Pre-Calculus A
Polynomials Quiz Review

Name: Key
Date: ______ Period: ____

1. Find the zeros of the function $f(x) = x^4 - x^3 - 2x^2$ algebraically.

$x^2(x^2 - x - 2)$ ← Factor out GCF of x^2 first

$x^2(x-2)(x+1)$

$\boxed{x=0,\ x=-1,\ x=2}$

** 2. Determine the complete factorization of the polynomial $f(x) = 3x^3 - 10x^2 + 12x - 22$ given one factor $(x-4)$.

Skip #2 :)

3. Find the possible rational zeros and then determine the complete factorization of the function $g(x) = 2x^3 + 11x^2 - 21x - 90$.

$$\begin{array}{r|rrrr} 3 & 2 & 11 & -21 & -90 \\ & & 6 & 51 & 90 \\ \hline & 2 & 17 & 30 & 0 \end{array}$$

$(x-3)(2x^2+17x+30)$

$\boxed{(x-3)(2x+5)(x+6)}$

4. Use the Rational Zero Test to list all possible rational zeros of the function $h(x) = 4x^3 - 11x^2 + 10x - 3$.

$\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4} = \boxed{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}}$

5. Use synthetic division to divide $(24x^2 - x - 8) \div (3x - 2)$.

$3x - 2 = 0 \Rightarrow 3x = 2 \Rightarrow x = 2/3$

↑ this # goes on outside of synthetic

$$\begin{array}{r|rrr} \frac{2}{3} & 24 & -1 & -8 \\ & & 16 & 10 \\ \hline & 24 & 15 & 2 \end{array}$$

$\boxed{24x + 15 + \dfrac{2}{3x-2}}$

6. Use synthetic division and one given zero to factor the polynomial completely and find all real zeros of the function. $2x^3 - 15x^2 + 27x - 10 = 0,\ x = \dfrac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$(2x-1)(2x^2-14x+20)$

$2(2x-1)(x^2-7x+10)$

$2(2x-1)(x-5)(x-2)$

$\boxed{x = 1/2,\ x=5,\ x=2}$

$\dfrac{14 \pm \sqrt{36}}{4} = \dfrac{14 \pm 6}{4} < \begin{matrix} 5 \\ 2 \end{matrix}$

7. State all possible rational zeros of the function $h(x) = 2x^3 - 3x^2 - 3x + 2$, then find all real zeros.

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -3 & 2 \\ & & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

$(x+1)(2x^2-5x+2)$

$(x+1)(2x-1)(x-2)$

$\boxed{x=-1,\ x=1/2,\ x=2}$

8. State all possible rational zeros of the function $f(x) = x^3 - 4x^2 - 2x + 3$, then find all real zeros.

$$\begin{array}{r|rrrr} -1 & 1 & -4 & -2 & 3 \\ & & -1 & 5 & -3 \\ \hline & 1 & -5 & 3 & 0 \end{array}$$

$(x+1)(x^2-5x+3)$

$\dfrac{5 \pm \sqrt{13}}{2}$

$\boxed{x=-1,\ x=\dfrac{5+\sqrt{13}}{2},\ x=\dfrac{5-\sqrt{13}}{2}}$

discriminant: b^2-4ac

$=(-5)^2-4(1)(3)=13$

9. Use the Remainder Theorem to evaluate the function $g(x) = 2x^3 - 7x + 3$ at each given value.

a. $g(1) = 2(1)^3 - 7(1) + 3$ $\boxed{-2}$

b. $g(-2) = 2(-2)^3 - 7(-2) + 3$ $\boxed{1}$

c. $g(2) = 2(2)^3 - 7(2) + 3$ $\boxed{5}$

For questions #10 – 12, perform the following operations and write all answers in standard form.

10. $(8+5i)-(2+3i)$ $\quad 8+5i-2-3i = \boxed{6+2i}$

11. $(1+2i)(6-3i)$ $= 6-3i+12i-6i^2 = 6+12i+6 = \boxed{12+12i}$

12. $\dfrac{4+3i}{3-2i} \cdot \dfrac{(3+2i)}{(3+2i)} = \dfrac{12+8i+9i+6i^2}{9-4i^2} = \dfrac{6+17i}{13} = \boxed{\dfrac{6}{13}+\dfrac{17}{13}i}$

For questions #13 – 14, write the complex conjugate of the number, then multiply the two quantities.

13. $-2+3i$ $(2-3i)$ = $4-9i^2$ = $\boxed{13}$

14. $6-2i$ $(6+2i)$

$36-4i^2 = \boxed{40}$

15. Show that $2-3i$ is a solution of $x^2-4x+13=0$.

discriminant: $b^2-4ac = (-4)^2-4(1)(13) = -36$

$$x = \frac{4\pm\sqrt{-36}}{2(1)} = \frac{4\pm\sqrt{36}i}{2} = \frac{4\pm 6i}{2} = 2\pm 3i$$

$\boxed{x=2-3i,\ x=2+3i}$

16. Find all real and/or imaginary solutions of the function $f(x)=x^3-5x^2+11x-15$.

3	1	-5	11	-15
		3	-6	15
	1	-2	5	0

$(x-3)(x^2-2x+5)$

discriminant: $b^2-4ac = (-2)^2-4(1)(5) = -16$

$$\frac{2\pm\sqrt{-16}}{2} = \frac{2\pm 4i}{2} = 1\pm 2i$$

$\boxed{x=3,\ x=1+2i,\ x=1-2i}$

17. If 2 – 3i is a root of a polynomial equation, then ___2+3i___ is also a root.

18. Find a second-degree polynomial equation with real coefficients that has a root of $-3i$.

$x=-3i$: $(x+3i)$

$x=3i$: $(x-3i)$

$(x-3i)(x+3i)$

$x^2+\cancel{3ix}-\cancel{3ix}-9i^2$

$x^2-9(-1)$

$\boxed{f(x)=x^2+9}$

19. Find a second-degree polynomial equation with real coefficients that has a root of $1 - i$.

$x = 1 - i : (x - (1 - i))$

$x = 1 + i : (x - (1 + i))$

$(x - (1 - i))(x - (1 + i))$

$(x - 1 + i)(x - 1 - i)$

$x^2 - x - \cancel{ix} - x + 1 + \cancel{i} + \cancel{ix} - \cancel{i} - i^2$

$x^2 - 2x + 1 - (-1) = \boxed{x^2 - 2x + 2 = j(x)}$

20. Find a fourth degree polynomial equation with real coefficients that has roots of 4, -2, and $4i$.

$(x - 4)(x + 2)(x - 4i)(x + 4i)$

$(x^2 - 2x - 8)(x^2 - 16i^2)$

$(x^2 - 2x - 8)(x^2 + 16)$

$x^4 + 16x^2 - 2x^3 - 32x - 8x^2 - 128 \Rightarrow \boxed{x^4 + 8x^2 - 2x^3 - 32x - 128 = g(x)}$

$x = 4 : (x - 4)$

$x = -2 : (x + 2)$

$x = 4i : (x - 4i)$

$x = -4i : (x + 4i)$

21. Find a polynomial equation with real coefficients that has the given zeros: -3, 0, 1, 4

$x(x + 3)(x - 1)(x - 4)$

$(x^2 + 3x)(x^2 - 5x + 4)$

$x^4 - 5x^3 + 4x^2 + 3x^3 - 15x^2 + 12x$

$\boxed{x^4 - 2x^3 - 11x^2 + 12x = f(x)}$

$x = -3 : (x + 3)$

$x = 0 : (x + 0) = x$

$x = 1 : (x - 1)$

$x = 4 : (x - 4)$