#6, #12, #14

Pre-Calculus A
Chapter 2 Test Review

Name : Key
Date : ____________ Period : ______

1) a. Use synthetic division to show that $x=-4$ is a solution of $2x^3+9x^2+3x-4=0$.

$$\begin{array}{r|rrrr} -4 & 2 & 9 & 3 & -4 \\ & \downarrow & -8 & -4 & 4 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

← x = -4 is a solution since the remainder is zero

b. Use the result from part (a) to factor the polynomial **completely.** → ALL factors!

$-2 \begin{smallmatrix} 2 \\ -1 \end{smallmatrix}$

$2x^2+x-1$

$2x^2+2x-1x-1$

$2x(x+1)-1(x+1)$

$\boxed{(2x-1)(x+1)(x+4)}$

c. List **all** real solutions of the equation. → ALL x-values!

$x=-4,\ x=-1,\ x=1/2$

2) Write $\dfrac{8-7i}{1-2i}$ in standard form.

$$\frac{8-7i}{1-2i}\frac{(1+2i)}{(1+2i)} = \frac{8+16i-7i-14i^2}{1-4i^2} = \frac{8+9i-14(-1)}{1-4(-1)} = \frac{22+9i}{5} = \boxed{\frac{22}{5}+\frac{9}{5}i}$$

3) Write $\dfrac{4+2i}{3-i}$ in standard form.

$$\frac{4+2i}{3-i}\frac{(3+i)}{(3+i)} = \frac{12+4i+6i+2i^2}{9-i^2} = \frac{12+10i+2(-1)}{9-(-1)} = \frac{10+10i}{10} = \boxed{1+i}$$

4) Find the product $\left(6-\sqrt{-20}\right)\left(-2+\sqrt{-45}\right)$

$\sqrt{-45}=\sqrt{45}\,i=\sqrt{9}\sqrt{5}\,i=3\sqrt{5}\,i$

$\sqrt{-20}=\sqrt{20}\,i=\sqrt{4}\sqrt{5}\,i=2\sqrt{5}\,i$

$-12+6\sqrt{-45}+2\sqrt{-20}-\sqrt{-20}\sqrt{-45}$

$-12+6(3\sqrt{5}i)+2(2\sqrt{5}i)-(2\sqrt{5}i)(3\sqrt{5}i)$

$-12+18\sqrt{5}i+4\sqrt{5}i-6(\sqrt{5})^2i^2$

$-12+22\sqrt{5}i-6(5)(-1)$

$-12+22\sqrt{5}i+30$

$\boxed{18+22\sqrt{5}i}$

5) Find the product $\left(4-\sqrt{-27}\right)\left(-2+\sqrt{-12}\right)$ $\sqrt{-27}=\sqrt{27}i=\sqrt{9}\sqrt{3}i=3\sqrt{3}i$

$\sqrt{-12}=\sqrt{12}i=\sqrt{4}\sqrt{3}i=2\sqrt{3}i$

$-8+4\sqrt{-12}+2\sqrt{-27}-\sqrt{-27}\sqrt{-12}$

$-8+4(2\sqrt{3}i)+2(3\sqrt{3}i)-(3\sqrt{3}i)(2\sqrt{3}i)$

$-8+8\sqrt{3}i+6\sqrt{3}i-6(\sqrt{3})^2i^2$

$-8+14\sqrt{3}i-6(3)(-1) \Rightarrow -8+14\sqrt{3}i+18 \Rightarrow \boxed{10+14\sqrt{3}i}$

* 6) Find all real and/or imaginary solutions of the function $f(x)=x^3-5x^2+11x-15$

Possible rational zeros: $\pm1, \pm3, \pm5, \pm15$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 11 & -15 \\ & \downarrow & 3 & -6 & 15 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$(x-3)(x^2-2x+5)$

$\downarrow$

discrim: $(-2)^2-4(1)(5)$

$=-16$ imag. solution

$$\boxed{\text{Solutions: } x=3,\ x=1+2i,\ x=1-2i}$$

x^2-2x+5: $\dfrac{2\pm\sqrt{-16}}{2(1)}$

$=\dfrac{2\pm\sqrt{16}i}{2}$

$=\dfrac{2\pm4i}{2}$

$=\dfrac{2}{2}\pm\dfrac{4}{2}i=1\pm2i$

7) Find a ***third*** degree polynomial function with integer coefficients that has -2 and 6i as zeros. $x=-2,\ x=6i,\ x=-6i$

$(x+2)(x-6i)(x+6i) \leftarrow$ multiply the two imaginary factors first

$(x+2)(x^2+6ix-6ix-36i^2)$

$(x+2)(x^2-36(-1))$

$(x+2)(x^2+36) = x^3+36x+2x^2+72 = \boxed{x^3+2x^2+36x+72}$

Please (a) state the equations of all asymptotes of the function (horizontal, vertical and slant), (b) the coordinates of any holes, (c) decide whether the function is continuous or discontinuous, and (d) state the domain.

8) $h(x)=\dfrac{x^2+2x-15}{2x^2-7x+3} \Rightarrow \dfrac{(x+5)\cancel{(x-3)}}{(2x-1)\cancel{(x-3)}}$

$2x^2-6x-1x+3$

$2x(x-3)-1(x-3)$

$(2x-1)(x-3)$

(d) Domain: $(2x-1)(x-3)=0$

$2x-1=0 \quad x-3=0 \Rightarrow (-\infty,0.5)\cup(0.5,3)\cup(3,\infty)$

$x=1/2 \quad x=3$

(c) Discontinuous; the graph skips over 0.5 and 3 in the domain

(b) $x-3=0 \Rightarrow x=3$

$\dfrac{x+5}{2x-1}=\dfrac{3+5}{2(3)-1}=\dfrac{8}{6-1}=\dfrac{8}{5} \Rightarrow (3, 8/5)$

(a) HA: $y=\dfrac{1}{2}$

VA: $x=1/2$

Slant: None

9) $j(x)=\dfrac{x^2-15x+56}{x-3} = \dfrac{(x-8)(x-7)}{x-3}$

$$\begin{array}{r|rrr} 3 & 1 & -15 & 56 \\ & \downarrow & 3 & -36 \\ \hline & 1 & -12 & 20 \end{array}$$

(d) Domain: $(-\infty,3)\cup(3,\infty)$

(c) Discontinuous

(b) No hole

(a) HA: None

VA: $x=3$

Slant: $y=x-12$

10) $k(x)=\dfrac{x-3}{x^2+x-12}$ $\dfrac{\cancel{(x-3)}}{\cancel{(x-3)}(x+4)} = \dfrac{1}{x+4}$

(d) Domain: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$

(c) Discontinuous

(b) $x-3=0 \Rightarrow x=3$ $\Rightarrow (3, 1/7)$

$\dfrac{1}{3+4}=\dfrac{1}{7}$

(a) HA: $y=0$

VA: $x=-4$

Slant: None

11) $f(x)=\dfrac{3x^2-11x-4}{x^2+2}$ $x^2=-2 \Rightarrow x=\pm\sqrt{-2}$ ← not possible

(d): Domain: $(-\infty,\infty)$

(c): Continuous

(b). No holes

(a): HA: $y=3$

VA: None

Slant: None

** 12) Find the slant asymptote of the following function. How do you know the function has a slant asymptote?

$$f(x)=\frac{5x^3-8}{x^2+3x-1}$$

$$f(x)=\frac{5x^2-8}{x-2}$$

$$\begin{array}{r|rrr} 2 & 5 & 0 & -8 \\ & \downarrow & 10 & 20 \\ \hline & 5 & 10 & 12 \end{array}$$

$\boxed{y=5x+10}$

Graph the following functions. Be sure to identify all asymptotes, coordinates of holes and x- and y-intercepts.

13) $f(x) = \dfrac{-3+x}{5-x}$

domain: $5-x=0 \Rightarrow x=5$

$(-\infty,5)\cup(5,\infty)$

HA: $y=-1$

VA: $x=5$

Slant: None

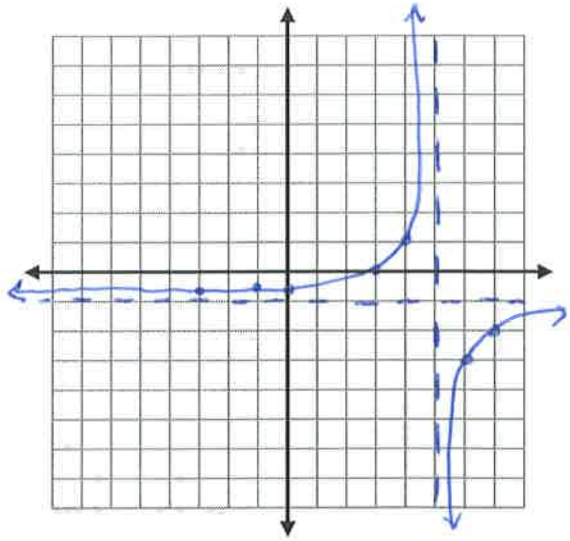
Holes: None

y-int: $f(0) = \dfrac{-3+0}{5-0} = \dfrac{-3}{5} \Rightarrow (0,^{-3}/_5)$

x-int: $-3+x=0 \Rightarrow x=3 \Rightarrow (3,0)$

range: $(-\infty,-1)\cup(-1,\infty)$

x	y
-3	-0.75
1	-0.5
3	0
4	1
6	-3
7	-2



14) $k(x) = \dfrac{x^2+9x-36}{x^2+x-12}$ $\dfrac{(x+12)\cancel{(x-3)}}{(x+4)\cancel{(x-3)}} = \dfrac{x+12}{x+4}$

domain: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$

HA: $y=1$

VA: $x=-4$

Holes: $x=3$

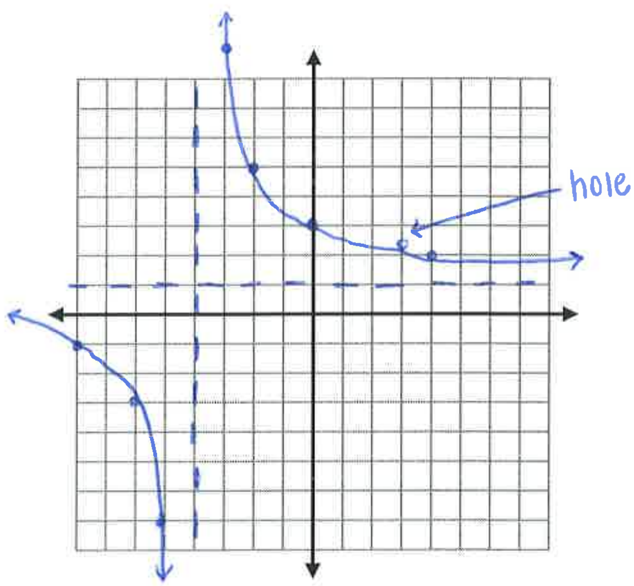
$\dfrac{x+12}{x+4} = \dfrac{3+12}{3+4} = \dfrac{15}{7} \Rightarrow (3,{}^{15}/_7)$

Slant: None

y-int: $f(0) = \dfrac{0+12}{0+4} = \dfrac{12}{4} = 3$ $(0,3)$

x-int: $x+12=0$
$x=-12 \Rightarrow (-12,0)$

range:



x	y
-8	-1
-6	-3
-5	-7
-3	9
-2	5
0	3
4	2

15) There were a group of horses introduced to new game lands. The population N of the herd is given by the function below, where *t* is the time in years.

$$N = \frac{40(5+3t)}{1+0.04t} = \frac{200+120t}{1+0.04t} = \frac{120t+200}{0.04t+1}$$

a) What is the domain of the function?

$t \geq 0$

Time cannot be negative

b) Find the initial number of horses introduced to the game lands.

$N(0) = \frac{40(5+3(0))}{1+0.04(0)} = \frac{40(5)}{1} = \frac{200}{1} = 200$ horses

c) Find the population after 5, 10, and 25 years.

$N(5) = \frac{40(5+15)}{1+0.04(5)} = \frac{800}{1.2} = 666$ horses

$N(25) = \frac{3200}{2} = 1600$ horses

$N(10) = \frac{1400}{1.4} = 1000$ horses

d) What is the maximum number of horses introduced as time passes? Explain your reasoning.

HA: $\frac{120}{.04} = 3000$ horses

Answer Key :

1. a. Show synthetic division
   b. $(2x-1)(x+1)(x+4)$
   c. $x=-4,\ x=-1,\ x=\frac{1}{2}$

2. $\frac{22}{5}+\frac{9}{5}i$

3. $1+i$

4. $18+22\sqrt{5}i$

5. $10+14\sqrt{3}i$

6. $x=+3,\ x=1+2i,\ x=1-2i$
7. $f(x)=x^3+2x^2+36x+72$
8. (a) H.A. : $y=\frac{1}{2}$, V.A. : $x=\frac{1}{2}$, Slant : None
   (b) $\left(3,\frac{8}{5}\right)$
   (c) Discontinuous
   (d) $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},3\right)\cup(3,\infty)$

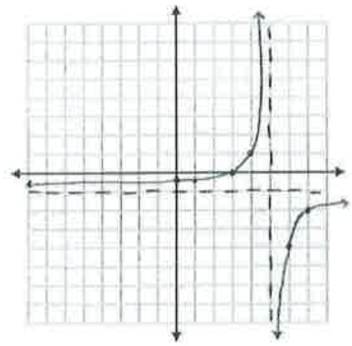
9. (a) H.A. : None , V.A. : $x=3$, Slant : $y=x-12$
   (b) No holes
   (c) Discontinuous
   (d) $(-\infty,3)\cup(3,\infty)$

10. (a) H.A. : $y=0$, V.A. : $x=-4$, Slant : None
    (b) $\left(3,\frac{1}{7}\right)$
    (c) Discontinuous
    (d) $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$

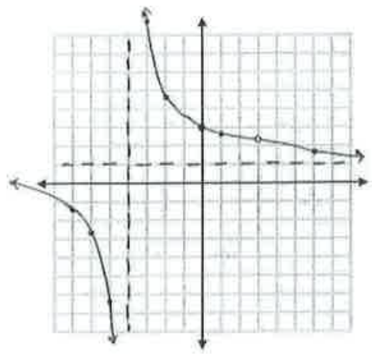
11. (a) H.A. : $y=3$, V.A. : None , Slant : None
    (b) No holes
    (c) Continuous
    (d) $(-\infty,\infty)$

12. The slant asymptote occurs at ~~$y = 5x - 15$~~ $y=5x+10$. The function has a slant asymptote because the degree of the numerator is exactly one more than the degree of the denominator.

13. H.A. : $y = -1$ ✓
V.A. : $x = 5$ ✓
Slant : None, Holes : None ✓
Y-int : $\left(0, -\frac{3}{5}\right)$ ✓
X-int : $(3, 0)$ ✓



14. H.A. : $y = 1$ ✓
V.A. : $x = -4$ ✓
Slant : None ✓
Hole : $\left(3, \frac{15}{7}\right)$ ✓
Y-int : $(0, 3)$ ✓
X-int : $(-12, 0)$ ✓



15. (a) Domain : $t \geq 0$, time cannot be negative ✓
(b) 200 horses ✓
(c) t = 5 : 666 horses , t = 10 : 1000 horses , t = 25 : 1600 horses ✓
(d) The horizontal asymptote is the cap on the amount of horses introduced. Since the horizontal asymptote occurs at $y = 3000$, the maximum number of horses introduced will be 3,000. ✓