

Properties of Exponents : Simplify the expression and write without negative exponents.

$$1. y^9 \cdot y^1$$

$$= y^{9+1}$$

$$= \boxed{y^{10}}$$

$$2. (5pq)^3$$

$$= 5^3 p^3 q^3$$

$$= \boxed{125p^3q^3}$$

$$3. (-8m^4)^2 \cdot m^3$$

$$= (-8)^2 m^{4 \cdot 2} m^3$$

$$= 64 m^8 m^3 = 64 m^{8+3}$$

$$= \boxed{64m^{11}}$$

$$4. (3x^5)^3 (2x^7)^2$$

$$= 3^3 x^{5 \cdot 3} \cdot 2^2 x^{7 \cdot 2}$$

$$= 27 x^{15} \cdot 4 x^{14}$$

$$= 108 x^{15+14} = \boxed{108x^{29}}$$

$$5. (-3x^2y)^3 (11x^3y^5)^2$$

$$= (-3)^3 x^6 y^3 \cdot 11^2 x^6 y^{10}$$

$$= -27 x^{6+6} y^{3+10} \cdot 121$$

$$= \boxed{-3267 x^{12} y^{13}}$$

$$6. \left(\frac{7x^3}{2y^4}\right)^2$$

$$= \frac{7^2 x^6}{2^2 y^8} = \boxed{\frac{49x^6}{4y^8}}$$

$$7. \left(\frac{2f^{-2}g^3}{3fg^{-1}}\right)^4 = \frac{2^4 f^{-8} g^{12}}{3^4 f^4 g^{-4}}$$

$$= \frac{16 f^{-8} g^{12}}{81 f^4 g^{-4}} = \frac{16 g^{12} g^4}{81 f^4 f^8}$$

$$= \boxed{\frac{16g^{16}}{81f^{12}}}$$

$$8. \frac{2s^3t^3}{st^{-2}} \cdot \frac{(3st)^3}{s^{-2}t}$$

$$= \frac{2s^3t^3t^2}{s} \cdot \frac{3^3s^3t^3s^2}{t}$$

$$= \frac{2s^3t^5}{s} \cdot \frac{27s^5t^3}{t}$$

$$= 2s^2t^5 \cdot 27s^5t^2 = \boxed{54s^7t^7}$$

$$9. 2^{-2} \cdot 2^{-3}$$

$$= 2^{-3+2}$$

$$= 2^{-1}$$

$$= \frac{1}{2^1}$$

$$= \boxed{\frac{1}{2}}$$

$$10. 7^{-6} \cdot 7^4$$

$$= 7^{-6+4}$$

$$= 7^{-2}$$

$$= \frac{1}{7^2} = \boxed{\frac{1}{49}}$$

$$11. (2^{-1})^5$$

$$= 2^{-5}$$

$$= \frac{1}{2^5}$$

$$= \boxed{\frac{1}{32}}$$

$$12. \frac{6^{-3}}{6^{-5}} = \frac{6^5}{6^3} = 6^{5-3} = 6^2$$

$$= \boxed{36}$$

$$13. 3^{-2} \cdot \left(\frac{5}{7^0}\right)$$

$$= \frac{1}{3^2} \cdot \frac{5}{1}$$

$$= \frac{1}{9} \cdot \frac{5}{1}$$

$$= \boxed{\frac{5}{9}}$$

$$14. \frac{(3x)^{-3}y^4}{x^2y^{-6}}$$

$$= \frac{(3)^{-3}x^{-3}y^4}{x^2y^{-6}} = \frac{y^4}{3^3x^3x^2y^{-6}}$$

$$= \frac{y^4y^6}{27x^{3+2}}$$

$$= \frac{y^{10}}{27x^5} = \boxed{\frac{y^{10}}{27x^5}}$$

$$15. \frac{12x^8y^{-7}}{(4x^{-2}y^{-6})^2} = \frac{12x^8y^{-7}}{4^2x^{-4}y^{-12}}$$

$$= \frac{12x^8x^4y^{12}}{16y^7} = \frac{12x^{12}y^{12}}{16y^7}$$

$$= \frac{3x^{12}y^{12}}{4y^7}$$

$$= \boxed{\frac{3x^{12}y^5}{4}}$$

Fractional Exponents : Simplify the expression and write without negative or fractional exponents.

$$16. \sqrt{9x^3}$$

$$= (9x^3)^{1/2}$$

$$= 9^{1/2} x^{3/2}$$

$$= \boxed{3\sqrt{x^3}}$$

$$17. x^{3/5} \cdot x^{-1/2}$$

$$= x^{3/5 + (-1/2)}$$

$$= x^{6/10 - 5/10}$$

$$= x^{1/10} = \boxed{10\sqrt{x}}$$

$$18. (16x^3)^{1/2}$$

$$= 16^{1/2} x^{3/2}$$

$$= 4x^{3/2}$$

$$= \boxed{4\sqrt{x^3}}$$

$$19. \frac{15x^{3/2}}{3x^{1/4}}$$

$$= \frac{5x^{3/2}}{x^{1/4}} = 5x^{3/2 - 1/4}$$

$$= 5x^{6/4 - 1/4}$$

$$= 5x^{5/4}$$

$$= \boxed{5\sqrt[4]{x^5}}$$

$$20. \sqrt[4]{25} \cdot \sqrt{5}$$

$$= 4\sqrt[4]{5^2} \cdot \sqrt{5}$$

$$= (5^2)^{1/4} \cdot 5^{1/2}$$

$$= 5^{2/4} \cdot 5^{1/2}$$

$$= 5^{1/2} \cdot 5^{1/2}$$

$$= 5^1 = \boxed{5}$$

$$21. \frac{(64x^4y^4)^{1/2}}{4y^2}$$

$$= \frac{64^{1/2} x^{4/2} y^{4/2}}{4y^2}$$

$$= \frac{(8x^2y^2)}{4y^2} = \frac{2x^2y^2}{y^2} = \boxed{2x^2}$$

$$22. (5^{1/3} \cdot x^{1/4})^3$$

$$= 5^{3/3} \cdot x^{3/4}$$

$$= 5x^{3/4}$$

$$= \boxed{5\sqrt[4]{x^3}}$$

$$23. \sqrt[4]{81x^5y^2z^8}$$

$$= (81x^5y^2z^8)^{1/4}$$

$$= 81^{1/4} x^{5/4} y^{2/4} z^{8/4}$$

$$= 3x^{5/4} y^{1/2} z^2$$

$$= \boxed{3\sqrt[4]{x^5} \sqrt{y} z^2}$$

$$24. (x^{2/5})^{3/4}$$

$$= x^{6/20}$$

$$= x^{3/10}$$

$$= \boxed{10\sqrt{x^3}}$$

$$25. \sqrt{\frac{4x^4}{25y^6}}$$

$$\left(\frac{4x^4}{25y^6}\right)^{1/2} = \frac{4^{1/2} x^{4/2}}{25^{1/2} y^{6/2}}$$

$$= \frac{2x^2}{5y^3}$$

$$= \boxed{\frac{2x^2}{5y^3}}$$

$$26. \sqrt[5]{x^{15}y^5}$$

$$= (x^{15}y^5)^{1/5}$$

$$= x^{15/5} y^{5/5}$$

$$= \boxed{x^3y}$$

$$27. (49x^{-4/3}y^{2/3})^{-3/2}$$

$$= 49^{-3/2} x^{12/6} y^{-6/6}$$

$$= 49^{-3/2} x^2 y^{-1}$$

$$= \frac{x^2}{49^{3/2} y} = \frac{x^2}{\sqrt{49^3} y}$$

$$= \frac{x^2}{\sqrt{117649} y} = \boxed{\frac{x^2}{343y}}$$

Write each of the following logarithmic equations in exponential form.

28. $\log_3 243 = 5$

$$\boxed{3^5 = 243}$$

29. $\log_a N = x$

$$\boxed{a^x = N}$$

30. $\log_{10} 1 = 0$

$$\boxed{10^0 = 1}$$

Find the value of each of the following logarithms. It may help to write in exponential form first.

31. $\log_5 25 = x$

$$5^x = 25$$

$$\boxed{x = 2}$$

since $5^2 = 25$

32. $\log_7 7 = x$

$$7^x = 7$$

$$\boxed{x = 1}$$

since $7^1 = 7$

33. $\log_6 1 = x$

$$6^x = 1$$

$$\boxed{x = 0}$$

since $6^0 = 1$

34. $\log_{27} \sqrt[3]{27} = x$

$$27^x = \sqrt[3]{27}$$

$$\boxed{x = \frac{1}{3}}$$

since $27^{1/3} = \sqrt[3]{27}$

35. $\log_2 \frac{1}{8} = x$

$$2^x = \frac{1}{8}$$

$$\boxed{x = -3}$$

since $2^{-3} = \frac{1}{8}$

36. $\log_{16} 4 = x$

$$16^x = 4$$

$$\boxed{x = \frac{1}{2}}$$

since $16^{1/2} = 4$
or
 $\sqrt{16} = 4$

Using the properties of logarithms, please evaluate each expression.

37. $\log_3 3$ ← inverse prop

$$= \boxed{1}$$

38. $5^{\log_5 7}$ ← inverse

$$= \boxed{7}$$

39. $\log_4 4^5$ ← inverse

$$= \boxed{5}$$

Write each of the following natural logarithmic equations in exponential form.

40. $\ln 1 = 0$

$$\boxed{e^0 = 1}$$

41. $\ln e = 1$

$$\boxed{e^1 = e}$$

42. $\ln 7.3891 = 2$

$$\boxed{e^2 = 7.3891}$$

Using the properties of natural logarithms, please evaluate each expression.

43. $e^{\ln 7}$ ↙ inverse
 $\boxed{7}$

44. $21 \ln 1$
 $= 21(0)$
 $= \boxed{0}$

45. $\frac{8}{\ln e}$
 $= \frac{8}{1} = \boxed{8}$

Use the properties of logarithms to condense the following logarithmic expressions.

46. $\log_6 7 - \log_6 y$

$\boxed{\log_6 \frac{7}{y}}$

47. $\log_3 q + \log_3 r$

$\boxed{\log_3 qr}$

48. $6 \log_a t - 7 \log_a t$

$\log_a t^6 - \log_a t^7$
 $= \log_a \frac{t^6}{t^7} = \boxed{\log_a \frac{1}{t}}$

49. $(\log_a x - \log_a r) + 5 \log_a p$

$= \log_a \frac{x}{r} + \log_a p^5$

$= \log_a \frac{x}{r} \cdot \frac{p^5}{1}$

$= \boxed{\log_a \frac{x p^5}{r}}$

50. $2 \log_m q - 3 \log_m y^2$

$= \log_m q^2 - \log_m y^{2 \cdot 3}$

$= \log_m q^2 - \log_m y^6$

$= \boxed{\log_m \frac{q^2}{y^6}}$

51. $5 \log_2 (2x+6) + 4 \log_2 (4x-4)$

$= \log_2 (2x+6)^5 + \log_2 (4x-4)^4$

$= \boxed{\log_2 (2x+6)^5 (4x-4)^4}$

52. $6 \ln x + \frac{1}{2} \ln y - (2 \ln x + 3 \ln 2)$

$= \ln x^6 + \ln y^{1/2} - (\ln x^2 + \ln 2^3)$

$= \ln x^6 + \ln \sqrt{y} - (\ln x^2 + \ln 8)$

$= \ln x^6 \sqrt{y} - (\ln 8 x^2)$

$= \ln \frac{x^6 \sqrt{y}}{8 x^2} = \boxed{\ln \frac{x^4 \sqrt{y}}{8}}$

53. $\frac{1}{2} [\ln(x-1) + \ln(x+2)]$

$= \frac{1}{2} [\ln(x-1)(x+2)]$

$= \frac{1}{2} [\ln(x^2+x-2)]$

$= \ln(x^2+x-2)^{1/2}$

$= \boxed{\ln \sqrt{x^2+x-2}}$

54. $\log x - (2 \log y + 3 \log z)$

$= \log x - (\log y^2 + \log z^3)$

$= \log x - \log y^2 z^3$

$= \boxed{\log \frac{x}{y^2 z^3}}$