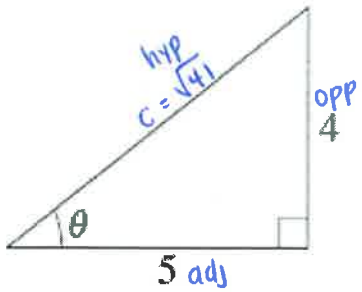
Pre-Calculus A
Section 4.3 – 4.4 Quiz Review

Name : Key
Date : _____ Period : _____

Please find the exact values of the six trigonometric functions of the angle θ shown in the figure.

1.

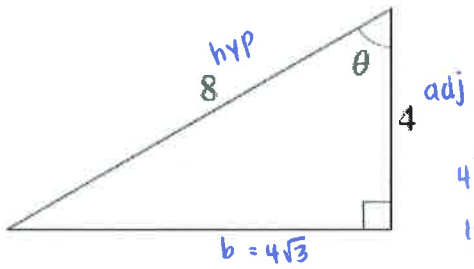


$4^2+5^2=c^2$
$16+25=c^2$
$41=c^2$
$c=\sqrt{41}$

$\sin\theta = \frac{opp}{hyp} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$ $\tan\theta = \frac{opp}{adj} = \frac{4}{5}$ $\sec\theta = \frac{hyp}{adj} = \frac{\sqrt{41}}{5}$

$\cos\theta = \frac{adj}{hyp} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$ $\csc\theta = \frac{hyp}{opp} = \frac{\sqrt{41}}{4}$ $\cot\theta = \frac{adj}{opp} = \frac{5}{4}$

2.

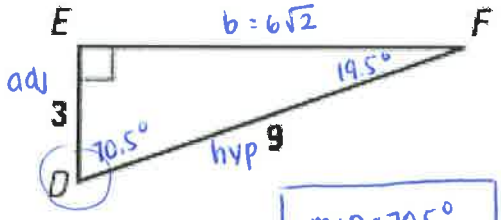


$4^2+b^2=8^2$
$16+b^2=64$
$b^2=48$
$b=\sqrt{48}$
$b=4\sqrt{3}$

* work on last page for #2

Please **solve** the right triangles below. Please write side lengths in simplest radical form, if possible.

3.



$3^2+b^2=9^2$
$9+b^2=81$
$b^2=72$
$b=\sqrt{72}$
$b=6\sqrt{2}$

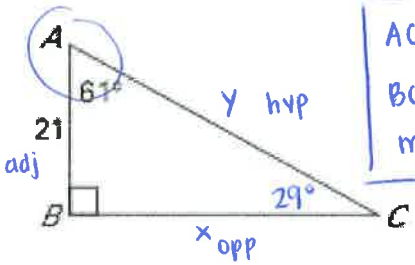
$\cos D = \frac{3}{9}$

$m\angle D = \cos^{-1}\left(\frac{3}{9}\right)$

$m\angle D = 70.5°$

$m\angle D = 70.5°$
$m\angle F = 19.5°$
$EF = 6\sqrt{2}$

4.



$AC = 43.3$
$BC = 37.9$
$m\angle C = 29°$

$\frac{\tan 61°}{1} = \frac{x}{21}$

$x = 21 \cdot \tan 61°$

$x = 37.9$

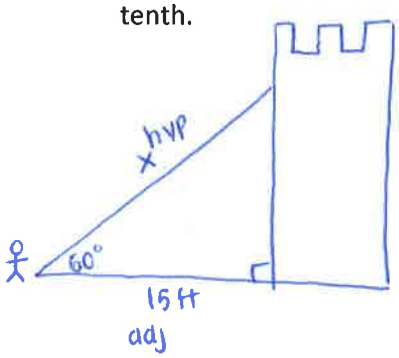
$\frac{\cos 61°}{1} = \frac{21}{y}$

$21 = y \cdot \cos 61°$

$y = \frac{21}{\cos 61°}$

$y = 43.3$

5. A damsel is in distress and is being held captive in a tower. Her knight in shining armor is on the ground below with a ladder. When the knight stands 15 feet from the base of the tower and looks up at his precious damsel, the angle of elevation to her window is 60 degrees. How long does the ladder have to be for him to be able to rescue her? Round your answer to the nearest tenth.



$\frac{\cos 60°}{1} = \frac{15}{x}$

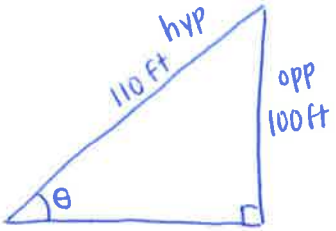
$x\cos 60° = 15$

$x = \frac{15}{\cos 60°}$

$x = 30$

The ladder must be 30 ft long

6. A fire department's longest ladder is 110 feet long, and the safety regulation states that they can use it for rescues up to 100 feet off the ground. What is the maximum safe angle of elevation for the rescue ladder? Round your answer to the nearest tenth.



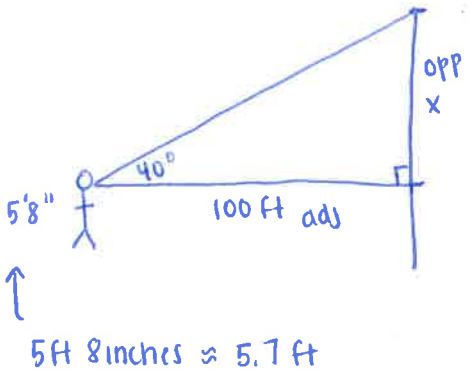
$\sin\theta = \frac{100}{110}$

$\theta = \sin^{-1}\left(\frac{100}{110}\right)$

$\theta = 65.4°$

The max. safe angle of elevation is 65.4°

7. You want to find the height of a tower used to transmit cell phone calls. You stand 100 feet away from the base of the tower and at eye-level, you measure the angle of elevation to be 40°. How high is the tower if your eye-level is 5 feet 8 inches off the ground? Round your answer to the nearest tenth.



5 ft 8 inches ≈ 5.7 ft

$\frac{\tan 40°}{1} = \frac{x}{100}$

$x = 100 \tan 40°$

$x = 83.9 \text{ ft} + 5.7 \text{ ft (height of person)}$

$= 89.6 \text{ ft}$

The tower is about 89.6 ft tall

In questions #8 – 11, state the quadrant in which θ lies.

8. $\sin\theta > 0$ and $\cos\theta > 0$ Quadrant I

9. $\sin\theta < 0$ and $\sec\theta < 0$ Quadrant III
   $\cos\theta < 0$

10. $\sin\theta > 0$ and $\cos\theta < 0$ Quadrant II

11. $\sec\theta > 0$ and $\cot\theta < 0$ Quadrant IV
    $\cos\theta > 0$ $\tan\theta < 0$

Sine +	All +
Tan +	Cos +

The point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

12. (5, 12) ⇒ Quad I

$x=5,\ y=12,\ r=13$

$r=\sqrt{(5)^2+(12)^2}$

$r=\sqrt{169}$

$r=13$

$\sin\theta=\frac{y}{r}=\frac{12}{13}$ $\quad \csc\theta=\frac{r}{y}=\frac{13}{12}$

$\cos\theta=\frac{x}{r}=\frac{5}{13}$ $\quad \sec\theta=\frac{r}{x}=\frac{13}{5}$

$\tan\theta=\frac{y}{x}=\frac{12}{5}$ $\quad \cot\theta=\frac{x}{y}=\frac{5}{12}$

13. (-4, 10) ⇒ Quad II

$x=-4,\ y=10,\ r=2\sqrt{29}$

$r=\sqrt{(-4)^2+(10)^2}$

$r=\sqrt{116}$

$r=2\sqrt{29}$

$\sin\theta=\frac{y}{r}=\frac{10}{2\sqrt{29}}=\frac{5}{\sqrt{29}}=\frac{5\sqrt{29}}{29}$ $\quad \csc\theta=\frac{r}{y}=\frac{2\sqrt{29}}{10}=\frac{\sqrt{29}}{5}$

$\cos\theta=\frac{x}{r}=\frac{-4}{2\sqrt{29}}=\frac{-2}{\sqrt{29}}=-\frac{2\sqrt{29}}{29}$ $\quad \sec\theta=\frac{r}{x}=\frac{+2\sqrt{29}}{-4}=\frac{\sqrt{29}}{2}$

$\tan\theta=\frac{y}{x}=-\frac{10}{4}=-\frac{5}{2}$ $\quad \cot\theta=\frac{x}{y}=\frac{-4}{10}=-\frac{2}{5}$

14. (-5, -2) ⇒ Quad III

$x=-5,\ y=-2,\ r=\sqrt{29}$

$r=\sqrt{(-5)^2+(-2)^2}$

$r=\sqrt{29}$

$\sin\theta=\frac{y}{r}=\frac{-2}{\sqrt{29}}=-\frac{2\sqrt{29}}{29}$ $\quad \csc\theta=\frac{r}{y}=\frac{\sqrt{29}}{-2}$

$\cos\theta=\frac{x}{r}=\frac{-5}{\sqrt{29}}=-\frac{5\sqrt{29}}{29}$ $\quad \sec\theta=\frac{r}{x}=\frac{\sqrt{29}}{-5}$

$\tan\theta=\frac{y}{x}=\frac{-2}{-5}=\frac{2}{5}$ $\quad \cot\theta=\frac{x}{y}=\frac{-5}{-2}=\frac{5}{2}$

Find the values of the six trigonometric functions of θ with the given constraint.

15. $\cos\theta=\frac{8}{17}$ (x = 8, r = 17) and $\tan\theta<0$ Quad IV

$r=\sqrt{x^2+y^2}$

$17=\sqrt{(8)^2+y^2}$

$17=\sqrt{64+y^2}$

$289=64+y^2$

$225=y^2$

$y=15 \Rightarrow \boxed{y=-15}$

$\sin\theta=\frac{y}{r}=\frac{-15}{17}$ $\quad \csc\theta=\frac{r}{y}=-\frac{17}{15}$

$\cos\theta=\frac{8}{17}$ $\quad \sec\theta=\frac{17}{8}$

$\tan\theta=\frac{y}{x}=\frac{-15}{8}$ $\quad \cot\theta=\frac{x}{y}=-\frac{8}{15}$

16. $\tan\theta=-\frac{15}{8}$ (y = 15, x = −8) and $\sin\theta>0$ Quad II : $x=-8,\ y=15,\ r=17$

$r=\sqrt{x^2+y^2}$

$r=\sqrt{(-8)^2+(15)^2}$

$r=\sqrt{64+225}$

$r=\sqrt{289}$

$r=17$

$\sin\theta=\frac{y}{r}=\frac{15}{17}$ $\quad \csc\theta=\frac{r}{y}=\frac{17}{15}$

$\cos\theta=\frac{x}{r}=-\frac{8}{17}$ $\quad \sec\theta=\frac{r}{x}=\frac{17}{-8}$

$\tan\theta=-\frac{15}{8}$ $\quad \cot\theta=-\frac{8}{15}$

sin: − ⇒ Quad III

cos: −

17. $\csc\theta = -\frac{5}{3}$ and $\cos\theta < 0$ $\quad x=-4,\ y=-3,\ r=5$

$r = \sqrt{x^2+y^2}$

$5 = \sqrt{x^2+(-3)^2}$

$5 = \sqrt{x^2+9}$

$25 = x^2+9$

$16 = x^2$

$x = 4$

$\boxed{x=-4}$

in Quad III

$\sin\theta = \frac{y}{r} = \frac{-3}{5}$ $\quad \csc\theta = \frac{r}{y} = -\frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{-4}{5}$ $\quad \sec\theta = \frac{r}{x} = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$ $\quad \cot\theta = \frac{x}{y} = \frac{-4}{-3} = \frac{4}{3}$

18. $\cot\theta = -3$ and $\frac{\pi}{2} < \theta < \pi$ $\quad x=-3,\ y=1,\ r=\sqrt{10}$

tan −

Quad II

$\cot\theta = -\frac{3}{1}$ (x over y)

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-3)^2+(1)^2}$

$r = \sqrt{9+1}$

$r = \sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $\quad \csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$ $\quad \sec\theta = \frac{r}{x} = -\frac{\sqrt{10}}{3}$

$\tan\theta = \frac{y}{x} = -\frac{1}{3}$ $\quad \cot\theta = \frac{x}{y} = -\frac{3}{1} = -3$

* Work for #2

$\sin\theta = \frac{opp}{hyp} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $\quad \csc\theta = \frac{hyp}{opp} = \frac{8}{4\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{adj}{hyp} = \frac{4}{8} = \frac{1}{2}$ $\quad \sec\theta = \frac{hyp}{adj} = \frac{8}{4} = \frac{2}{1} = 2$

$\tan\theta = \frac{opp}{adj} = \frac{4\sqrt{3}}{4} = \sqrt{3}$ $\quad \cot\theta = \frac{adj}{opp} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$