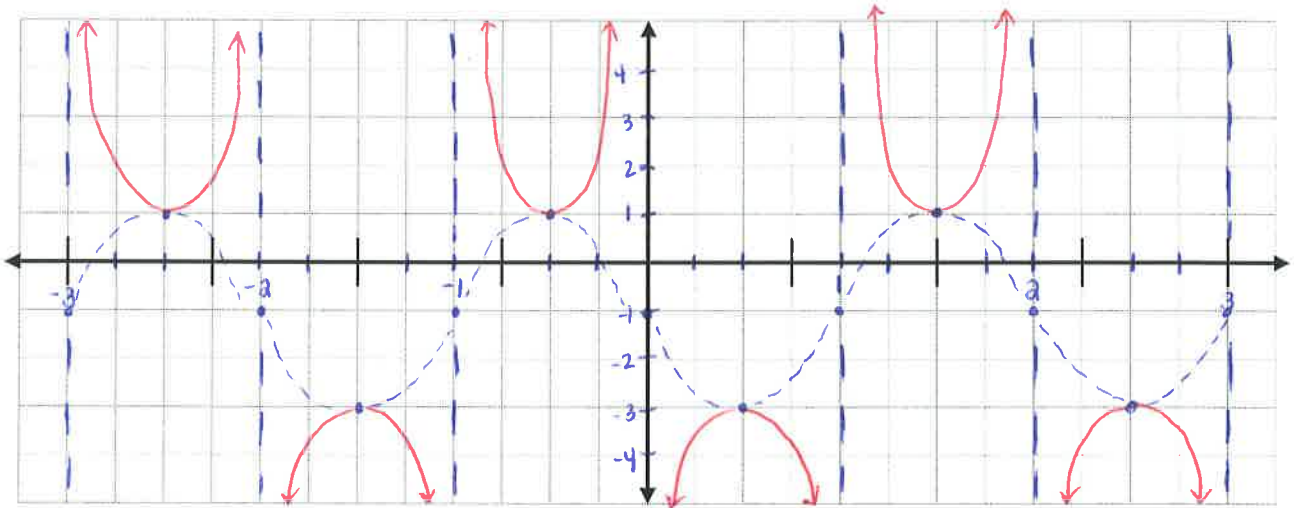


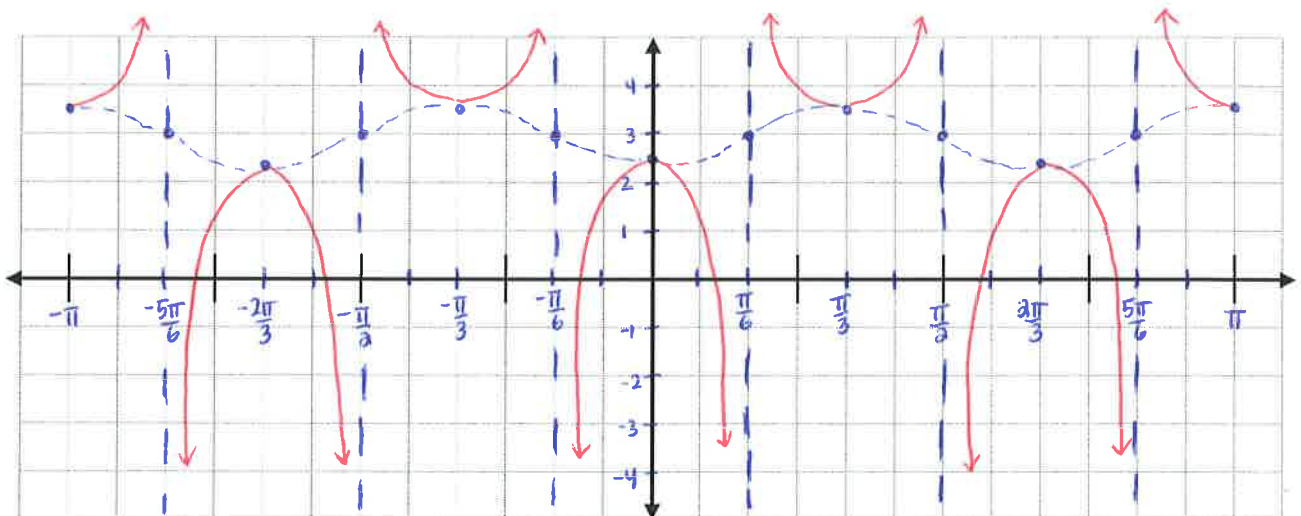
Please graph TWO FULL PERIODS of the following functions. Be sure to identify your asymptotes and find the domain, range and period. Do all work on a separate sheet of paper.

1. $y = -1 - 2 \sec\left(\pi x - \frac{\pi}{2}\right)$



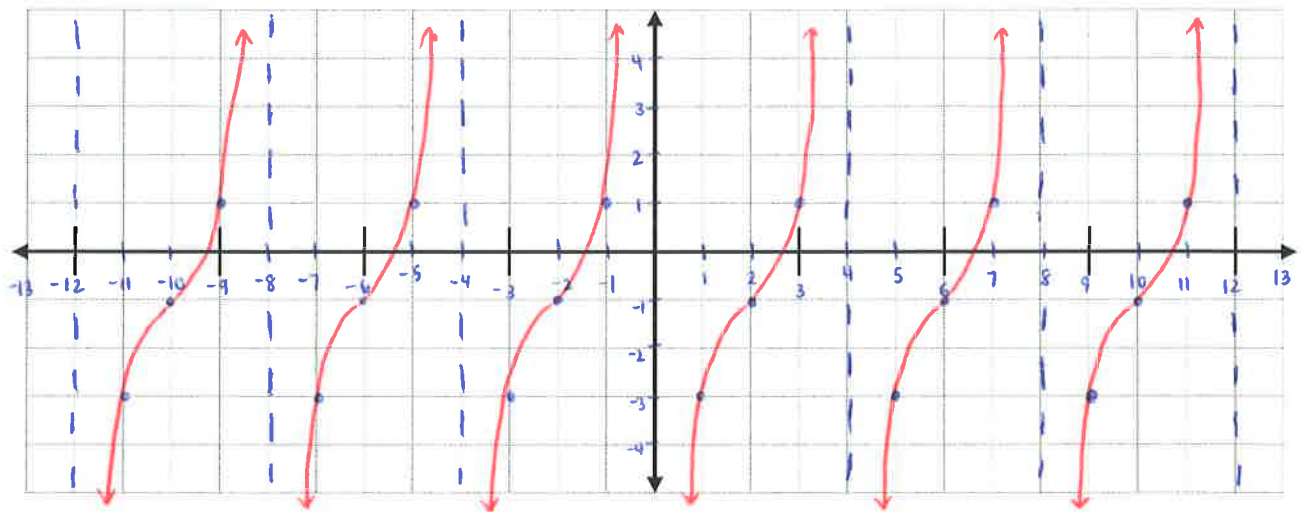
Period: $\frac{2\pi}{\frac{\pi}{1}} = 2$ Range: $(-\infty, -3] \cup [1, \infty)$ Domain: \mathbb{R} except \mathbb{Z}

2. $y = \frac{1}{2} \csc\left(3x - \frac{\pi}{2}\right) + 3$



Period: $\frac{2\pi}{\frac{3}{1}} = \frac{2\pi}{3}$ Range: $(-\infty, 2.5] \cup [3.5, \infty)$ Domain: \mathbb{R} except $\frac{\pi}{6} + \frac{2\pi}{3}n$

3. $y = -1 + 2 \tan\left(\frac{\pi x}{4} - \frac{\pi}{2}\right)$ ↗ increasing

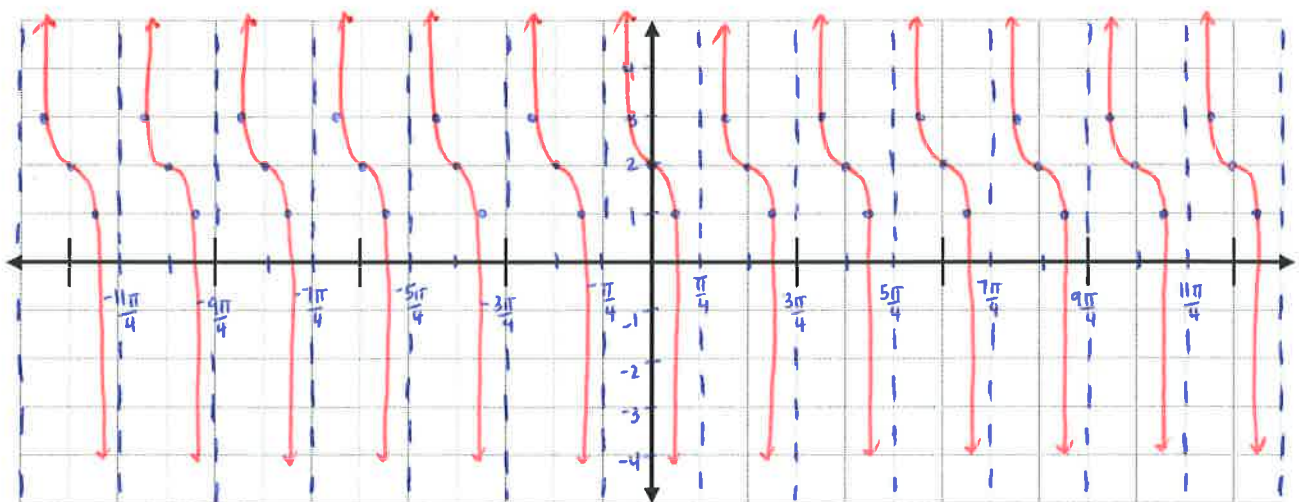


Period = $4 - 0 = 4$

Range: $(-\infty, \infty)$

Domain: \mathbb{R} except $4n$

4. $y = \cot\left(2x - \frac{\pi}{2}\right) + 2$

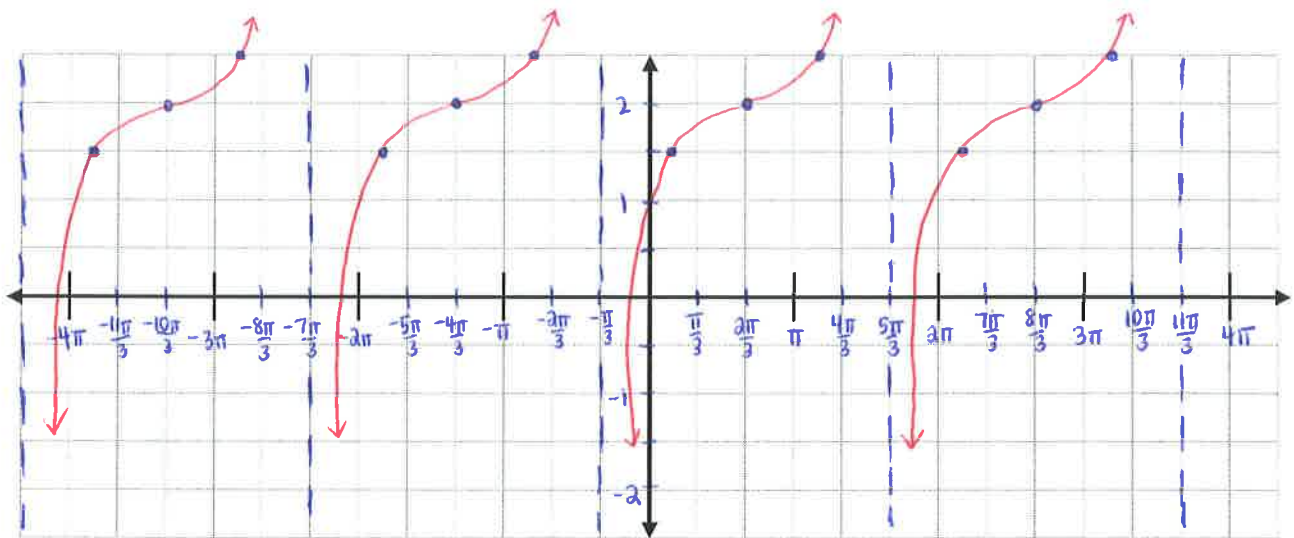


Period = $\frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

Range: $(-\infty, \infty)$

Domain: \mathbb{R} except $\frac{\pi}{4} + \frac{\pi}{2}n$

5. $y = \frac{1}{2} \tan\left(\frac{x}{2} + \frac{2\pi}{3}\right) + 2$

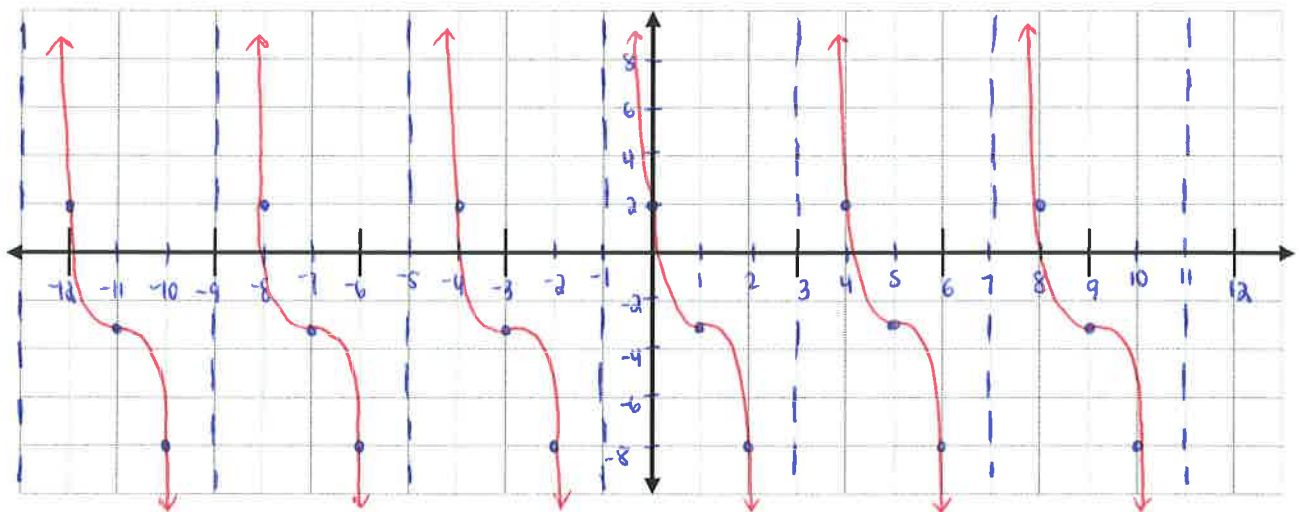


period = $\frac{11\pi}{3} - \frac{5\pi}{3} = \frac{6\pi}{3} = 2\pi$

Range: $(-\infty, \infty)$

Domain: \mathbb{R} except $\frac{5\pi}{3} + 2n\pi$

6. $y = 5 \cot\left(\frac{\pi}{4}x + \frac{\pi}{4}\right) - 3$



period = $7 - 3 = 4$

Range: $(-\infty, \infty)$

Domain: \mathbb{R} except $3 + 4n$

① \rightarrow cosine
 $-1 - a \sec(\pi x - \frac{\pi}{2})$

$$\begin{array}{ccccc} \pi x - \frac{\pi}{2} = 0 & \pi x - \frac{\pi}{2} = \frac{\pi}{2} & \pi x - \frac{\pi}{2} = \pi & \pi x - \frac{\pi}{2} = \frac{3\pi}{2} & \pi x - \frac{\pi}{2} = 2\pi \\ \frac{1}{\pi} \cdot \frac{\pi x - \frac{\pi}{2}}{1} = \frac{\pi}{2} \cdot \frac{1}{\pi} & \frac{\pi x - \frac{\pi}{2}}{\pi} = \frac{\pi}{\pi} & \frac{1}{\pi} \cdot \frac{\pi x - \frac{\pi}{2}}{1} = \frac{3\pi}{2} \cdot \frac{1}{\pi} & \frac{\pi x - \frac{\pi}{2}}{\pi} = \frac{2\pi}{\pi} & \frac{1}{\pi} \cdot \frac{\pi x - \frac{\pi}{2}}{1} = \frac{5\pi}{2} \cdot \frac{1}{\pi} \\ x = \frac{1}{2} & x = 1 & x = \frac{3}{2} & x = 2 & x = \frac{5}{2} \end{array}$$

	new x	x axis	y axis	-2y	-2y-1
	$\frac{1}{2}$	0	1	-2	-3
asymptote \rightarrow	1	$\frac{\pi}{2}$	0	0	-1
	$\frac{3}{2}$	π	-1	2	1
asymptote \rightarrow	2	$\frac{3\pi}{2}$	0	0	-1
	$\frac{5}{2}$	2π	1	-2	-3

② \rightarrow sine
 $\frac{1}{2} \csc(3x - \frac{\pi}{2}) + 3$

$$\begin{array}{ccccc} 3x - \frac{\pi}{2} = 0 & 3x - \frac{\pi}{2} = \frac{\pi}{2} & 3x - \frac{\pi}{2} = \pi & 3x - \frac{\pi}{2} = \frac{3\pi}{2} & 3x - \frac{\pi}{2} = 2\pi \\ 3x = \frac{\pi}{2} & 3x = \pi & 3x = \frac{3\pi}{2} & 3x = 2\pi & 3x = \frac{5\pi}{2} \\ x = \frac{\pi}{2} \cdot \frac{1}{3} & x = \frac{\pi}{3} & x = \frac{3\pi}{2} \cdot \frac{1}{3} & x = \frac{2\pi}{3} & x = \frac{5\pi}{2} \cdot \frac{1}{3} \\ x = \frac{\pi}{6} & & x = \frac{\pi}{2} & & x = \frac{5\pi}{6} \end{array}$$

	new x	x axis	y axis	$\frac{1}{2}y$	$\frac{1}{2}y+3$
asymptote \rightarrow	$\frac{\pi}{6}$	0	0	0	3
	$\frac{\pi}{3}$	$\frac{\pi}{2}$	1	0.5	3.5
asymptote \rightarrow	$\frac{\pi}{2}$	π	0	0	3
	$\frac{2\pi}{3}$	$\frac{3\pi}{2}$	-1	-0.5	2.5
asymptote \rightarrow	$\frac{5\pi}{6}$	2π	0	0	3

③ $-1 + a \tan(\frac{\pi x}{4} - \frac{\pi}{2})$

$$\begin{array}{cc} \frac{\pi x}{4} - \frac{\pi}{2} = -\frac{\pi}{2} & \frac{\pi x}{4} - \frac{\pi}{2} = \frac{\pi}{2} \\ \frac{\pi x}{4} = 0 & \frac{\pi x}{4} = \pi \\ 0 = \pi x & \frac{4\pi}{\pi} = \frac{\pi x}{\pi} \\ x = 0 & x = 4 \end{array}$$

consecutive asymptotes at $x=0, x=4$
 midpoint: -1
 1st Q. Point: $-1 \cdot a = -a - 1 = -3$
 2nd Q. Point: $1 \cdot a = a - 1 = 1$

$$④ \cot\left(2x - \frac{\pi}{2}\right) + a$$

$$2x - \frac{\pi}{2} = 0 \quad 2x - \frac{\pi}{2} = \pi$$

$$2x = \frac{\pi}{2} \quad 2x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{2} \cdot \frac{1}{2} \quad x = \frac{3\pi}{4}$$

$$x = \frac{\pi}{4}$$

Consecutive Asymptotes at $x = \frac{\pi}{4}, x = \frac{3\pi}{4}$

midpoint: a

$$1^{\text{st}} \text{ Q. Point: } 1 + a = 3$$

$$2^{\text{nd}} \text{ Q. Point: } -1 + a = 1$$

$$⑤ \frac{1}{a} \tan\left(\frac{x}{a} + \frac{a\pi}{3}\right) + a$$

$$\frac{x}{a} + \frac{a\pi}{3} = -\frac{\pi}{a}$$

$$\frac{x}{a} + \frac{a\pi}{3} = \frac{\pi}{a}$$

$$\frac{x}{a} = -\frac{\pi}{a} - \frac{a\pi}{3}$$

$$\frac{x}{a} = \frac{\pi}{a} - \frac{a\pi}{3}$$

$$\frac{x}{a} = -\frac{3\pi}{6} - \frac{4\pi}{6}$$

$$\frac{x}{a} = \frac{3\pi}{6} - \frac{4\pi}{6}$$

$$\frac{x}{a} = -\frac{7\pi}{6} \cdot \frac{a}{1}$$

$$\frac{x}{a} = \frac{-\pi \cdot a}{6 \cdot 1}$$

$$x = -\frac{14\pi}{6} = -\frac{7\pi}{3}$$

$$x = -\frac{a\pi}{6} = -\frac{\pi}{3}$$

Consecutive Asymptotes at $x = -\frac{7\pi}{3}, x = -\frac{\pi}{3}$

midpoint: a

$$1^{\text{st}} \text{ Q. Point: } -1 \times \frac{1}{a} = -\frac{1}{a} + a = 1.5$$

$$2^{\text{nd}} \text{ Q. Point: } 1 \times \frac{1}{a} = \frac{1}{a} + a = 2.5$$

$$⑥ 5 \cot\left(\frac{\pi}{4}x + \frac{\pi}{4}\right) - 3$$

$$\frac{\pi}{4}x + \frac{\pi}{4} = 0$$

$$\frac{\pi}{4}x + \frac{\pi}{4} = \pi$$

$$\frac{\pi}{4}x = -\frac{\pi}{4} \cdot \frac{4}{\pi}$$

$$\frac{\pi}{4}x = \frac{3\pi}{4} \cdot \frac{4}{\pi}$$

$$x = -\frac{4\pi}{4\pi} = -1$$

$$x = 3$$

Consecutive Asymptotes: $x = -1, x = 3$

midpoint: -3

$$1^{\text{st}} \text{ Q. Point: } 1 \times 5 = 5 - 3 = 2$$

$$2^{\text{nd}} \text{ Q. Point: } -1 \times 5 = -5 - 3 = -8$$