado and the distance d (in miles) the tornado travels are related by the model $S = 93\log d + 65$. On March 18, 1925, a large tornado struck portions of Missouri, Illinois, and Indiana with a wind speed at the center of about 283 miles per hour. Approximate the distance traveled by this tornado to the nearest tenth.

$283 = 93\log d + 65$

$218 = 93\log d$

$\frac{218}{93} = \log d$

$10^{218/93} = d \Rightarrow \boxed{d \approx 220.8 \text{ miles}}$

39. How long will it take a sample of radioactive substance to decay to half of its of its original amount, if it decays according to the function $A(t) = 500e^{-0.000436t}$ where t is the time in years? Round to the nearest hundredth.

$250 = 500e^{-0.000436t}$

$0.5 = e^{-0.000436t}$

$\ln 0.5 = -0.000436t$

$t = \frac{\ln 0.5}{-0.000436} \Rightarrow \boxed{t = 1589.79 \text{ years}}$

$P_0 = 100,000$

$P = 500,000$ when $t = 20$

40. The population of Phoenix, Arizona has grown exponentially since 1950 according to the equation $P = P_0 e^{kt}$. In 1950 (t = 0), the population was 100,000 and in 1970 (t = 20) it was 500,000.

a) How many people lived in Phoenix in 1980 (t = 30) ?

$500,000 = 100,000 e^{k(20)}$

$5 = e^{20k}$

$\ln(5) = 20k$

$k = \frac{\ln 5}{20} \Rightarrow k \approx 0.0805$

$P = 100,000 e^{.0805(30)}$

$\boxed{P = 1,189,977}$

b) During what year will the population of Phoenix reach 5,000,000 assuming growth continues exponentially?

$5000000 = 100000 e^{.0805t}$

$50 = e^{.0805t}$

$\ln(50) = .0805t$

$t = \frac{\ln 50}{.0805} \Rightarrow t = 48.5$

$t = 0$: 1950

$t = 30$: 1980

$t = 48.5$: $\boxed{1998}$

Answer Key :

1) $\log_4 1024 = 5$
2) $\ln 15 = 2.708$
3) $7^{-2} = \frac{1}{49}$
4) $10^{-4} = 0.001$
5) $e^{1.6094} = 5$
6) 1
7) $\frac{1}{2}$
8) -2
9) $\frac{1}{6}$
10) 0
11) 2.5951
12) -2.3666
13) 1.613
14) 1.3166
15) $\ln 8 - (\ln 6 + \ln x)$
16) $\log_7 6 + 2\log_7 x + \log_7 y - (3\log_7 t + \log_7 z)$
17) $1 + \frac{1}{2}\log_3 x - 2\log_3 y$
18) $\ln 16 + 2\ln x + \frac{2}{5}\ln y + \frac{3}{5}\ln z$
19) $\log_3 210$
20) $\log_2 27$
21) $\ln 3x^3$
22) $\log_4 \frac{\sqrt[3]{x}}{y^2\sqrt{z}}$
23) $x = \frac{1}{2}$
24) $x \approx 0.1105$
25) $x = -\frac{1}{2}$
26) $x \approx -1.3863$
27) $x \approx 8.6391$
28) $x = 5$
29) $x = 13$
30) $x \approx 364.8777$
31) $x = 53.75$
32) $x = 12$
33) $x = -\frac{1}{5}$
34) $x = -6,\ x = 2$
35) 49.3 years
36) 11.1%
37) 2037
38) 220.8 miles
39) 1589.79 years
40) a. 1,118,033 people b. 1998