Pre-Calculus A
4.5 Graphing Sine and Cosine - the **b and c** values

Name: Key

Period of Sine and Cosine Functions:

→ how long it takes for the graph to complete one full cycle

Let b be a positive real number. The period of $f(x) = d + a\sin(bx - c)$ and $g(x) = d + a\cos(bx - c)$ is given by $P = \frac{2\pi}{b}$.

(a max ; a min)

Example: Determine the fundamental period of $f(x) = \overset{d}{-1} + \overset{a}{\frac{1}{2}}\sin\left(\overset{b}{2}x + \overset{c}{\frac{\pi}{3}}\right)$ $\left(2x - -\frac{\pi}{3}\right)$ $c = -\frac{\pi}{3}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \boxed{\pi}$

↑ It will take π units for the graph to complete a full cycle

Transformations from the b and c values:

If $0 < b < 1$, the period is greater than 2π, and there is a horizontal stretch.

If $b > 1$, the period is less than 2π and represents a horizontal shrink.

Example: Determine the horizontal transformation of $f(x) = -1 + \frac{1}{2}\sin\left(2x + \frac{\pi}{3}\right)$.

$b = 2$ $c = -\frac{\pi}{3}$

$b = 2$: horizontal shrink (it will take less time to complete a full cycle than the parent sine function)

$c = -\frac{\pi}{3}$: horizontal shift: left $\frac{\pi}{3}$ units

A horizontal shift is called a phase shift. It is determined by calculating $\left|\frac{c}{b}\right|$. Indicate direction.

Example: Determine the phase shift of $f(x) = -1 + \frac{1}{2}\sin\left(2x + \frac{\pi}{3}\right)$.

P.S. $= \left|\frac{c}{b}\right| = \left|\frac{-\frac{\pi}{3}}{2}\right| = \left|-\frac{\pi}{3} \div \frac{2}{1}\right| = \left|-\frac{\pi}{3} \cdot \frac{1}{2}\right| = \left|-\frac{\pi}{6}\right| = \frac{\pi}{6}$

Since the c-value is negative, the phase shift will be to the left,

so P.S. $= \frac{\pi}{6}$ units to the left

When looking at a graph with a horizontal shift and/or a horizontal stretch/shrink, to determine the x-value of the "new" key points, we set the x-value of the parent graph's key points equal to the argument $(bx-c)$. There should be five key points.

$\hookrightarrow 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\hookrightarrow$ b & c values change the x-values of the table of values

Example: Determine the x-values of the new key points of $f(x)=-1+\frac{1}{2}\sin\left(2x+\frac{\pi}{3}\right)$.

set equal to 5 key points to find new x-values

$2x+\frac{\pi}{3}=0$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{-\frac{\pi}{3}}{2}$

$x=-\frac{\pi}{3}\div\frac{2}{1}$

$x=-\frac{\pi}{3}\cdot\frac{1}{2}$

$\boxed{x=-\frac{\pi}{6}}$

$2x+\frac{\pi}{3}=\frac{\pi}{2}$

$2x=\frac{\pi}{2}-\frac{\pi}{3}$

$2x=\frac{3\pi}{6}-\frac{2\pi}{6}$

$2x=\frac{\pi}{6}$

$x=\frac{\pi}{6}\cdot\frac{1}{2}$

$\boxed{x=\frac{\pi}{12}}$

$2x+\frac{\pi}{3}=\pi$

$2x=\frac{\pi}{1}-\frac{\pi}{3}$

$2x=\frac{3\pi}{3}-\frac{\pi}{3}$

$2x=\frac{2\pi}{3}$

$x=\frac{2\pi}{3}\cdot\frac{1}{2}$

$x=\frac{2\pi}{6}$

$\boxed{x=\frac{\pi}{3}}$

$2x+\frac{\pi}{3}=\frac{3\pi}{2}$

$2x=\frac{3\pi}{2}-\frac{\pi}{3}$

$2x=\frac{9\pi}{6}-\frac{2\pi}{6}$

$2x=\frac{7\pi}{6}$

$x=\frac{7\pi}{6}\cdot\frac{1}{2}$

$\boxed{x=\frac{7\pi}{12}}$

$2x+\frac{\pi}{3}=2\pi$

$2x=\frac{2\pi}{1}-\frac{\pi}{3}$

$2x=\frac{6\pi}{3}-\frac{\pi}{3}$

$2x=\frac{5\pi}{3}$

$x=\frac{5\pi}{3}\cdot\frac{1}{2}$

$\boxed{x=\frac{5\pi}{6}}$

Example: Describe how the graph of $f(x)=-1+\frac{1}{2}\sin\left(2x+\frac{\pi}{3}\right)$ is different from the parent graph.

$d=-1$: graph is shifted down 1 unit

$a=\frac{1}{2}$: vertical shrink (graph is flatter than the parent)

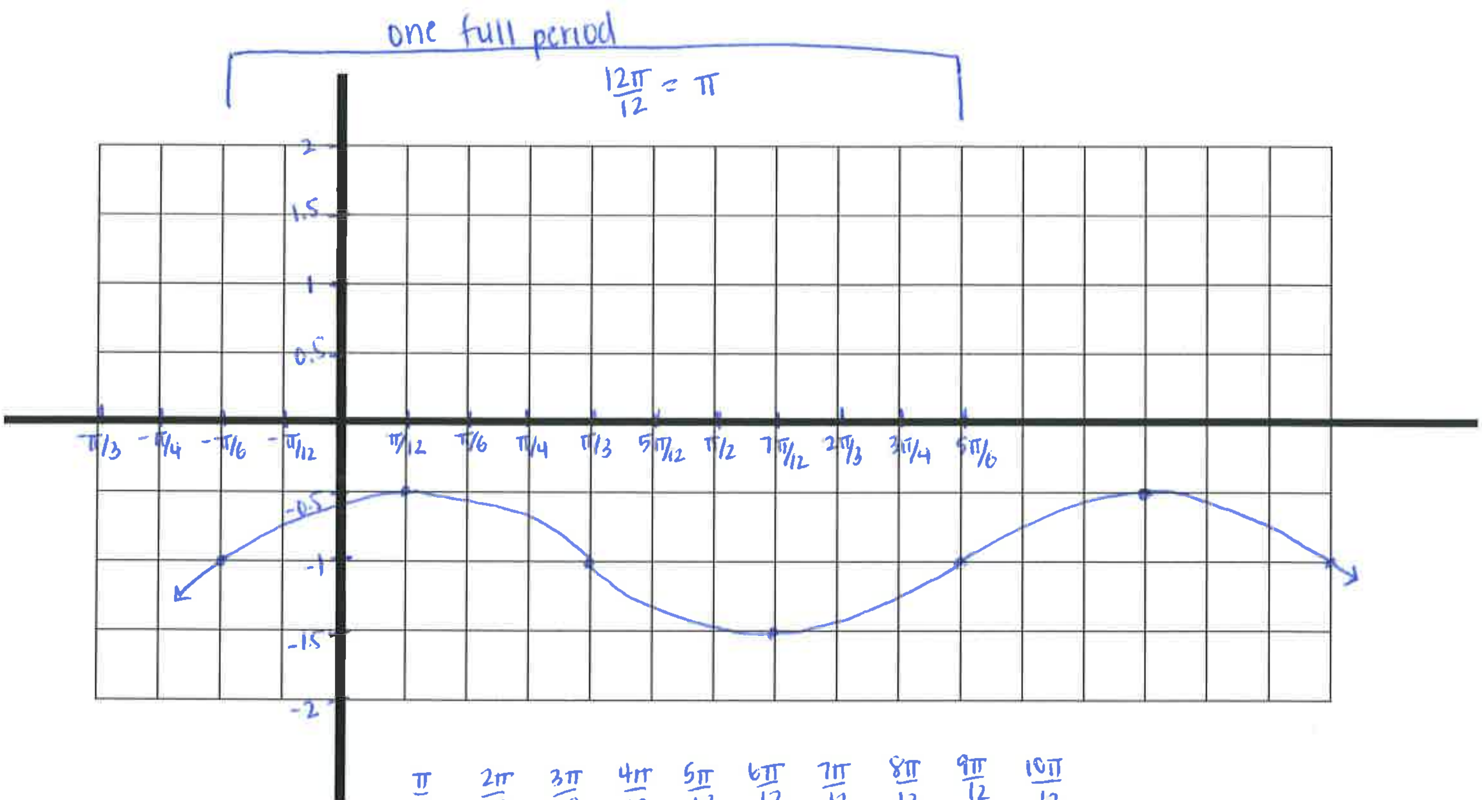
b & c: phase shift $\frac{\pi}{6}$ units to the left

Example: Graph the function $f(x) = -1 + \frac{1}{2}\sin\left(2x + \frac{\pi}{3}\right)$.

* always start with the parent sine function table of values

new x	x-axis	y-axis	$\frac{1}{2}y$	$\frac{1}{2}y - 1$
$-\frac{\pi}{6}$	0	0	0	-1
$\frac{\pi}{12}$	$\frac{\pi}{2}$	1	1/2	-0.5
$\pi/3$	π	0	0	-1
$7\pi/12$	$\frac{3\pi}{2}$	-1	-1/2	-1.5
$5\pi/6$	2π	0	0	-1

parent sine table (x-axis, y-axis) ↑ multiply a-value first ($\frac{1}{2}y$) ↑ add/subtract d-value last ($\frac{1}{2}y - 1$)



$\frac{\pi}{12}$ $\quad \frac{2\pi}{12} \to \frac{\pi}{6}$ $\quad \frac{3\pi}{12} \to \frac{\pi}{4}$ $\quad \frac{4\pi}{12} \to \frac{\pi}{3}$ $\quad \frac{5\pi}{12}$ $\quad \frac{6\pi}{12} \to \frac{\pi}{2}$ $\quad \frac{7\pi}{12}$ $\quad \frac{8\pi}{12} \to \frac{2\pi}{3}$ $\quad \frac{9\pi}{12} \to \frac{3\pi}{4}$ $\quad \frac{10\pi}{12} \to \frac{5\pi}{6}$

domain: $(-\infty, \infty)$ range: $[-1.5, 0.5]$ amp $= \frac{1}{2}$ period $= \pi$

no a or d values so y will not change

1. Graph the function $n(x) = \sin\left(2x - \frac{\pi}{2}\right)$ ← $b = 2,\ c = \frac{\pi}{2}$

To find new x-values:

$2x - \frac{\pi}{2} = 0$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{2} \cdot \frac{1}{2}$

$\boxed{x = \frac{\pi}{4}}$

$2x - \frac{\pi}{2} = \frac{\pi}{2}$

$2x = \pi$

$\boxed{x = \frac{\pi}{2}}$

$2x - \frac{\pi}{2} = \pi$

$2x = \frac{\pi}{1} + \frac{\pi}{2}$

$2x = \frac{2\pi}{2} + \frac{\pi}{2}$

$2x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} \cdot \frac{1}{2}$

$\boxed{x = \frac{3\pi}{4}}$

$2x - \frac{\pi}{2} = \frac{3\pi}{2}$

$2x = \frac{3\pi}{2} + \frac{\pi}{2}$

$2x = \frac{4\pi}{2}$

$2x = 2\pi$

$\boxed{x = \pi}$

$2x - \frac{\pi}{2} = 2\pi$

$2x = \frac{\pi}{2} + \frac{2\pi}{1}$

$2x = \frac{\pi}{2} + \frac{4\pi}{2}$

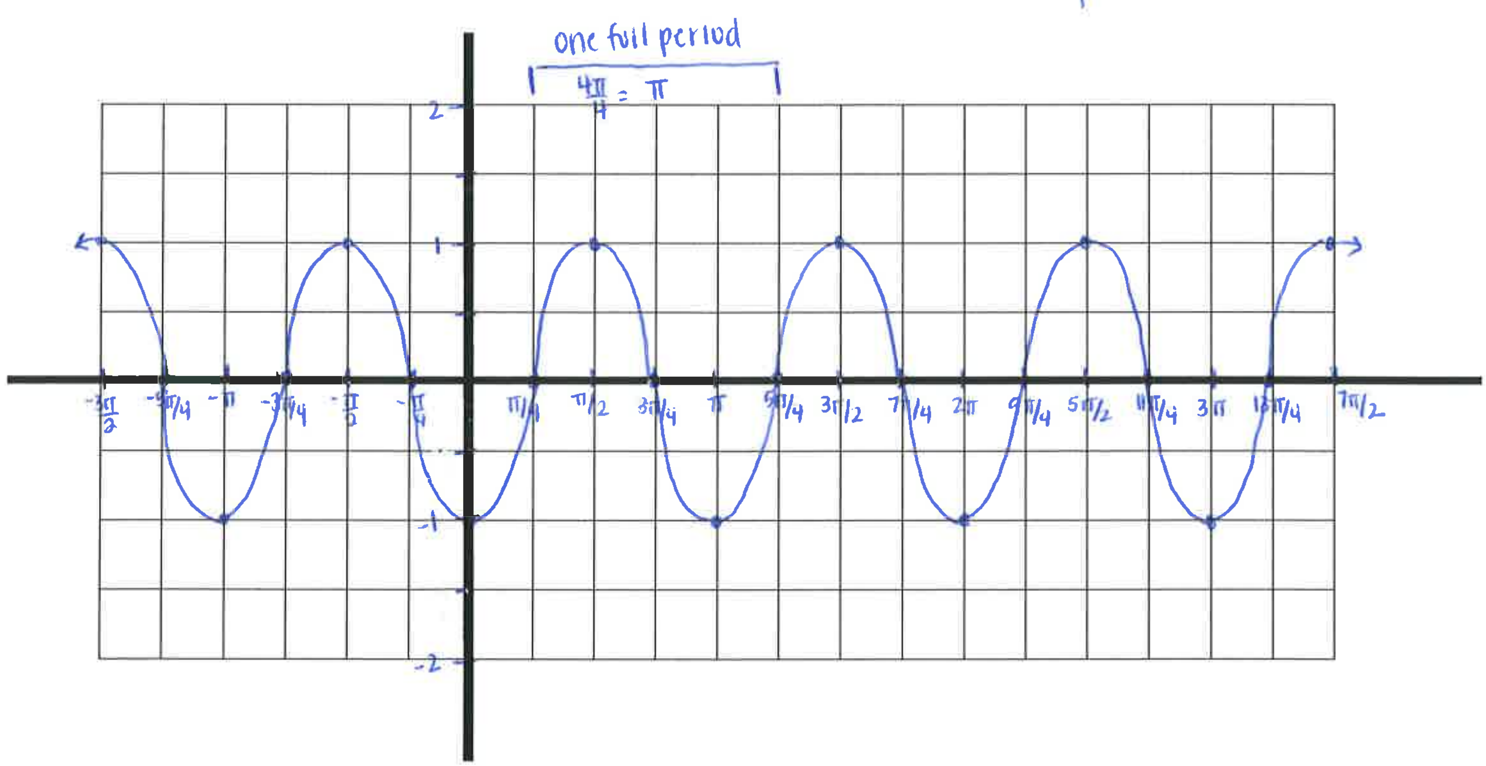
$2x = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} \cdot \frac{1}{2}$

$\boxed{x = \frac{5\pi}{4}}$

new x	x-axis	y-axis
$\pi/4$	0	0
$\pi/2$	$\pi/2$	1
$3\pi/4$	π	0
π	$3\pi/2$	-1
$5\pi/4$	2π	0

parent sine



domain: $(-\infty, \infty)$ range: $[-1, 1]$ amp: 1 period: $\frac{2\pi}{2} = \pi$ phase shift $= \left|\frac{c}{b}\right| = \left|\frac{\frac{\pi}{2}}{2}\right| = \left|\frac{\pi}{2} \cdot \frac{1}{2}\right| = \left|\frac{\pi}{4}\right|$

$= \frac{\pi}{4}$ units right