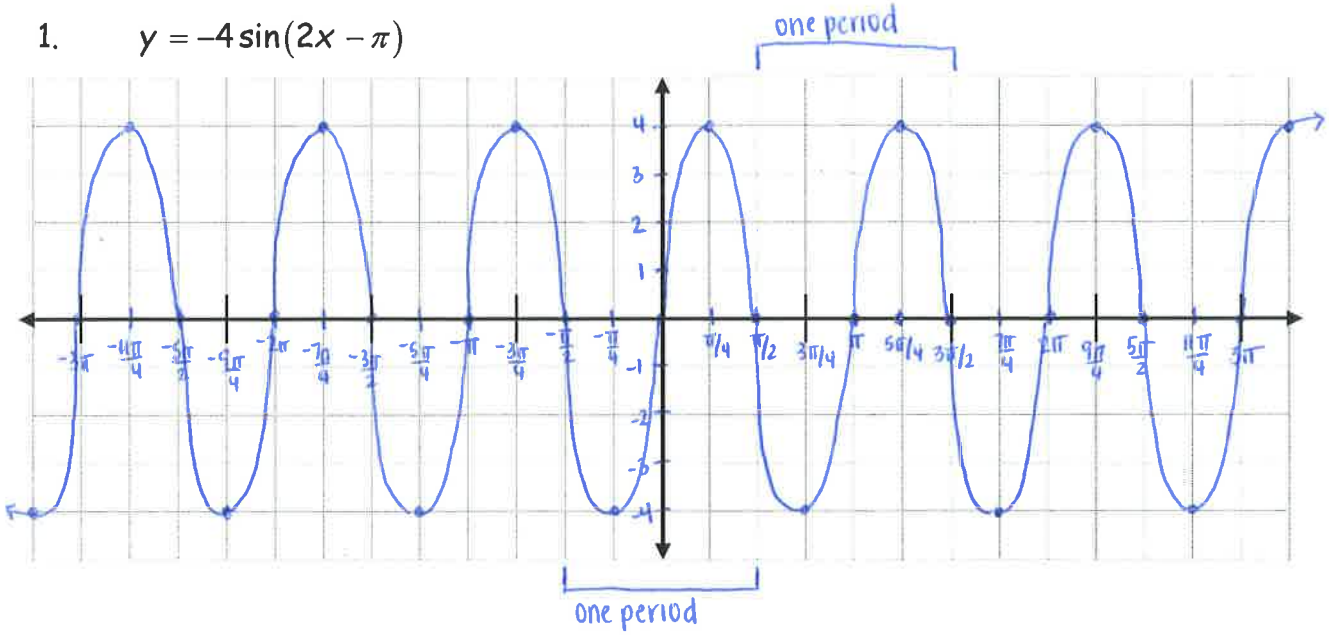
Pre-Calculus (A)
Do the Wave!

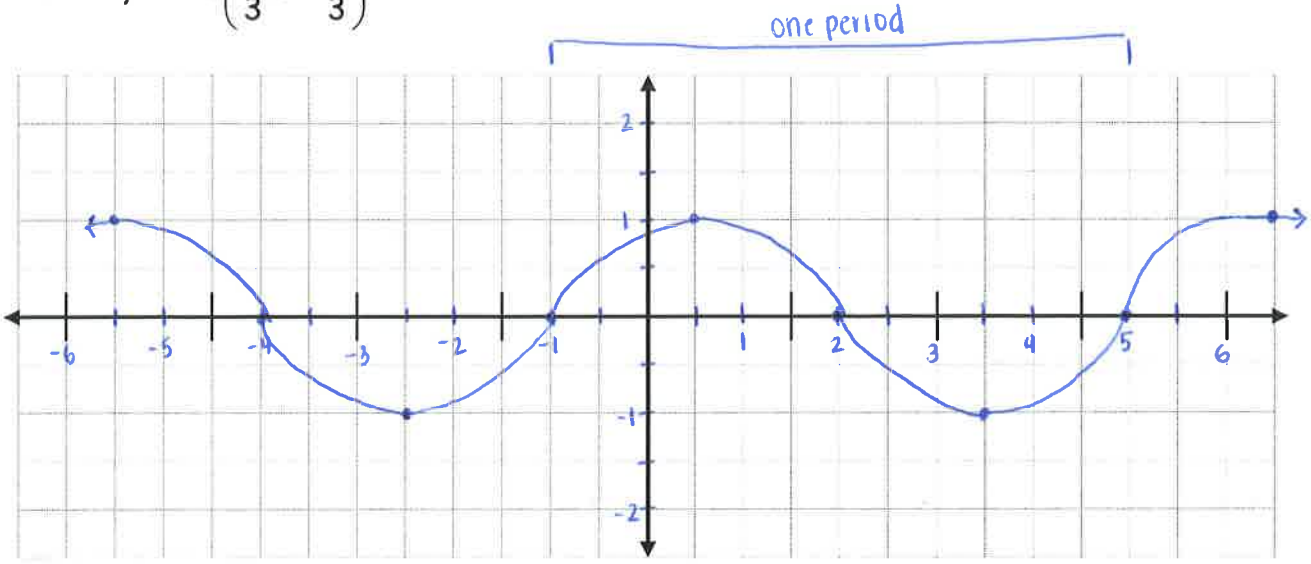
Name Key
Date

Graph **two** full periods of the following curves. Put your work on a separate sheet of paper.

1. $y = -4\sin(2x - \pi)$



2. $y = \sin\left(\frac{\pi}{3}x + \frac{\pi}{3}\right)$



$a=-4 \quad b=2 \quad c=\pi$

① $y = -4\sin(2x-\pi)$

To find new x-values:

$2x-\pi=0$ | $2x-\pi=\frac{\pi}{2}$ | $2x-\pi=\pi$ | $2x-\pi=\frac{3\pi}{2}$ | $2x-\pi=2\pi$

$2x=\pi$ | $2x=\frac{\pi}{2}+\frac{2\pi}{2}$ | $2x=2\pi$ | $2x=\frac{3\pi}{2}+\frac{2\pi}{2}$ | $2x=3\pi$

$\boxed{x=\frac{\pi}{2}}$ | $2x=\frac{3\pi}{2}$ | $\boxed{x=\pi}$ | $2x=\frac{5\pi}{2}$ | $\boxed{x=\frac{3\pi}{2}}$

$x=\frac{3\pi}{2}\cdot\frac{1}{2}$ | $x=\frac{5\pi}{2}\cdot\frac{1}{2}$

$\boxed{x=3\pi/4}$ | $\boxed{x=5\pi/4}$

new x	x-axis	y-axis	-4y
$\pi/2$	0	0	0
$3\pi/4$	$\pi/2$	1	-4
π	π	0	0
$5\pi/4$	$3\pi/2$	-1	4
$3\pi/2$	2π	0	0

domain: $(-\infty, \infty)$

range: $[-4, 4]$

amplitude $= |-4| = 4$

period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift $= \left|\frac{c}{b}\right| = \left|\frac{\pi}{2}\right| = \frac{\pi}{2}$ units to the right since $c>0$

② $y = \sin\left(\frac{\pi}{3}x + \frac{\pi}{3}\right)$

To find new x-values:

$\frac{\pi}{3}x + \frac{\pi}{3} = 0$

$\frac{\pi}{3}x = -\frac{\pi}{3}$

$\boxed{x = -1}$

$\frac{\pi}{3}x + \frac{\pi}{3} = \frac{\pi}{2}$

$\frac{\pi}{3}x = \frac{\pi}{2} - \frac{\pi}{3}$

$\frac{\pi}{3}x = \frac{3\pi}{6} - \frac{2\pi}{6}$

$\frac{\pi}{3}x = \frac{\pi}{6}$

$x = \frac{\pi}{6} \cdot \frac{3}{\pi}$

$\boxed{x = 1/2}$

$\frac{\pi}{3}x + \frac{\pi}{3} = \pi$

$\frac{\pi}{3}x = \frac{3\pi}{3} - \frac{\pi}{3}$

$\frac{\pi}{3}x = \frac{2\pi}{3}$

$x = \frac{2\pi}{3} \cdot \frac{3}{\pi}$

$\boxed{x = 2}$

$\frac{\pi}{3}x + \frac{\pi}{3} = \frac{3\pi}{2}$

$\frac{\pi}{3}x = \frac{3\pi}{2} - \frac{\pi}{3}$

$\frac{\pi}{3}x = \frac{9\pi}{6} - \frac{2\pi}{6}$

$\frac{\pi}{3}x = \frac{7\pi}{6}$

$x = \frac{7\pi}{6} \cdot \frac{3}{\pi}$

$x = 21/6$

$\boxed{x = 7/2}$

$\frac{\pi}{3}x + \frac{\pi}{3} = 2\pi$

$\frac{\pi}{3}x = \frac{6\pi}{3} - \frac{\pi}{3}$

$\frac{\pi}{3}x = \frac{5\pi}{3}$

$x = \frac{5\pi}{3} \cdot \frac{3}{\pi}$

$\boxed{x = 5}$

new x	x-axis	y-axis
-1	0	0
0.5	$\pi/2$	1
2	π	0
3.5	$3\pi/2$	-1
5	2π	0

domain: $(-\infty, \infty)$

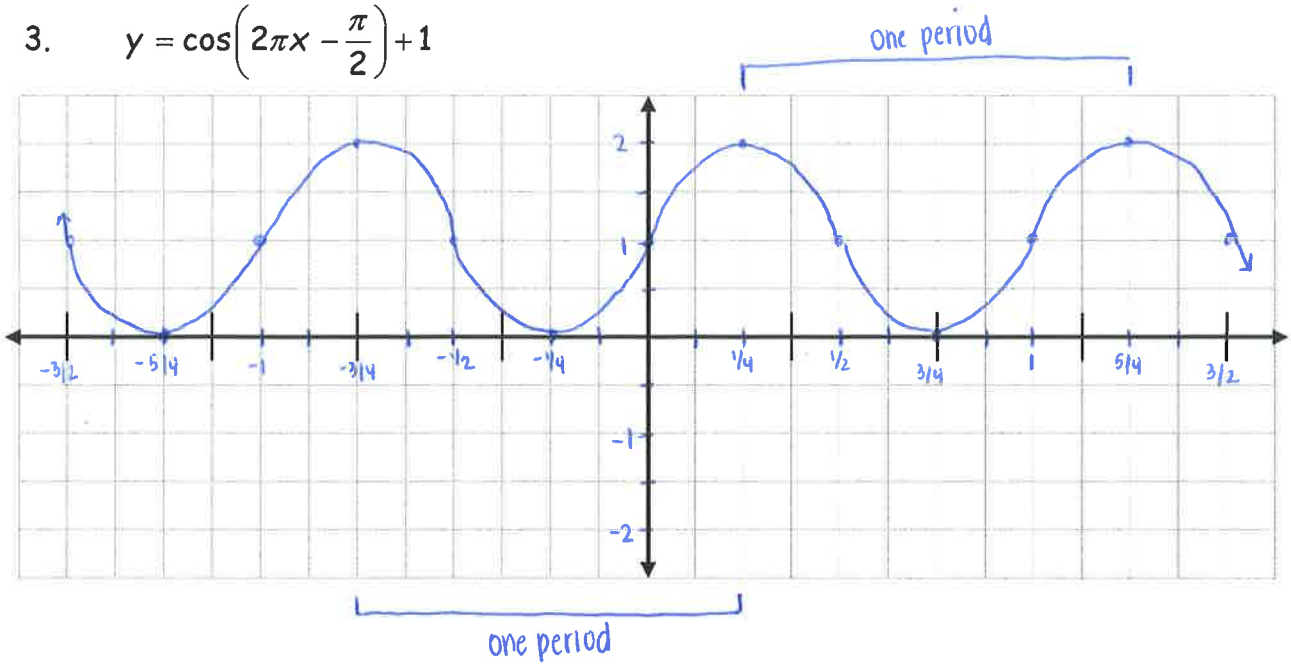
range: $[-1, 1]$

amp: 1

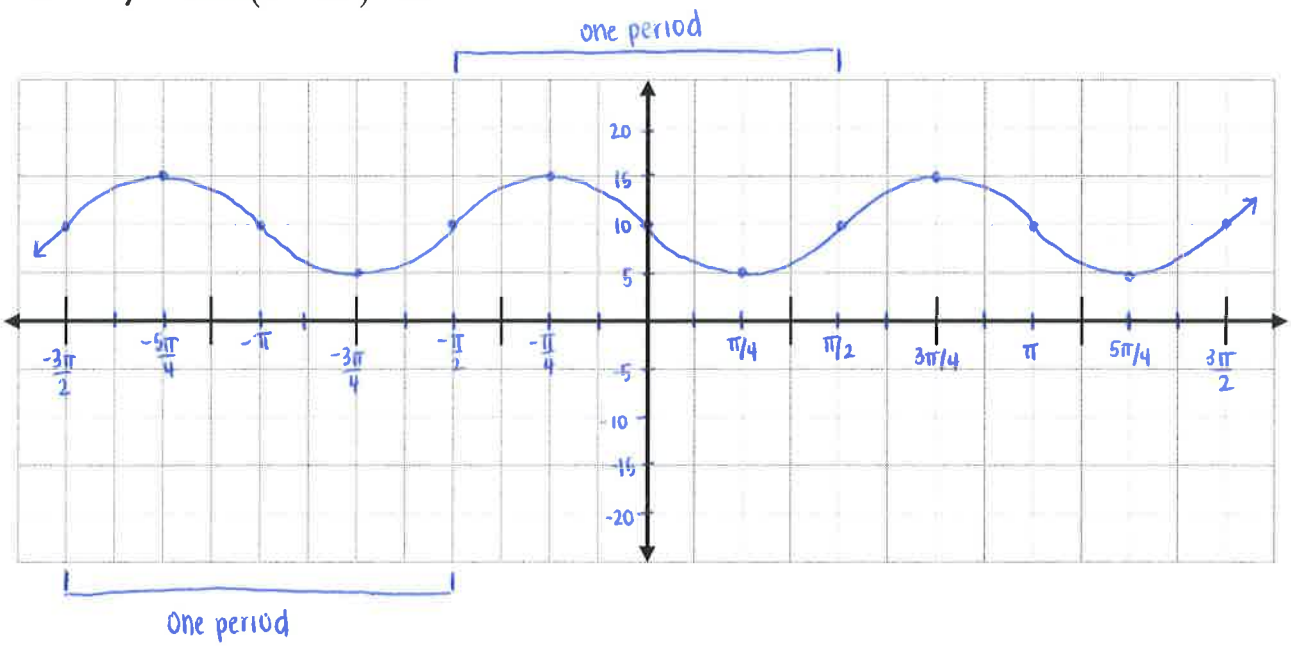
period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/3} = \frac{2\pi}{1} \div \frac{\pi}{3} = \frac{2\pi}{1} \cdot \frac{3}{\pi} = 6$

phase shift: $\left|\frac{c}{b}\right| = \left|\frac{-\frac{\pi}{3}}{\frac{\pi}{3}}\right| = |-1| = 1$ unit to the left since $c < 0$

3. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$



4. $y = 5\sin(\pi + 2x) + 10$



③ $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$

To find new x-values:

$2\pi x - \frac{\pi}{2} = 0$

$2\pi x = \frac{\pi}{2}$

$x = \frac{\pi}{2} \div \frac{2\pi}{1}$

$x = \frac{\pi}{2} \cdot \frac{1}{2\pi}$

$\boxed{x = \frac{1}{4}}$

$2\pi x - \frac{\pi}{2} = \frac{\pi}{2}$

$2\pi x = \pi$

$x = \frac{\pi}{2\pi}$

$\boxed{x = \frac{1}{2}}$

$2\pi x - \frac{\pi}{2} = \pi$

$2\pi x = \frac{2\pi}{2} + \frac{\pi}{2}$

$2\pi x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} \div \frac{2\pi}{1}$

$x = \frac{3\pi}{2} \cdot \frac{1}{2\pi}$

$\boxed{x = 3/4}$

$2\pi x - \frac{\pi}{2} = \frac{3\pi}{2}$

$2\pi x = \frac{4\pi}{2}$

$2\pi x = 2\pi$

$\boxed{x = 1}$

$2\pi x - \frac{\pi}{2} = 2\pi$

$2\pi x = \frac{4\pi}{2} + \frac{\pi}{2}$

$2\pi x = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} \div \frac{2\pi}{1}$

$x = \frac{5\pi}{2} \cdot \frac{1}{2\pi}$

$\boxed{x = 5/4}$

new x	x-axis	y-axis	y+1
1/4	0	1	2
1/2	$\pi/2$	0	1
3/4	π	-1	0
1	$3\pi/2$	0	1
5/4	2π	1	2

domain: $(-\infty, \infty)$

range: $[0, 2]$

amp: 1

period: $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$

phase shift: $\left|\frac{c}{b}\right| = \left|\frac{\frac{\pi}{2}}{2\pi}\right| = \left|\frac{\pi}{2} \div \frac{2\pi}{1}\right| = \left|\frac{\pi}{2} \cdot \frac{1}{2\pi}\right| = \left|\frac{1}{4}\right| = \frac{1}{4}$ to the right since $c > 0$.

④ $y = 5\sin(\pi + 2x) + 10$

To find new x-values:

$\pi + 2x = 0$

$2x = -\pi$

$\boxed{x = -\frac{\pi}{2}}$

$\pi + 2x = \frac{\pi}{2}$

$2x = \frac{\pi}{2} - \frac{\pi}{1}$

$2x = \frac{\pi}{2} - \frac{2\pi}{2}$

$2x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} \cdot \frac{1}{2}$

$\boxed{x = -\frac{\pi}{4}}$

$\pi + 2x = \pi$

$2x = 0$

$\boxed{x = 0}$

$\pi + 2x = \frac{3\pi}{2}$

$2x = \frac{3\pi}{2} - \frac{\pi}{1}$

$2x = \frac{3\pi}{2} - \frac{2\pi}{2}$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{2} \cdot \frac{1}{2}$

$\boxed{x = \pi/4}$

$\pi + 2x = 2\pi$

$2x = \pi$

$\boxed{x = \frac{\pi}{2}}$

new x	x-axis	y-axis	5y	5y+10
$-\pi/2$	0	0	0	10
$-\pi/4$	$\frac{\pi}{2}$	1	5	15
0	π	0	0	10
$\pi/4$	$\frac{3\pi}{2}$	-1	-5	5
$\pi/2$	2π	0	0	10

domain: $(-\infty, \infty)$

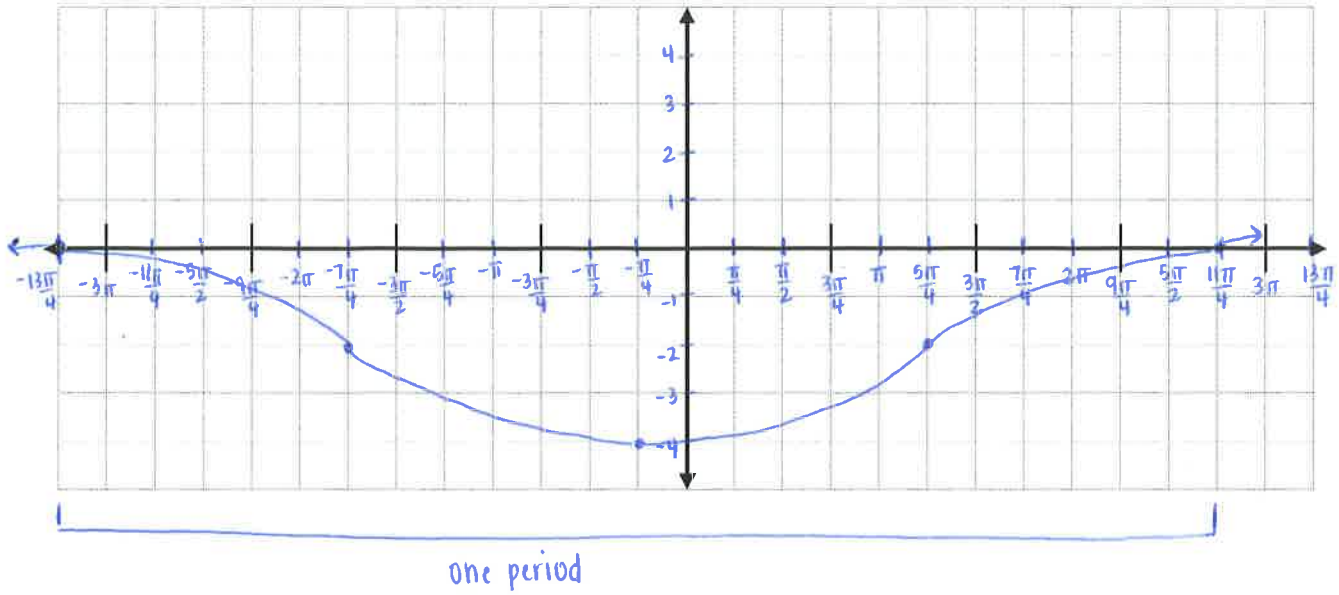
range: $[5, 15]$

amp = 5

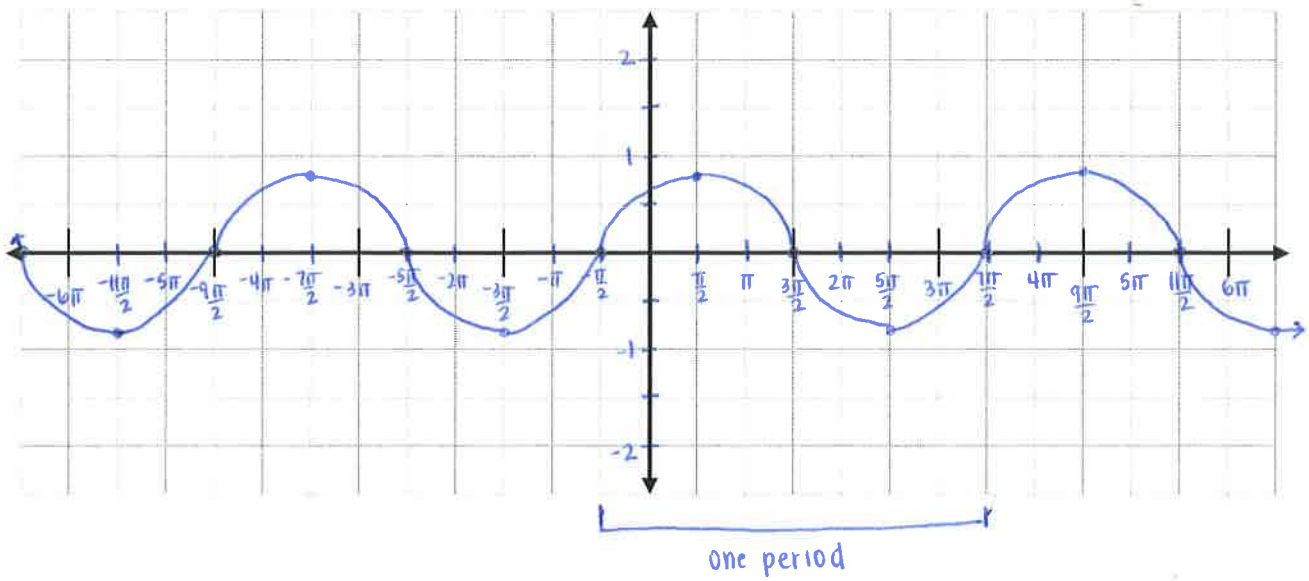
period = $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift = $\left|\frac{c}{b}\right| = \left|\frac{\pi}{2}\right| = \frac{\pi}{2}$ units to the left

5. $y = -2\cos\left(\frac{1}{3}x + \frac{\pi}{12}\right) - 2$



6. $y = \frac{2}{3}\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$



⑤ $y = -2\cos\left(\frac{1}{3}x + \frac{\pi}{12}\right) - 2$

To find new x-values:

$\frac{1}{3}x + \frac{\pi}{12} = 0$

$\frac{1}{3}x = -\frac{\pi}{12}$

$x = -\frac{\pi}{12} \cdot \frac{3}{1}$

$x = -\frac{3\pi}{12}$

$\boxed{x = -\frac{\pi}{4}}$

$\frac{1}{3}x + \frac{\pi}{12} = \frac{\pi}{2}$

$\frac{1}{3}x = \frac{\pi}{2} - \frac{\pi}{12}$

$\frac{1}{3}x = \frac{6\pi}{12} - \frac{\pi}{12}$

$\frac{1}{3}x = \frac{5\pi}{12}$

$x = \frac{5\pi}{12} \cdot \frac{3}{1}$

$x = \frac{15\pi}{12}$

$\boxed{x = \frac{5\pi}{4}}$

$\frac{1}{3}x + \frac{\pi}{12} = \frac{\pi}{1}$

$\frac{1}{3}x = \frac{12\pi}{12} - \frac{\pi}{12}$

$\frac{1}{3}x = \frac{11\pi}{12}$

$x = \frac{11\pi}{12} \cdot \frac{3}{1}$

$x = \frac{33\pi}{12}$

$\boxed{x = \frac{11\pi}{4}}$

$\frac{1}{3}x + \frac{\pi}{12} = \frac{3\pi}{2}$

$\frac{1}{3}x = \frac{3\pi}{2} - \frac{\pi}{12}$

$\frac{1}{3}x = \frac{18\pi}{12} - \frac{\pi}{12}$

$\frac{1}{3}x = \frac{17\pi}{12}$

$x = \frac{17\pi}{12} \cdot \frac{3}{1}$

$x = \frac{51\pi}{12}$

$\boxed{x = \frac{17\pi}{4}}$

$\frac{1}{3}x + \frac{\pi}{12} = \frac{2\pi}{1}$

$\frac{1}{3}x = \frac{24\pi}{12} - \frac{\pi}{12}$

$\frac{1}{3}x = \frac{23\pi}{12}$

$x = \frac{23\pi}{12} \cdot \frac{3}{1}$

$x = \frac{69\pi}{12}$

$\boxed{x = \frac{23\pi}{4}}$

new x	x-axis	y-axis	-2y	-2y-2
$-\pi/4$	0	1	-2	-4
$5\pi/4$	$\pi/2$	0	0	-2
$11\pi/4$	π	-1	2	0
$17\pi/4$	$3\pi/2$	0	0	-2
$23\pi/4$	2π	1	-2	-4

domain: $(-\infty, \infty)$

range: $[-4, 0]$

amplitude $= |-2| = 2$

period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = \frac{2\pi}{1} \div \frac{1}{3} = \frac{2\pi}{1} \cdot \frac{3}{1} = 6\pi$

phase shift $= \left|\frac{c}{b}\right| = \left|\frac{-\frac{\pi}{12}}{\frac{1}{3}}\right| = \left|-\frac{\pi}{12} \div \frac{1}{3}\right| = \left|-\frac{\pi}{12} \cdot \frac{3}{1}\right| = \left|-\frac{3\pi}{12}\right| = \left|-\frac{\pi}{4}\right| = \frac{\pi}{4}$ units to the left since $c < 0$

⑥ $y = \frac{2}{3}\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

To find new x-values:

$\frac{1}{2}x - \frac{\pi}{4} = 0$

$\frac{1}{2}x = \frac{\pi}{4}$

$x = \frac{\pi}{4} \cdot \frac{2}{1}$

$x = \frac{2\pi}{4}$

$\boxed{x = \frac{\pi}{2}}$

$\frac{1}{2}x - \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{1}{2}x = \frac{\pi}{2} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{2\pi}{4} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} \cdot \frac{2}{1}$

$x = \frac{6\pi}{4}$

$\boxed{x = \frac{3\pi}{2}}$

$\frac{1}{2}x - \frac{\pi}{4} = \frac{\pi}{1}$

$\frac{1}{2}x = \frac{4\pi}{4} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} \cdot \frac{2}{1}$

$x = \frac{10\pi}{4}$

$\boxed{x = \frac{5\pi}{2}}$

$\frac{1}{2}x - \frac{\pi}{4} = \frac{3\pi}{2}$

$\frac{1}{2}x = \frac{3\pi}{2} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{6\pi}{4} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{7\pi}{4}$

$x = \frac{7\pi}{4} \cdot \frac{2}{1}$

$x = \frac{14\pi}{4}$

$\boxed{x = \frac{7\pi}{2}}$

$\frac{1}{2}x - \frac{\pi}{4} = \frac{2\pi}{1}$

$\frac{1}{2}x = \frac{2\pi}{1} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{8\pi}{4} + \frac{\pi}{4}$

$\frac{1}{2}x = \frac{9\pi}{4}$

$x = \frac{9\pi}{4} \cdot \frac{2}{1}$

$x = \frac{18\pi}{4}$

$\boxed{x = \frac{9\pi}{2}}$

new x	x-axis	y-axis	$\frac{2}{3}y$
$\pi/2$	0	1	$2/3$
$3\pi/2$	$\pi/2$	0	0
$5\pi/2$	π	-1	$-2/3$
$7\pi/2$	$3\pi/2$	0	0
$9\pi/2$	2π	1	$2/3$

domain: $(-\infty, \infty)$

range: $[-2/3, 2/3]$

amplitude $= \frac{2}{3}$

period $= \frac{2\pi}{b} = \frac{2\pi}{1/2} = 2\pi \div \frac{1}{2} = 2\pi \cdot \frac{2}{1} = 4\pi$

phase shift $= \left|\frac{c}{b}\right| = \left|\frac{\frac{\pi}{4}}{\frac{1}{2}}\right| = \left|\frac{\pi}{4} \div \frac{1}{2}\right| = \left|\frac{\pi}{4} \cdot \frac{2}{1}\right| = \left|\frac{2\pi}{4}\right| = \left|\frac{\pi}{2}\right| = \frac{\pi}{2}$ units to the right since $c > 0$