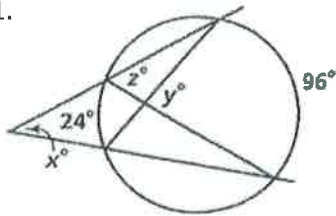


Please solve for the indicated variable(s).

1.

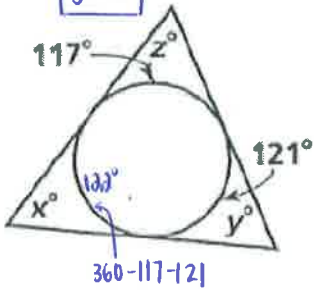


$$x = \frac{1}{2}(96 - 24) \quad y = \frac{1}{2}(24 + 96) \quad z = \frac{1}{2}(96)$$

$$x = \frac{1}{2}(72) \quad y = \frac{1}{2}(120) \quad z = 48$$

$$\boxed{x = 36} \quad \boxed{y = 60} \quad \boxed{z = 48}$$

3.



$$x = \frac{1}{2}(117 + 121 - 122)$$

$$x = \frac{1}{2}(116)$$

$$\boxed{x = 58}$$

$$y = \frac{1}{2}(122 + 117 - 121)$$

$$y = \frac{1}{2}(118)$$

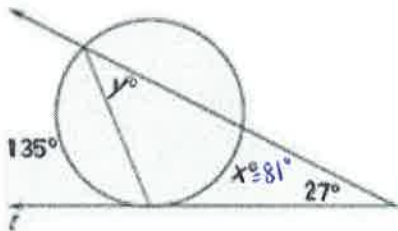
$$\boxed{y = 59}$$

$$z = \frac{1}{2}(122 + 121 - 117)$$

$$z = \frac{1}{2}(126)$$

$$\boxed{z = 63}$$

5.



$$135 = \frac{1}{2}(135 - x)$$

$$54 = 135 - x$$

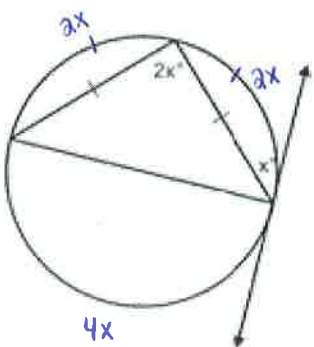
$$-81 = -x$$

$$\boxed{x = 81}$$

$$y = \frac{1}{2}(81)$$

$$\boxed{y = 40.5}$$

7.

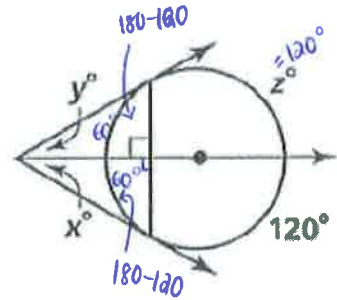


$$4x + 2x + 2x = 360$$

$$8x = 360$$

$$\boxed{x = 45}$$

2.



$$90 = \frac{1}{2}(60 + z)$$

$$180 = 60 + z$$

$$\boxed{z = 120}$$

$$x = \frac{1}{2}(120 - 60)$$

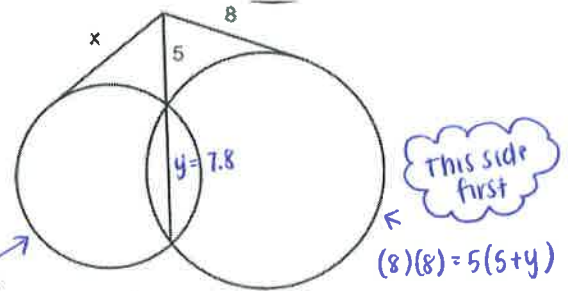
$$x = \frac{1}{2}(60)$$

$$\boxed{x = 30}$$

$$y = \frac{1}{2}(120 - 60)$$

$$y = \frac{1}{2}(60) \Rightarrow \boxed{y = 30}$$

4.



$$(x)(x) = 5(5 + 7.8)$$

$$x^2 = 5(12.8)$$

$$x^2 = 64$$

$$\boxed{x = 8}$$

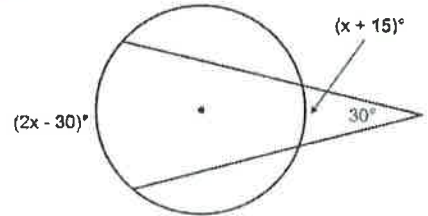
$$(8)(8) = 5(5 + y)$$

$$64 = 25 + 5y$$

$$39 = 5y$$

$$\boxed{y = 7.8}$$

6.



$$30 = \frac{1}{2}(2x - 30 - (x + 15))$$

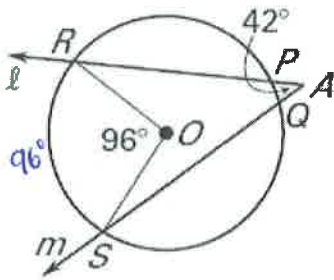
$$30 = \frac{1}{2}(2x - 30 - x - 15)$$

$$30 = \frac{1}{2}(x - 45)$$

$$60 = x - 45$$

$$\boxed{x = 105}$$

8. The secants l and m intersect at point A and \widehat{PQ} and \widehat{RS} are the intercepted arcs. If $m\angle PAQ = 42^\circ$ and $m\angle ROS = 96^\circ$, find $m\widehat{PQ}$.



$$42 = \frac{1}{2}(96 - m\widehat{PQ})$$

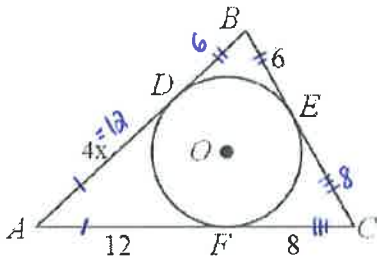
$$84 = 96 - m\widehat{PQ}$$

$$-12 = -m\widehat{PQ}$$

$m\widehat{PQ} = 12^\circ$

In #9 – 10, please solve for the indicated variable(s) and find the perimeter of the triangle or quadrilateral.

9.



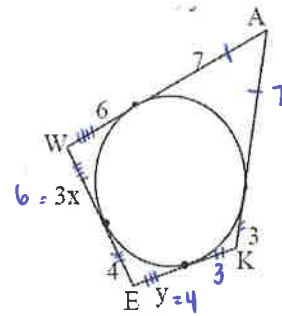
$$4x = 12$$

$x = 3$

$$p = 12 + 6 + 6 + 8 + 8 + 12$$

$P = 52 \text{ units}$

10.



$$6 = 3x$$

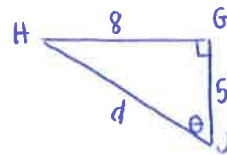
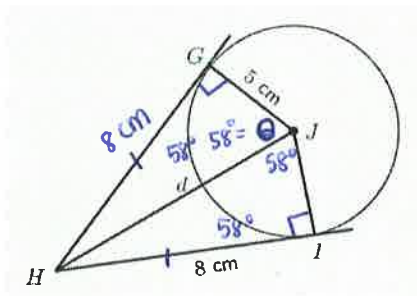
$x = 2$

$$y = 4$$

$$p = 6 + 6 + 7 + 7 + 3 + 3 + 4 + 4$$

$P = 40 \text{ units}$

11. Given that \overline{GH} and \overline{IH} are tangents to $\odot J$, please find the value of d and find $m\widehat{GI}$.



$$\tan \theta = \frac{8}{5}$$

$$\theta = \tan^{-1}\left(\frac{8}{5}\right)$$

$$\theta = 58^\circ$$

$$8^2 + 5^2 = d^2$$

$$64 + 25 = d^2$$

$$89 = d^2$$

$d = \sqrt{89}$

$$m\widehat{GI} = 58 + 58$$

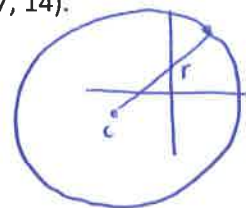
$m\widehat{GI} = 116^\circ$

12. Write the standard equation of the circle with center $(-5, -2)$ and a point on the circle is $(7, 14)$.

$$\text{radius} = \sqrt{(-5-7)^2 + (-2-14)^2} = \sqrt{(-12)^2 + (-16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

$$(x+5)^2 + (y+2)^2 = 20^2$$

$$(x+5)^2 + (y+2)^2 = 400$$



13. Write the standard equation of the circle and find the center and the radius given the general equation $x^2 + y^2 - 4x - 18y = -81$.

$$\frac{1}{2}(-4) = -2 = 4$$

$$\frac{1}{2}(-18) = -9 = 81$$

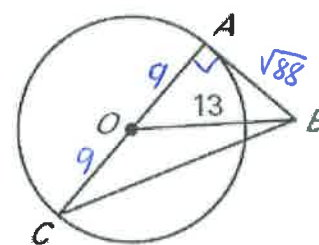
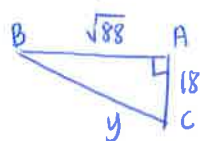
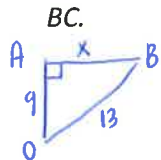
$$x^2 - 4x + 4 + y^2 - 18y + 81 = -81 + 4 + 81$$

$$(x-2)(x-2) + (y-9)(y-9) = 4$$

$$(x-2)^2 + (y-9)^2 = 4$$

center : $(2, 9)$
radius = $\sqrt{4} = 2$

14. In the figure below, $OB = 13$ and \overline{AB} is tangent to $\odot O$, whose diameter \overline{AC} has length 18. Find BC .



$$x^2 + 9^2 = 13^2$$

$$x^2 + 81 = 169$$

$$x^2 = 88$$

$$x = \sqrt{88}$$

$$(\sqrt{88})^2 + 18^2 = y^2$$

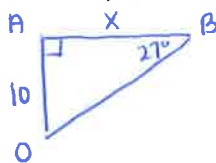
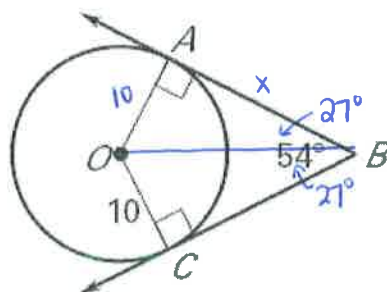
$$88 + 324 = y^2$$

$$412 = y^2$$

$$y = \sqrt{412} \approx 20.3$$

$BC \approx 20.3$ units

15. In the figure below, $OC = 10$, $m\angle ABC = 54^\circ$, and \overline{BA} and \overline{BC} are tangents to $\odot O$. Find BC .



$$\tan 27^\circ = \frac{10}{x}$$

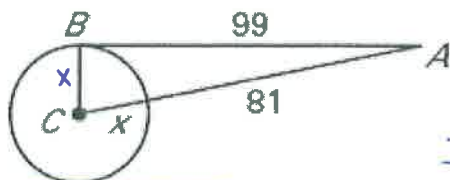
$$10 = x \tan 27^\circ$$

$$x = \frac{10}{\tan 27^\circ}$$

$$x \approx 19.6$$

$BC \approx 19.6$ units

16. Find the length of the radius of $\odot C$.



$$x^2 + 99^2 = (x+81)^2$$

$$x^2 + 9801 = (x+81)(x+81)$$

$$\cancel{x^2} + 9801 = \cancel{x^2} + 162x + 6561$$

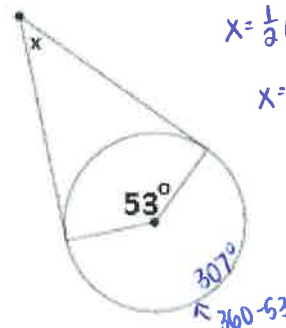
$$9801 = 162x + 6561$$

$$3240 = 162x$$

$$x = 20$$

So the radius = 20 units

17. Please solve for x .



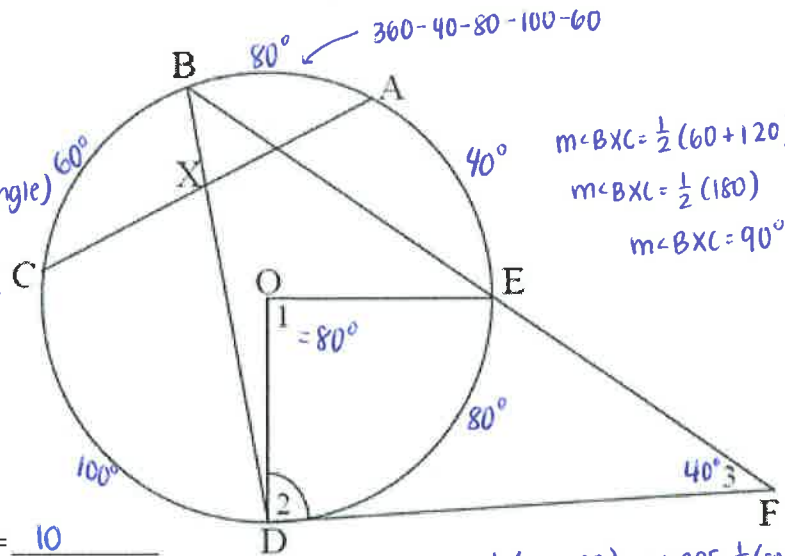
$$x = \frac{1}{2}(307 - 53)$$

$$x = \frac{1}{2}(254)$$

$x = 127$

18. Given $\odot O$, radii \overline{OD} and \overline{OE} , tangent \overline{DF} , secant \overline{BF} , $m\widehat{BC} = 60^\circ$, $m\widehat{DC} = 100^\circ$, $m\widehat{DE} = 80^\circ$, and $m\widehat{EA} = 40^\circ$, find:

- $m\widehat{AB} = 80^\circ$
- $m\angle 1 = 80^\circ$ (same as arc ED since it is a central angle)
- $m\angle 2 = 90^\circ$ (\overline{DF} is tangent to the circle)
- $m\angle BXC = 90^\circ$ (angle w/ vertex INSIDE)
- $m\angle 3 = 40^\circ$
- $m\angle BDF = 100^\circ$ (angle w/ vertex ON)



$$m\angle BXC = \frac{1}{2}(60 + 120)$$

$$m\angle BXC = \frac{1}{2}(180)$$

$$m\angle BXC = 90^\circ$$

$$m\angle 3 = \frac{1}{2}(160 - 80)$$

$$m\angle 3 = \frac{1}{2}(80)$$

$$m\angle 3 = 40^\circ$$

$$m\angle BDF = \frac{1}{2}(80 + 40 + 80)$$

$$m\angle BDF = \frac{1}{2}(200)$$

$$m\angle BDF = 100^\circ$$

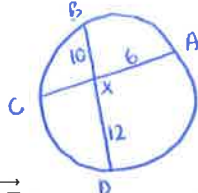
- If $BE = 21$ and $EF = 4$, then $DF = 10$
- If $BD = 22$, $BX = 10$, and $AX = 6$, then $CX = 20$

$$(DF)(OF) = 4(4+21)$$

$$DF^2 = 4(25)$$

$$DF^2 = 100$$

$$DF = 10$$



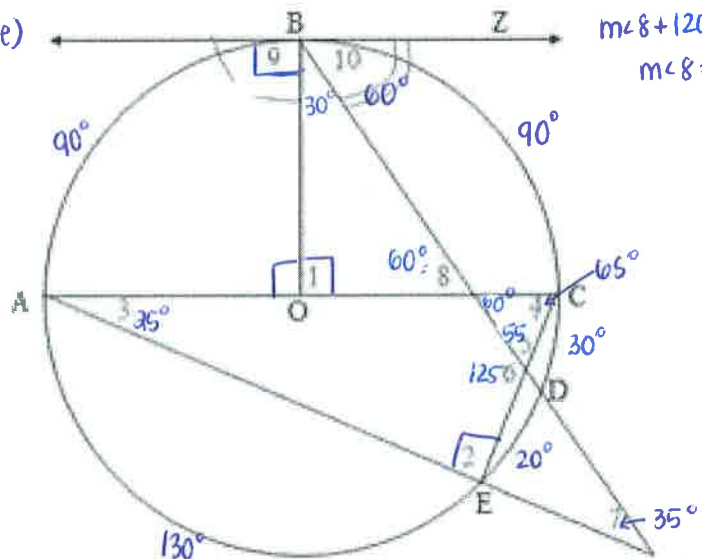
$$(10)(12) = (6)(CX)$$

$$120 = 6CX$$

$$CX = 20$$

19. Given \overline{BZ} is tangent to $\odot O$, \overline{AC} is a diameter, $m\widehat{BC} = 90^\circ$, $m\widehat{CD} = 30^\circ$, and $m\widehat{DE} = 20^\circ$, find all marked angles.

- $m\angle 1 = 90^\circ$ (central angle to \widehat{BC})
- $m\angle 2 = 90^\circ$ (inscribed to a semicircle)
- $m\angle 3 = 25^\circ$
- $m\angle 4 = 65^\circ$ (inscribed to \widehat{AE})
- $m\angle 5 = 55^\circ$
- $m\angle 6 = 125^\circ$
- $m\angle 7 = 35^\circ$
- $m\angle 8 = 60^\circ$
- $m\angle 9 = 90^\circ$
- $m\angle 10 = 60^\circ$



$$m\angle 1 + m\angle 8 + 30 = 180 \text{ (}\Delta\text{sum)}$$

$$90 + m\angle 8 + 30 = 180$$

$$m\angle 8 + 120 = 180$$

$$m\angle 8 = 60^\circ$$

$$m\angle 7 = \frac{1}{2}(90 - 20)$$

$$m\angle 7 = \frac{1}{2}(70)$$

$$m\angle 7 = 35^\circ$$

$$m\angle 2 + m\angle 3 + m\angle 4 = 180 \text{ (}\Delta\text{sum)}$$

$$90 + m\angle 3 + 65 = 180$$

$$m\angle 3 + 155 = 180$$

$$m\angle 3 = 25^\circ$$

$$\Delta\text{sum}$$

$$m\angle 5 + 60 + 65 = 180$$

$$m\angle 5 + 125 = 180$$

$$m\angle 5 = 55^\circ$$

$$m\angle 5 + m\angle 6 = 180 \text{ (linear pair)}$$

$$55 + m\angle 6 = 180$$

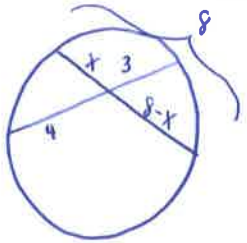
$$m\angle 6 = 125^\circ$$

$$m\angle 10 = \frac{1}{2}(90 + 30)$$

$$m\angle 10 = \frac{1}{2}(120)$$

$$m\angle 10 = 60^\circ$$

20. Please use the diagram below to solve for the indicated variables.



a. If $a+d=8$, $b=4$, and $c=3$, find a .

$$(x)(8-x) = (3)(4)$$

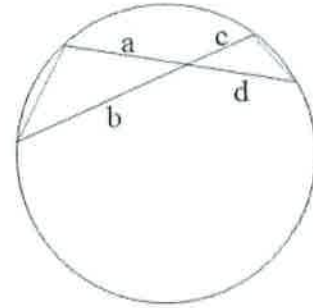
$$8x - x^2 = 12$$

$$0 = x^2 - 8x + 12$$

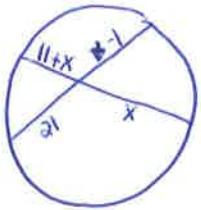
$$0 = (x-6)(x-2)$$

$$x=6, x=2$$

$$a=6 \text{ or } a=2$$



b. If $b=21$, a is 11 more than d , and c is 1 less than d , then solve for a , b , c , and d .



$$21(x-1) = x(11+x)$$

$$21x - 21 = 11x + x^2$$

$$0 = x^2 - 10x + 21$$

$$0 = (x-7)(x-3)$$

$$x=7 \text{ or } x=3$$

$$a = 11+x = 11+7 \text{ or } 11+3$$

$$a = 18 \text{ or } 14$$

$$b = 21$$

$$c = x-1 = 11-1 \text{ or } 3-1$$

$$c = 10 \text{ or } 2$$

$$d = x = 7 \text{ or } 3$$

21. Please use the diagram below to solve for the indicated variables.

a. If $a+c=8$, $b=4$, and $d=6$, find a .

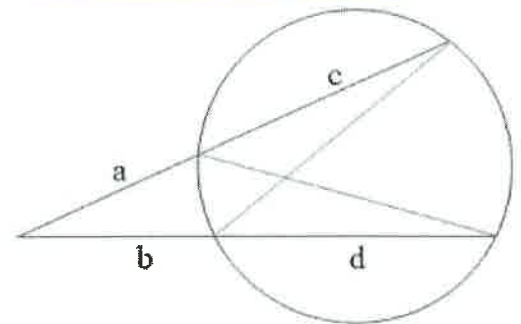
$$x(x+8-x) = 4(4+6)$$

$$x(8) = 4(10)$$

$$8x = 40$$

$$x = 5$$

$$a = 5$$



b. If $a = 12$, $b = 10$, and $c = 15$, find d .

$$12(12+15) = 10(10+d)$$

$$12(27) = 100 + 10d$$

$$324 = 100 + 10d$$

$$224 = 10d$$

$$d = 22.4$$

22. Write the equation of a circle with a center at (h, k) and a radius of $2\sqrt{5}$.

$$(x-5)^2 + (y-7)^2 = (2\sqrt{5})^2$$

$$(x-5)^2 + (y+7)^2 = 2^2 \sqrt{5}^2$$

$$(x-5)^2 + (y+7)^2 = 4(5)$$

$$(x-5)^2 + (y+7)^2 = 20$$