Pre-Calculus A
Section 3.4 Solving Puzzle

Name : ______________________
Date: ________________ Period : ___

## What Happened to the Snowman During the Heat Wave?

Solve each equation below and find your answer in the corresponding answer column. Write the letter of that exercise in the box that contains the number of the answer.

| | | | |
|---|---|---|---|
| E | $2\ln(x+3)-3=0$ | 9 | 395.4288 |
| S | $\log(-x-4)=2$ | 4 | 6 |
| O | $\ln\sqrt{x+8}=3$ | 6 | 1.3208 |
| L | $3^{x+3}=\frac{1}{81}$ | 15 | 1.4817 |
| A | $\ln(2x-1)=\ln(11)$ | 19 | -7 |
| F | Evaluate $\log_{16}0.63$ | 25 | -104 |
| E | $\frac{1025}{8+e^{4x}}=5$ | 28 | -0.1666 |

1

| | | | |
|---|---|---|---|
| L | Evaluate $\log_2\sqrt[3]{2}$ | 1 | 0.1931 |
| E | $\log_{1/4}(x-1)=0$ | 20 | 1.04 |
| D | $2(3^{x+1})-4=16$ | 13 | 2 |
| O | $\ln(x-1)+2\ln 5=0$ | 14 | -8 , 2 |
| M | $\log_{25}5=x$ | 5 | 1.0959 |
| H | $e^{2x+1}=4$ | 10 | 0.5 |
| T | $\log_5(3r^2+16)=\log_5(4r^2+6r)$ | 27 | 1/3 |

2

| | | | |
|---|---|---|---|
| O | $\ln x-\ln 3=2$ | 11 | 23 |
| A | $\log x+\log(x-15)=2$ | 17 | 22.1671 |
| I | $e^{4x}=e^{x^2+3}$ | 24 | No solution |
| P | $\log_4(3x-5)=3$ | 7 | 20 |
| E | $4^{2x+1}=\left(4^{-1}\right)^{2x+1}$ | 23 | 3 , 1 |
| M | $\ln(x-4)-\ln(x+1)=\ln 6$ | 21 | 0.7151 |
| F | $7^{3x}=65$ | 2 | ~~-1/6~~ -1/2 |

3

| | | | |
|---|---|---|---|
| E | $3+7\log_7(-2p+5)=-4$ | 18 | 0.3863 |
| M | $9^{x+1}=27^x$ | 26 | 2.4286 |
| O | $23-5e^{x+1}=3$ | 22 | 0.6065 |
| C | $\log_2 3+\log_2 x=\log_2 5+\log_2(x-2)$ | 16 | 3.3219 |
| H | $5+2\ln x=4$ | 3 | 2 |
| L | $\log_7(x+2)=-2$ | 12 | -1.9796 |
| P | $2^x+5=15$ | 8 | 5 |

4

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | E | M | A | D | E | A | C | O | M | P | L | E | T | E | P | O | O | L | O | F | H | I | M | S | E | L | F |

**More Application Practice!**

$A = Pe^{rt}$

1) You deposit $8500 into an account that pays 3.5% interest compounded continuously. How long will it take for the money to triple?

$A = 8500 \times 3 = 25500$

$P = 8500$

$r = .035$

$t = ?$

$25500 = 8500e^{.035t}$

$3 = e^{.035t}$

$\ln 3 = \cancel{\ln e}^{.035t}$

$\ln 3 = .035t$

$t = \frac{\ln 3}{.035} \approx \boxed{31.4 \text{ years}}$

2) The number N of bacteria in a culture is given by the model $N = 175e^{kt}$ where t is time in hours. If $N = 420$ when $t = 8$, estimate the time required for the population to double in size.

$N = 420$

$t = 8$

$K = ?$

$420 = 175e^{8K}$

$2.4 = e^{8K}$

$\ln 2.4 = 8K$

$K = \frac{\ln 2.4}{8} \approx .1094$

$N = 350$

$K = .1094$

$t = ?$

$350 = 175e^{.1094t}$

$2 = e^{.1094t}$

$\ln 2 = .1094t$

$t = \frac{\ln 2}{.1094} \approx \boxed{6.3 \text{ hours}}$

3) The logistic growth model $P(t) = \frac{1000}{1 + 32.33e^{-0.439t}}$ represents the population of a culture of bacteria after t hours. When will the amount of bacteria be 800?

$P = 800$

$t = ?$

$\frac{800}{1} = \frac{1000}{1 + 32.33e^{-0.439t}}$

$1000 = 800(1 + 32.33e^{-0.439t})$

$1.25 = 1 + 32.33e^{-0.439t}$

$0.25 = 32.33e^{-0.439t}$

$\frac{0.25}{32.33} = e^{-0.439t}$

$\ln\left(\frac{0.25}{32.33}\right) = -0.439t$

$t = \frac{\ln\left(\frac{0.25}{32.33}\right)}{-0.439} \approx \boxed{11.1 \text{ hrs}}$

4) The values y (in billions of dollars) of U.S. currency in circulation in the years 2000 through 2007 can be modeled by $y = -451 + 444\ln t$, where t represents the year, with $t = 10$ corresponding to 2000. During which year did the value of the U.S. currency in circulation exceed $690 billion?

$690 = -451 + 444\ln t$

$1141 = 444\ln t$

$\ln t = \frac{1141}{444}$

$e^{\left(\frac{1141}{444}\right)} = t$

$t \approx 13.1$

$t = 10 : 2000$

$t = 11 : 2001$

$t = 12 : 2002$

$\boxed{t = 13 : 2003}$

**Puzzle Answer Key :**

E: 15, S: 25, O: 9, L: 19, A: 4, F: 28, E: 6

O: 17, A: 7, I: 23, P: 11, E: 2, M: 24, F: 21

L: 27, E: 13, D: 5, O: 20, M: 10, H: 1, T: 14

E: 26, M: 3, O: 18, C: 8, H: 22, L: 12, P: 16

**Applications Answer Key:**

1) 31.4 years ✓

2) 6.3 hours

3) 11.1 hours ✓

4) 2003 ✓

3.4 Solving Puzzle : First Box

E $2\ln(x+3)=3$

$\ln(x+3)=3/2$

$e^{3/2}=x+3$

$x=e^{3/2}-3$

$x=1.4817$

S $10^{2}=-x-4$

$100=-x-4$

$104=-x$

$x=-104$

O $(e^{3})^{2}=(\sqrt{x+8})^{2}$

$e^{6}=x+8$

$x=e^{6}-8$

$x\approx 395.4288$

L $3^{x+3}=3^{-4}$

$x+3=-4$

$x=-7$

A $2x-1=11$

$2x=12$

$x=6$

F $\frac{\log 0.63}{\log 16}\approx -0.1666$

E $\frac{1025}{8+e^{4x}}=\frac{5}{1}$

$1025=5(8+e^{4x})$

$1025=40+5e^{4x}$

$985=5e^{4x}$

$197=e^{4x}$

$\ln 197=\cancel{\ln e}^{4x}$

$\ln 197=4x$

$x=\frac{\ln 197}{4}$

$x\approx 1.3208$

Second Box

L $\log_2 \sqrt[3]{2}$

$= \frac{\log \sqrt[3]{2}}{\log 2}$

$\approx 0.3333$ or $1/3$

E $\frac{1}{4}^0 = x-1$

$1 = x-1$

$x=2$

D $2(3^{x+1}) = 20$

$3^{x+1} = 10$

$\cancel{\log_3 3}^{x+1} = \log_3 10$

$x+1 = \log_3 10$

$x = \log_3 10 - 1$

$x = \frac{\log 10}{\log 3} - 1$

$x \approx 1.0959$

O $\ln(x-1) + \ln 5^2 = 0$

$\ln(x-1) + \ln 25 = 0$

$\ln 25(x-1) = 0$

$\ln 25x - 25 = 0$

$e^0 = 25x - 25$

$1 = 25x - 25$

$26 = 25x$

$x = 1.04$

M $25^x = 5$

$x = 1/2$

since $25^{1/2} = 5$

or $\sqrt{25} = 5$

H $\cancel{\ln e}^{2x+1} = \ln 4$

$2x+1 = \ln 4$

$2x = \ln 4 - 1$

$x = \frac{\ln 4 - 1}{2}$

$x \approx 0.1931$

T $3r^2+16 = 4r^2+6r$

$16 = r^2+6r$

$0 = r^2+6r-16$

$0 = (x+8)(x-2)$

$x=2,\ x=-8$

Check: $\log_5(3(2)^2+16) = \log_5(4(2)^2+6(2))$ ✓

$\log_5(3(-8)^2+16) = \log_5(4(-8)^2+6(-8))$

$= \log_5(208) = \log_5(208)$ ✓

Third Box

O $\ln \frac{x}{3} = 2$

$\frac{e^2}{1} = \frac{x}{3}$

$x = 3e^2$

$x = 22.1672$

A $\log(x^2 - 15x) = 2$

$10^2 = x^2 - 15x$

$100 = x^2 - 15x$

$0 = x^2 - 16x - 100$

$0 = (x-20)(x+5)$

$x = 20, \cancel{x = -5}$

Check: $\log(20) + \log(20-15) = 2$ ✓

$\log(-5) + \log(-5-15) = 2$ ×

I $4x = x^2 + 3$

$0 = x^2 - 4x + 3$

$0 = (x-3)(x-1)$

$x = 3, x = 1$

P $4^3 = 3x - 5$

$64 = 3x - 5$

$69 = 3x$

$x = 23$

E $4^{2x+1} = 4^{-2x-1}$

$2x + 1 = -2x - 1$

$4x + 1 = -1$

$4x = -2$

$x = -1/2$

M $\ln \frac{x-4}{x+1} = \ln 6$

$\frac{x-4}{x+1} = \frac{6}{1}$

$x - 4 = 6(x+1)$

$x - 4 = 6x + 6$

$-4 = 5x + 6$

$-10 = 5x$

$\cancel{x = -2}$ No solution

Check: $\ln(-2-4) - \ln(-2+1) = \ln 6$

$\ln(-6) - \ln(-1) = \ln 6$ ×

F $\cancel{\log_7 7}^{3x} = \log_7 65$

$3x = \log_7 65$

$x = \frac{\log_7 65}{3}$

$x = \left(\frac{\log 65}{\log 7}\right) \div 3$

$x \approx 0.7151$

Fourth Box

E $7\log_7(-2p+5)=-7$

$\log_7(-2p+5)=-1$

$7^{-1}=-2p+5$

$7^{-1}-5=-2p$

$p=\frac{7^{-1}-5}{-2}$

$p\approx 2.4286$

P $2^x=10$

$\cancel{\log_2} 2^x=\log_2 10$

$x=\log_2 10$

$x=\frac{\log 10}{\log 2}$

$x\approx 3.3219$

M $(3^2)^{x+1}=(3^3)^x$

$2x+2=3x$

$2=x$

O $-5e^{x+1}=-20$

$e^{x+1}=4$

$\cancel{\ln e}^{x+1}=\ln 4$

$x+1=\ln 4$

$x=\ln 4-1$

$x\approx 0.3863$

C $\log_2 3x=\log_2(5x-10)$

$3x=5x-10$

$-2x=-10$

$x=5$

H $2\ln x=-1$

$\ln x=-1/2$

$e^{-1/2}=x$

$x\approx 0.6065$

L $7^{-2}=x+2$

$x=7^{-2}-2$

$x\approx -1.9796$