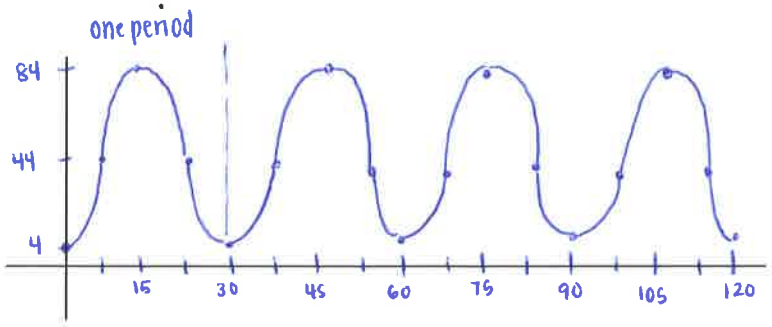
Name Key

February 7, 2017

Writing Sine and Cosine Functions Review

1) Suppose a Ferris wheel has a radius of 40 feet and operates at a speed of 2 revolutions per minute. The bottom car is 4 feet above the ground. The value of h is 4 feet when $t = 0$.



2 rev/1 min
1 rev/30 sec
$\frac{1}{2}$ rev/15 sec
$\frac{1}{4}$ rev/7.5 sec

a) Sketch a graph for a ride on this Ferris wheel (4 minutes). Identify the maximum, minimum, and amplitude on the graph. max = 84, min = 4, amp: $\frac{1}{2}(84-4) = \frac{1}{2}(80) = 40$

b) Where would the axis of wave or midline be for a graph of this model?

d = max − amp = 84 − 40 = 44

midline: $y = 44$

c) What is the period length for this scenario? What would the b-value be in a sine or cosine function for this scenario?

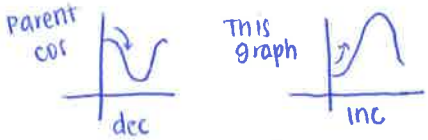
1 rev = 30 sec so period = 30

$\frac{2\pi}{b} = \frac{30}{1} \Rightarrow 30b = 2\pi \Rightarrow b = \frac{2\pi}{30} \Rightarrow b = \frac{\pi}{15}$

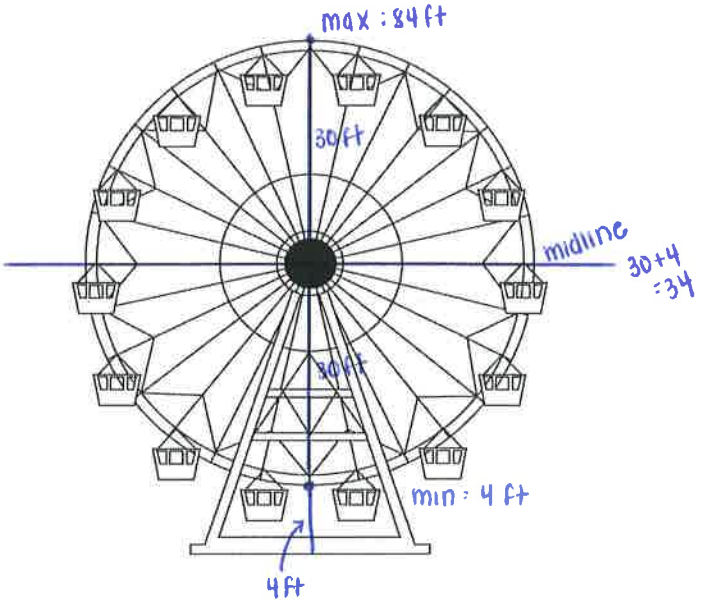
d) Write a cosine model for this scenario.

$y = -40\cos\left(\frac{\pi}{15}x\right) + 44$

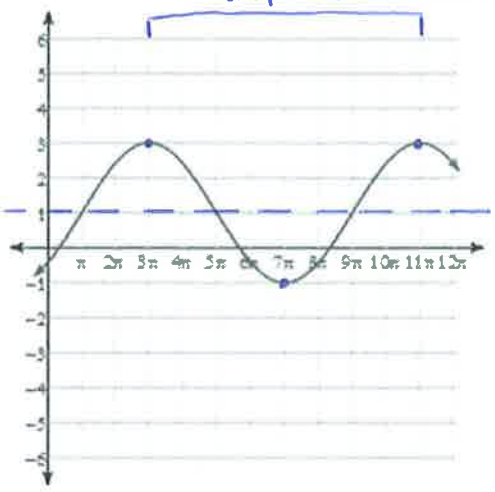
↑ amplitude is negative



opposite behavior so there is a reflection, and amplitude is negative



2)



max = 3 , min = −1

amp = $\frac{1}{2}$(max − min) = $\frac{1}{2}$(3 − −1) = $\frac{1}{2}$(3 + 1) = $\frac{1}{2}$(4) = 2

d-value: max − amp = 3 − 2 = 1

period = 11π − 3π = 8π

To find b: $\frac{2\pi}{b} = \frac{8\pi}{1}$ ⇒ $2\pi = 8\pi b$ ⇒ $b = \frac{2\pi}{8\pi} = \frac{1}{4}$

Vertical Shift: up 1 unit (d = 1)

Amplitude: 2

Period: 8π b-value: $\frac{1}{4}$

Phase shift: sine: Right π units; cosine: Right 3π units

Equation: $y = 2\sin\left(\frac{1}{4}x - \frac{\pi}{4}\right) + 1$

$y = 2\cos\left(\frac{1}{4}x - \pi\right) + 1$

Phase shift: sine: Right π units

Find c: $\frac{\pi}{1} = \frac{c}{b} \Rightarrow \frac{\pi}{1} = \frac{c}{1/4}$

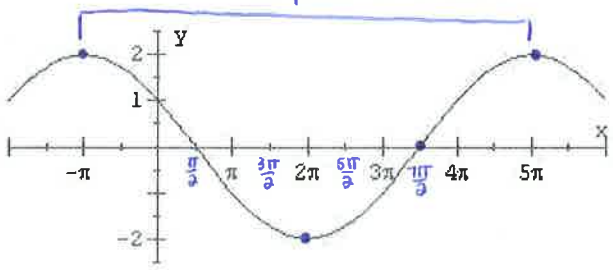
$\Rightarrow c = \frac{\pi}{4}$

cosine: Right 3π units

Find c: $\frac{3\pi}{1} = \frac{c}{b} \Rightarrow \frac{3\pi}{1} = \frac{c}{1/4}$

$\Rightarrow c = \frac{3\pi}{4}$

3)



max = 2 , min = −2

amp = $\frac{1}{2}$(2 − −2) = $\frac{1}{2}$(2 + 2) = $\frac{1}{2}$(4) = 2

d-value: max − amp = 2 − 2 = 0

period = 5π − −π = 5π + π = 6π

Find b: $\frac{2\pi}{b} = \frac{6\pi}{1} \Rightarrow 2\pi = 6\pi b \Rightarrow b = \frac{2\pi}{6\pi} = \frac{1}{3}$

Vertical Shift: none (d = 0)

Amplitude: 2

Period: 6π b-value: $\frac{1}{3}$

Phase shift: sine: Right $\frac{7\pi}{2}$ units; cosine: Left π units

Equation: $y = 2\sin\left(\frac{1}{3}x - \frac{7\pi}{6}\right)$

$y = 2\cos\left(\frac{1}{3}x + \frac{\pi}{3}\right)$

sine: Right $\frac{7\pi}{2}$ units

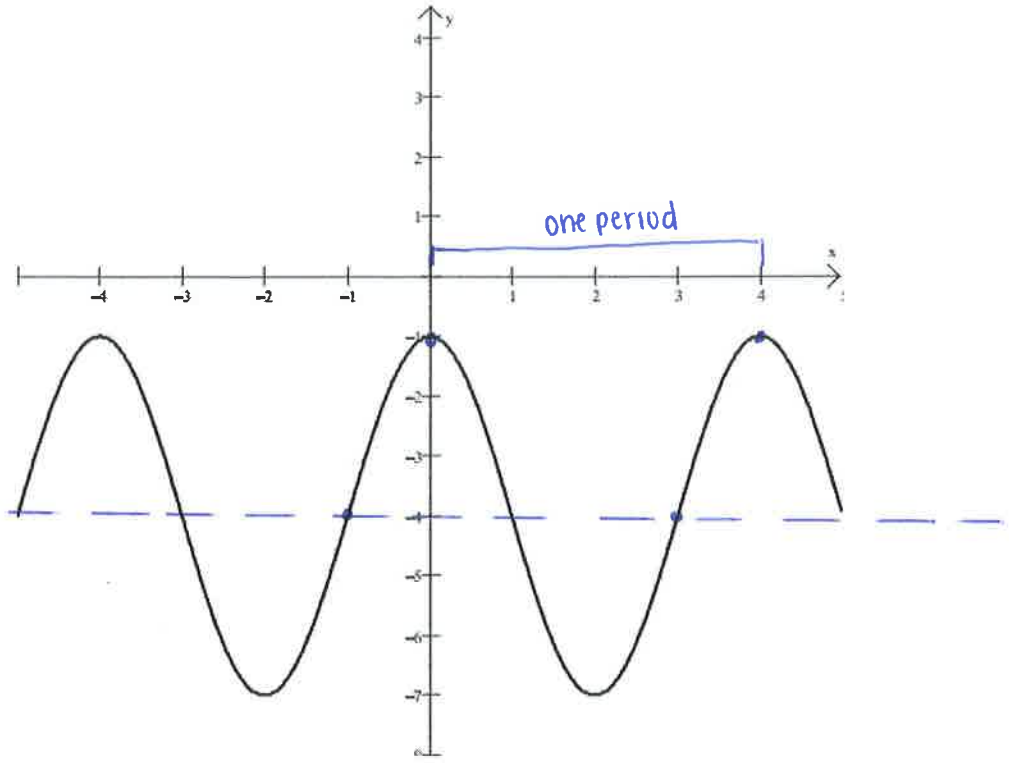
$\frac{7\pi}{2} = \frac{c}{1/3} \Rightarrow 2c = \frac{7\pi}{3}$

$c = \frac{7\pi}{3} \cdot \frac{1}{2} = \frac{7\pi}{6}$

cosine: Left π units

$-\frac{\pi}{1} = \frac{c}{1/3} \Rightarrow c = -\frac{\pi}{3}$

4)



max = -1

min = -7

amp = $\frac{1}{2}(-1--7) = \frac{1}{2}(-1+7) = \frac{1}{2}(6) = 3$

d-value: max - amp = $-1-3 = -4$

period = $4-0 = 4$

b-value: $\frac{2\pi}{b} = \frac{4}{1} \Rightarrow 4b = 2\pi \Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$

sine: Left 1 unit

$-\frac{1}{1} = \frac{c}{b} \Rightarrow -\frac{1}{1} = \frac{c}{\pi/2} \Rightarrow c = -\frac{\pi}{2}$

cosine: Right 4 units

$\frac{4}{1} = \frac{c}{b} \Rightarrow \frac{4}{1} = \frac{c}{\pi/2} \Rightarrow c = \frac{4\pi}{2} = 2\pi$

$y = 3\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right) - 4$

$y = 3\cos\left(\frac{\pi}{2}x - 2\pi\right) - 4$