Please verify, simplify or factor each of the following on a separate piece of paper. Make sure you show all work!

1. Verify $\csc x (\cos x + \sin x) = \cot x + 1$

LHS: $\dfrac{1}{\sin x}\left(\dfrac{\cos x}{1} + \dfrac{\sin x}{1}\right)$

$\Rightarrow \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\sin x}$

$\Rightarrow \cot x + 1$

$\cot x + 1 = \cot x + 1$

LHS = RHS

2. Verify $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = 1$

LHS: $\dfrac{\frac{1}{\cos\theta}}{\cos\theta} - \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$\Rightarrow \dfrac{1}{\cos\theta}\cdot\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow \dfrac{1 - \sin^2\theta}{\cos^2\theta}$

$\Rightarrow \dfrac{\cos^2\theta}{\cos^2\theta}$

$\Rightarrow 1$

$1 = 1$

LHS = RHS

3. Verify $\sec x - \tan x \sin x = \cos x$

LHS: $\Rightarrow \dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\cdot\dfrac{\sin x}{1}$

$\Rightarrow \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$\Rightarrow \dfrac{1 - \sin^2 x}{\cos x}$

$\Rightarrow \dfrac{\cos^2 x}{\cos x}$

$\Rightarrow \cos x$

$\cos x = \cos x$

LHS = RHS

4. Verify $\cot x (\tan x + \cot x) = \csc^2 x$

LHS: $\dfrac{\cos x}{\sin x}\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)$

$\Rightarrow \dfrac{\cos x \sin x}{\sin x \cos x} + \dfrac{\cos^2 x}{\sin^2 x}$

$\Rightarrow 1 + \cot^2 x$

$\Rightarrow \csc^2 x$

$\csc^2 x = \csc^2 x$

LHS = RHS

5. Verify $\sin x \tan x - \csc x \tan x = -\cos x$

LHS: $\dfrac{\sin x}{1}\cdot\dfrac{\sin x}{\cos x} - \dfrac{1}{\sin x}\cdot\dfrac{\sin x}{\cos x}$

$\Rightarrow \dfrac{\sin^2 x}{\cos x} - \dfrac{\sin x}{\sin x \cos x}$

$\Rightarrow \dfrac{\sin^2 x}{\cos x} - \dfrac{1}{\cos x}$

$\Rightarrow \dfrac{\sin^2 x - 1}{\cos x}$

$\Rightarrow \dfrac{-\cos^2 x}{\cos x}$

$\Rightarrow -\cos x$

$-\cos x = -\cos x$

LHS = RHS

6. Verify $\dfrac{\sec\theta \sin\theta}{\tan\theta + \cot\theta} = \sin^2\theta$

LHS: $\dfrac{1}{\cos\theta}\cdot\dfrac{\sin\theta}{1}$

LCD: $\cos\theta\sin\theta \rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta\cdot\sin\theta}{\sin\theta\cdot\cos\theta} + \frac{\cos\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}}$

$\Rightarrow \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} \div \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\rightarrow \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin\theta\cos\theta}{1}$

$\Rightarrow \sin^2\theta$

$\sin^2\theta = \sin^2\theta$

LHS = RHS

7. Verify $\cot\left(\dfrac{\pi}{2} - x\right)\cos x = \sin x$

LHS: $\tan x \cos x$

$\Rightarrow \dfrac{\sin x}{\cos x} \cdot \dfrac{\cos x}{1}$

$\Rightarrow \sin x$

$\sin x = \sin x$

LHS = RHS

8. Simplify $\cos x (\sec x - \cos x)$

$\Rightarrow \cos x \sec x - \cos^2 x$

$\Rightarrow \dfrac{\cos x}{1} \cdot \dfrac{1}{\cos x} - \cos^2 x$

$\Rightarrow 1 - \cos^2 x$

$\Rightarrow \boxed{\sin^2 x}$

9. Simplify $\sec x \cot x - \cot x \cos x$

$\Rightarrow \dfrac{1}{\cos x} \cdot \dfrac{\cos x}{\sin x} - \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{1}$

$\Rightarrow \dfrac{1}{\sin x} - \dfrac{\cos^2 x}{\sin x}$

$\Rightarrow \dfrac{1 - \cos^2 x}{\sin x}$

$\Rightarrow \dfrac{\sin^2 x}{\sin x}$

$\Rightarrow \boxed{\sin x}$

10. Simplify $\sin^3 x + \sin x \cos^2 x$ ← GCF: $\sin x$

$\Rightarrow \sin x (\sin^2 x + \cos^2 x)$

$\Rightarrow \sin x (1)$

$\Rightarrow \boxed{\sin x}$

11. Simplify $\cot^2 x - \cot^2 x \cos^2 x$ ← GCF: $\cot^2 x$

$\Rightarrow \cot^2 x (1 - \cos^2 x)$

$\Rightarrow \cot^2 x (\sin^2 x)$

$\Rightarrow \dfrac{\cos^2 x}{\sin^2 x} \cdot \dfrac{\sin^2 x}{1}$

$\Rightarrow \boxed{\cos^2 x}$

12. Simplify $\sec^4 x - \tan^4 x$

$\Rightarrow (\sec^2 x)(\sec^2 x) - \tan^4 x$

$\Rightarrow (1 + \tan^2 x)(1 + \tan^2 x) - \tan^4 x$

$\Rightarrow$ FOIL
$1 + \tan^2 x + \tan^2 x + \tan^4 x - \tan^4 x$

$\Rightarrow \boxed{1 + 2\tan^2 x}$

difference of two squares

13. Factor and simplify $\dfrac{\csc^2 x - 1}{\csc x - 1}$

$\dfrac{\csc^2 x - 1}{\csc x - 1} \Rightarrow \dfrac{(\csc x - 1)(\csc x + 1)}{\csc x - 1}$

$\Rightarrow \boxed{\csc x + 1}$

4 terms ⟹ factor by grouping

14. Factor $\sec^3 x - \sec^2 x - \sec x + 1$

$\sec^2 x (\sec x - 1) - 1(\sec x - 1)$

diff of 2 squares $\rightarrow (\sec^2 x - 1)(\sec x - 1)$

$\boxed{(\sec x - 1)(\sec x + 1)(\sec x - 1)}$

15. Factor $3\tan^2 x - 5\tan x - 2$

$\boxed{(3\tan x + 1)(\tan x - 2)}$

$3x^2 - 5x - 2$
$3x^2 - 6x + 1x - 2$
$3x(x-2) + 1(x-2)$
$(3x+1)(x-2)$

16. Factor $2\sin^2 x + 3\sin x - 2$

$\boxed{(2\sin x - 1)(\sin x + 2)}$

$2x^2 + 3x - 2$
$2x^2 + 4x - 1x - 2$
$2x(x+2) - 1(x+2)$
$(2x-1)(x+2)$

Answer Key (Simplifying and Factoring Problems Only):

8. $\sin^2 x$ ✓   9. $\sin x$ ✓   10. $\sin x$ ✓   11. $\cos^2 x$ 12. $1 + 2\tan^2 x$ ✓   13. $\csc x + 1$ ✓

14. $(\sec x - 1)(\sec x + 1)(\sec x - 1)$ ✓   15. $(3\tan x + 1)(\tan x - 2)$ ✓   16. $(\sin x + 2)(2\sin x - 1)$ ✓