Geometry H
Unit 6 Test Review

Name : Key
Date : _____ Period : ___

Section 8.1 : Interior and Exterior Angles of Convex and Regular Polygons

1. Find the sum of the measures of the exterior angles of a 15-gon.

Sum = 360°

2. Find the sum of the measures of the interior angles of a convex heptagon. n = 7

Sum = (7-2)·180 = **900°**

3. Determine if it is possible for a regular polygon to have an interior angle that is 75°. Explain.

$\frac{75}{1} = \frac{(n-2)\cdot 180}{n}$

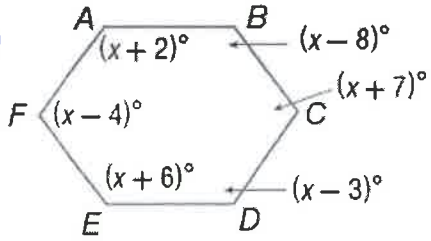
$75n = 180n - 360$

$-105n = -360$

$n = 3.4$

not possible because there can't be a decimal number of sides

4. Find the value of x.

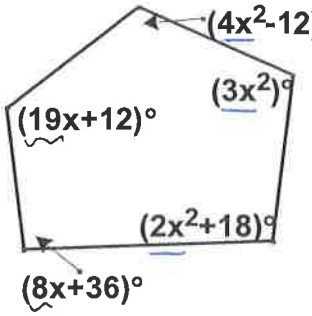


Sum = (6-2)·180 = 720°

$6x = 720$

$x = 120$

5. Find the possible value(s) of x.



Sum = (5-2)·180 = 540

$9x^2 + 27x + 54 = 540$

$9x^2 + 27x - 486 = 0$

$\frac{9(x^2+3x-54)}{9} = \frac{0}{9}$

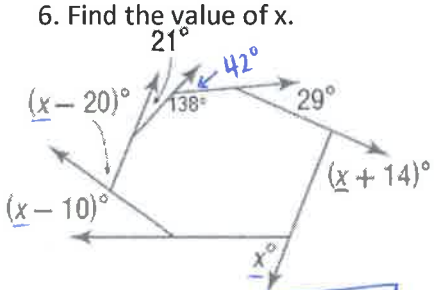
$x^2 + 3x - 54 = 0$

$(x+9)(x-6) = 0$

$x \neq -9$, **$x = 6$**

↳ doesn't work b/c if you sub back in to $19x+12 \Rightarrow 19(-9)+12 = -159°$

6. Find the value of x.



$4x + 76 = 360$

$4x = 284$

$x = 71$

7. Consider a regular polygon with a perimeter of 84 centimeters. What is the length of each side of the polygon if each interior angle measures 156°?

$\frac{156}{1} = \frac{(n-2)\cdot 180}{n}$

$156n = 180n - 360$

$-24n = -360$

$n = 15$

$15x = 84$

$x = 5.6$ cm

8. If each exterior angle of a regular polygon is 24°, how many sides does the polygon have?

$\frac{24^\circ}{1} = \frac{360}{n}$ $\boxed{n = 15}$

$24n = 360$

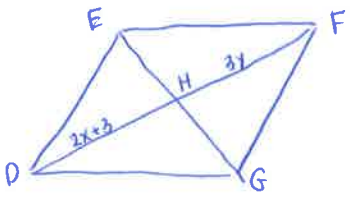
9. Find the measure of each interior and each exterior angle of a regular 30-gon.

Int: $\frac{(30-2)\cdot 180}{30} = \frac{5040}{30} = \boxed{168^\circ}$

Ext: $\frac{360}{30} = \boxed{12^\circ}$

Section 8.2/8.3 : Properties of Parallelograms

10. In parallelogram *DEFG*, the diagonals intersect at point *H*. If $DH = 2x + 3$, $HF = 3y$, $GH = 4x - 12$, and $HE = 2y + 3$, please find the values of x and y.



$2x + 3 = 3y \Rightarrow 2x = 3y - 3$

$4x - 12 = 2y + 3$

$4x = 2y + 15$

$2x = y + 7.5$

$3y - 3 = y + 7.5$

$2y = 10.5$

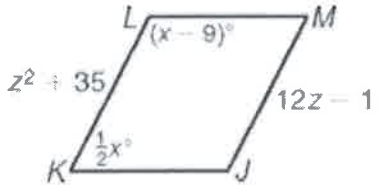
$\boxed{y = 5.25}$

$2x = 3(5.25) - 3$

$2x = 12.75$

$\boxed{x = 6.375}$

11. JKLM is a parallelogram. Find the m∠L, m∠K, and MJ.



$z^2 + 35 = 12z - 1$

$z^2 - 12z + 36 = 0$

$(z-6)(z-6) = 0$

$z = 6$

$MJ = 12(6) - 1 = \boxed{71 = MJ}$

$x - 9 + 0.5x = 180$

$1.5x - 9 = 180$

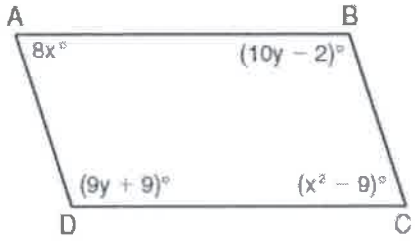
$1.5x = 189$

$x = 126$

$\boxed{m\angle L = 117^\circ}$

$\boxed{m\angle K = 63^\circ}$

12. Determine whether the figure is a parallelogram for $x = 9$ and $y = 11$. Explain your answer.



$x^2 - 9 = 8x$

$x^2 - 8x - 9 = 0$

$(x-9)(x+1) = 0$

$\boxed{x = 9}$ $x \neq -1$

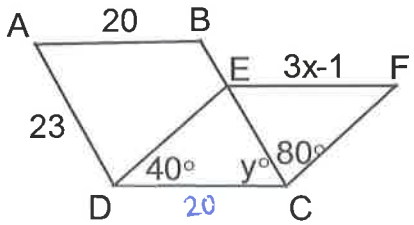
$9y + 9 = 10y - 2$

$9 = y - 2$

$\boxed{y = 11}$

Yes, $x = 9$ and $y = 11$ work bc if you set opp angles equal, you should get these values.

13. Given that *ABCD* and *CDEF* are parallelograms, please solve for x and y.



$40 + y + 80 = 180$

$y + 120 = 180$

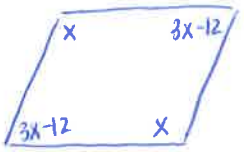
$\boxed{y = 60}$

$3x - 1 = 20$

$3x = 21$

$\boxed{x = 7}$

14. The measure of an angle of a parallelogram is 12 degrees less than 3 times the measure of a consecutive angle. Please find the measure of each angle in the parallelogram.



$x+3x-12=180$

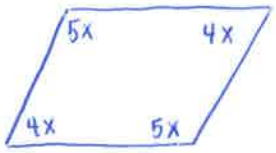
$4x-12=180$

$4x=192$

$x=48$

Angles: $48°, 48°, 132°, 132°$

15. The measures of two consecutive angles of a parallelogram are in the ratio of 5: 4. What is the measure of each angle in the parallelogram?



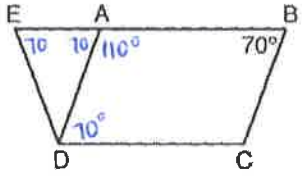
$5x+4x=180$

$9x=180$

$x=20$

Angles: $80°, 80°, 100°, 100°$

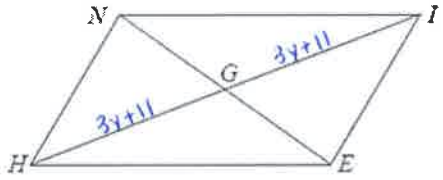
16. In the diagram, *ABCD* is a parallelogram. $\overline{DA} \cong \overline{DE}$, and $m\angle B = 70°$. Find the $m\angle DEA$ and $m\angle EDA$.



$m\angle DEA = 70°$

$m\angle EDA = 40°$

17. If $IG = 3y + 11$ and $IH = 8y$, use the diagram below to find the value of *y*.



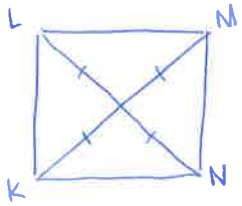
$3y+11+3y+11=8y$

$6y+22=8y$

$22=2y$

$y=11$

18. In parallelogram *KLMN*, $KM = LN$, $m\angle KLM = (2xy)°$ and $m\angle LMN = (9x + 9)°$. Please solve for *x* and *y*.



$2xy=90$

$xy=45$

$9y=45$

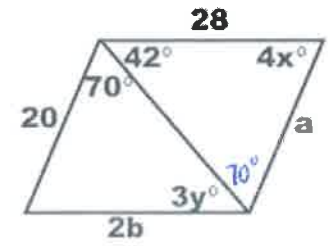
$y=5$

$9x+9=90$

$9x=81$

$x=9$

19. Please solve for *a*, *b*, *x*, and *y*.



$42=3y$

$y=14$

$28=2b$

$b=14$

$a=20$

$42+4x+70=180$

$4x+112=180$

$4x=68 \Rightarrow$ $x=17$

Section 8.4 : Properties of Rhombuses, Rectangles, and Squares

20. In a rectangle, the length is twice the width, and the perimeter is 48 inches. Find the area of the rectangle.

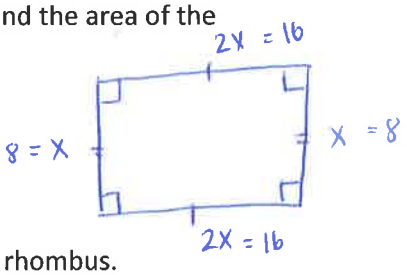
$6x = 48$

$x = 8$

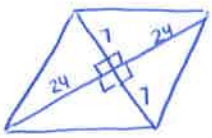
$\text{Area} = (\ell)(w)$

$= (8)(16)$

$A = 128\ in^2$



21. The diagonals of a rhombus are 14 and 48 cm. Find the length of a side of the rhombus.



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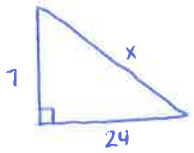


$7^2 + 24^2 = x^2$

$625 = x^2$

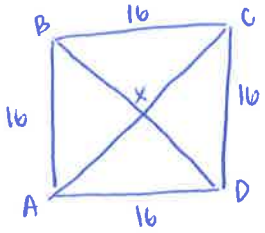
$x = 25$

One side is 25 cm

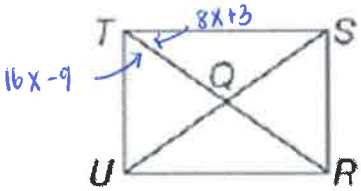
22. *ABCD* is a square whose diagonals intersect at *X*. If *AB* = 16 and *AC* = $16\sqrt{2}$, find the following:

a. BC = 16 b. BD = $16\sqrt{2}$ c. AD = 16

d. m∠DAC = 45° e. m∠AXB = 90° f. Perimeter of ΔABC = $32 + 16\sqrt{2}$



23. Quadrilateral *RUTS* is a rectangle. If $m\angle STR = (8x + 3)^\circ$ and $m\angle UTR = (16x - 9)^\circ$, find $m\angle STR$.



$8x + 3 + 16x - 9 = 90$

$24x - 6 = 90$

$24x = 96$

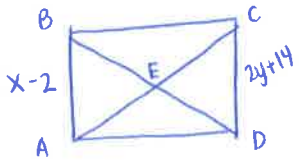
$x = 4$

$m\angle STR = 8(4) + 3$

$= 35^\circ$

24. *ABCD* is a rectangle whose diagonals intersect at *E*. In each scenario, please find the values of the variables.

a. $m\angle BCD = (18x - 3y)^\circ$, $AB = x - 2$, $CD = 2y + 14$



$18x - 3y = 90$

$-3y = 90 - 18x$

$y = -30 + 6x$

$x - 2 = 2y + 14$

$x = 2y + 16$

Substitution: $y = -30 + 6(2y + 16)$

$y = -30 + 12y + 96$

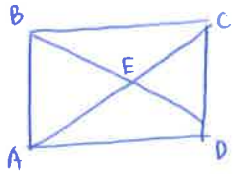
$-11y = 66$

$y = -6$

$x = 2(-6) + 16$

$x = 4$

b. $AC = 18$, $BE = x + y$, $BD = 3x - 2y$



$x + y + x + y = 3x - 2y$

$2x + 2y = 3x - 2y$

$2y = x - 2y$

$x = 4y$

$18 = 3x - 2y$

Substitution: $18 = 3(4y) - 2y$

$18 = 12y - 2y$

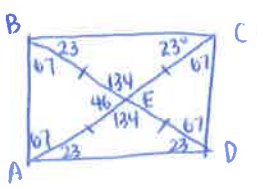
$18 = 10y$

$y = 1.8$

$x = 4(1.8)$

$x = 7.2$

c. $m\angle BCE = 23^\circ$. Find the $m\angle ADE$, $m\angle DEC$, and $m\angle BEC$.



$m\angle ADE = 23^\circ$

$m\angle DEC = 46^\circ$

$m\angle BEC = 134^\circ$

25. Solve the following questions given rhombus USCG whose diagonals intersect at A. (Each question is a separate scenario)

a. If $m\angle USA = 44°$, find $m\angle CGA$. $= 44°$

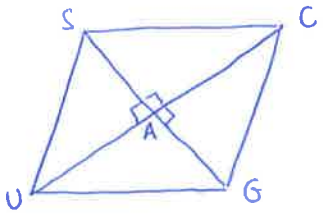
b. If $m\angle GUS = 102°$, find $m\angle ACG$. $= 51°$

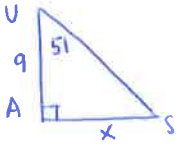
c. If $UC = 18$, and $m\angle AUS = 51°$, find SG. $= 22.2$

d. If $UC = 10$, find AC. $= 5$

e. If $m\angle SGU = 12°$, find $m\angle SCG$. $= 156°$

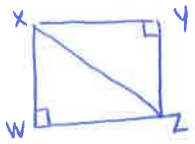
f. If $m\angle USC = 81°$, find $m\angle UAS$. $= 90°$





$\tan 51 = \frac{x}{9}$

26. $WXYZ$ is a rectangle. The perimeter of ΔXYZ is 24. $XY + YZ = 5x - 1$ and $XZ = 13 - x$. Please find WY.



$5x - 1 + 13 - x = 24$

$4x + 12 = 24$

$4x = 12$

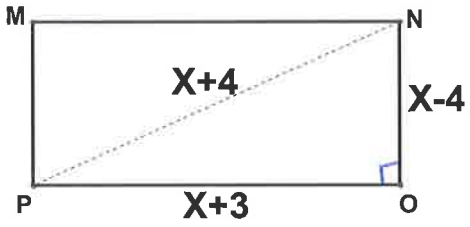
$x = 3$

$\overline{WY} \cong \overline{XZ}$

$XZ = 13 - 3 = 10$

$\boxed{WY = 10}$

27. $MNOP$ is a rectangle. Please find all possible values for x.



$(x-4)^2 + (x+3)^2 = (x+4)^2$

$x^2 - 8x + 16 + x^2 + 6x + 9 = x^2 + 8x + 16$

$2x^2 - 2x + 25 = x^2 + 8x + 16$

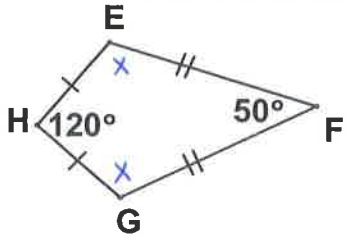
$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$\boxed{x = 9}$ ~~$x = 1$~~

Section 8.5 : Properties of Kites and Trapezoids

28. $EFGH$ is a kite. Please find the measure of angle G.



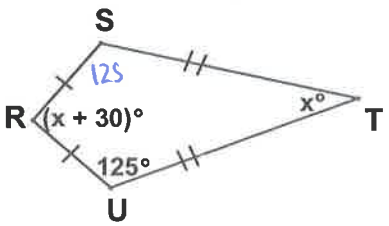
$2x + 170 = 360$

$2x = 190$

$x = 95$

$\boxed{m\angle G = 95°}$

29. $RSTU$ is a kite. Find the measures of angles R, S, and T.



$250 + 2x + 30 = 360$

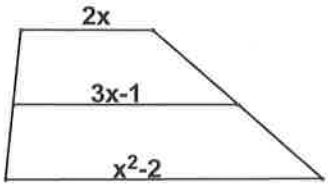
$2x + 280 = 360$

$2x = 80$

$x = 40$

$\boxed{\begin{array}{l} m\angle R = 70° \\ m\angle S = 125° \\ m\angle T = 40° \end{array}}$

30. The figure shown is a trapezoid with its midsegment. Please find all possible values of *x*.



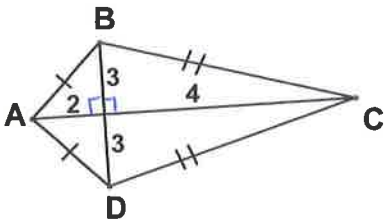
$3x-1=\frac{1}{2}(x^2+2x-2)$ $x\neq 0$, $\boxed{x=4}$

$6x-2=x^2+2x-2$

$x^2-4x=0$

$x(x-4)=0$

31. Please find the perimeter of the kite in simplest radical form.



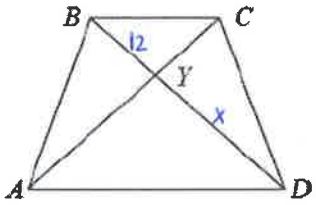
$3^2+4^2=BC^2$ $2^2+3^2=AB^2$

$25=BC^2$ $13=AB^2$

$BC=5$ $AB=\sqrt{13}$

$\boxed{P=10+2\sqrt{13}}$

32. Given isosceles trapezoid *ABCD* with $\overline{AB}\cong\overline{CD}$,



a. If *BY* = 12 and *AC* = 30, please find *YD*. $12+X=30$ $\boxed{YD=18}$

$X=18$

b. If $BD=a^2+27, AC=2a^2-54$, please find the value of *a*.

$a^2+27=2a^2-54$ $a^2=81$

$27=a^2-54$ $\boxed{a=-9 \text{ or } 9}$

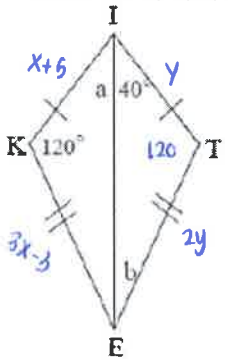
c. If $m\angle CBA=110°, m\angle BAD=(4x+10)°$, please find the value of x.

$110+4x+10=180$ $\boxed{x=15}$

$4x+120=180$

$4x=60$

33. Given kite *KITE*, *KI* = *x* + 5, *IT* = *y*, *TE* = 2*y*, *KE* = 3*x* − 3 and the measures given in the diagram, please solve for *a*, *b*, *x*, and *y*.



$\boxed{a=40}$

$x+5=y$ $y=13+5$ $\boxed{y=18}$

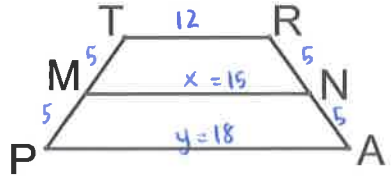
$2y=3x-3$ $40+120+b=180$

$2(x+5)=3x-3$ $160+b=180$

$2x+10=3x-3$ $\boxed{b=20}$

$\boxed{13=x}$

34. Isosceles trapezoid *TRAP* has legs of length 10 and midsegment *MN*. *TR* = 12 and the perimeter of *TRNM* is 37. Please find the perimeter of *NAPM*.



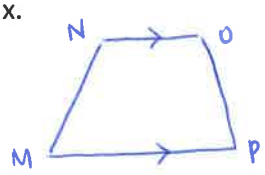
$12+10+X=37$ $15=\frac{1}{2}(12+y)$ Perimeter of NAPM $=15+10+18$

$22+X=37$ $15=6+\frac{1}{2}y$ $\boxed{P=43}$

$X=15$ $9=\frac{1}{2}y$

$\boxed{MN=15}$ $\boxed{18=y}$

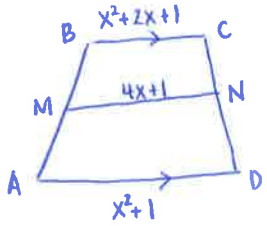
35. *MNOP* is an isosceles trapezoid with $\overline{NO} \parallel \overline{MP}$. If $m\angle M = 2x°$ and $m\angle P = (x+27)°$, find the value of x.



$2x = x+27$

$\boxed{x=27}$

36. In trapezoid *ABCD*, $\overline{AD} \parallel \overline{BC}$, M is the midpoint of $\overline{AB}$, and N is the midpoint of $\overline{DC}$. If $AD = x^2+1, MN = 4x+1$, and $BC = x^2+2x+1$, find AD, MN, and BC.



$4x+1 = \frac{1}{2}(x^2+2x+1+x^2+1)$

$4x+1 = \frac{1}{2}(2x^2+2x+2)$

$4x+1 = x^2+x+1$

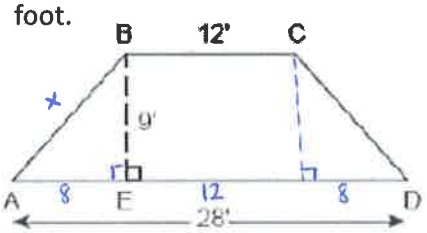
$0 = x^2-3x$

$0 = x(x-3)$

$x=0, \boxed{x=3}$

$\boxed{AD = 10}$
$\boxed{MN = 13}$
$\boxed{BC = 16}$

37. The cross section of an attic is in the shape of an isosceles trapezoid, as shown in the figure below. If the height of the attic is 9 feet, *BC* = 12 feet, and *AD* = 28 feet, find the length of $\overline{AB}$ to the nearest foot.

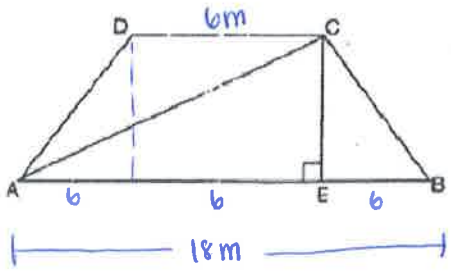


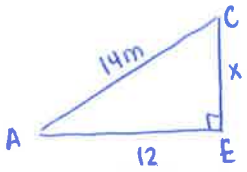
$8^2+9^2 = x^2$

$145 = x^2$

$x = \sqrt{145} \approx \boxed{12 \text{ ft}}$

38. In the diagram below, isosceles trapezoid has bases $\overline{DC}$ and $\overline{AB}$ with measures of 6 meters and 18 meters respectively. If altitude $\overline{CE}$ is drawn and $\overline{AC}$ has a measure of 14 meters, find the length of altitude $\overline{CE}$ in simplest radical form.





$x^2+12^2 = 14^2$

$x^2+144 = 196$

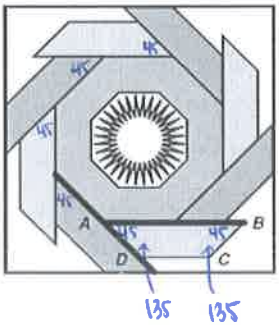
$x^2 = 52$

$x = \sqrt{52}$

$x = \sqrt{4}\sqrt{13}$

$\boxed{x = 2\sqrt{13}}$

39. In the design, eight isosceles trapezoids surround a regular octagon. What is the $m\angle C$ in trapezoid *ABCD*?



$\boxed{m\angle C = 135°}$

Quadrilaterals

Please complete the statement with always, sometimes, or never true.

40. A trapezoid is never a parallelogram.

41. Both pairs of opposite angles of a rhombus are always congruent.

42. Diagonals of a trapezoid are never perpendicular.

43. Consecutive angles of a rhombus are sometimes supplementary and congruent.

44. The diagonals of a parallelogram sometimes bisect the angles.

Please determine if the statement is true or false.

45. All quadrilaterals are squares. False

46. If a parallelogram has one right angle, then all of the other angles are right angles. True

47. The diagonals of a rectangle bisect opposite angles. False

48. A square is both a rectangle and a rhombus. True

49. The consecutive angles of a rhombus are congruent. False

50. If one pair of opposite sides of a quadrilateral is congruent, then the quadrilateral is a parallelogram. False

51. All parallelograms are quadrilaterals. True

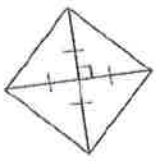
52. A rhombus and one of its diagonals form two isosceles triangles. True

53. Every parallelogram is a regular quadrilateral. False

54. All rhombuses are parallelograms. True

Multiple Choice : Please choose the best answer.

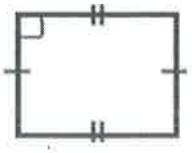
55. The most descriptive name for the figure at the right is a(n):



(a) Square b. Rectangle c. Parallelogram

d. Kite e. Trapezoid f. Isosceles Trapezoid

56. The most descriptive name for the figure at the right is a(n):



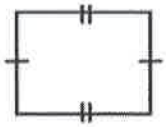
a. Square (b.) Rectangle c. Parallelogram

d. Kite e. Rhombus f. Isosceles Trapezoid

57. The most descriptive name for the figure at the right is a(n):

a. Square b. Rectangle (c.) Parallelogram

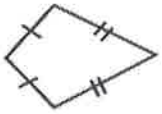
d. Kite e. Rhombus f. Isosceles Trapezoid



58. The most descriptive name for the figure at the right is a(n):

a. Square b. Rectangle c. Parallelogram

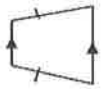
(d.) Kite e. Rhombus f. Isosceles Trapezoid



59. The most descriptive name for the figure at the right is a(n):

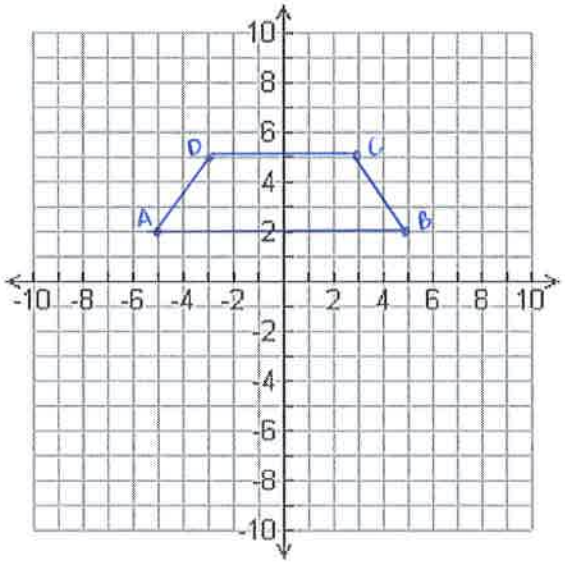
a. Square b. Trapezoid c. Parallelogram

d. Kite e. Rhombus (f.) Isosceles Trapezoid



Coordinate Proofs

60. Given coordinates *A*(-5,2),*B*(5,2), *C*(3,5) and *D*(-3,5), please determine what type of quadrilateral *ABCD* is. Justify your answer.



Slope Bases: $DC = \frac{5-5}{-3-3} = \frac{0}{-6} = 0$

$AB = \frac{2-2}{5+5} = \frac{0}{10} = 0$

$\rightarrow \overline{AB} \parallel \overline{DC}$

Dist Legs: $AD = \sqrt{(-3+5)^2+(5-2)^2} = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}$

$BC = \sqrt{(5-2)^2+(3-5)^2} = \sqrt{(3)^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

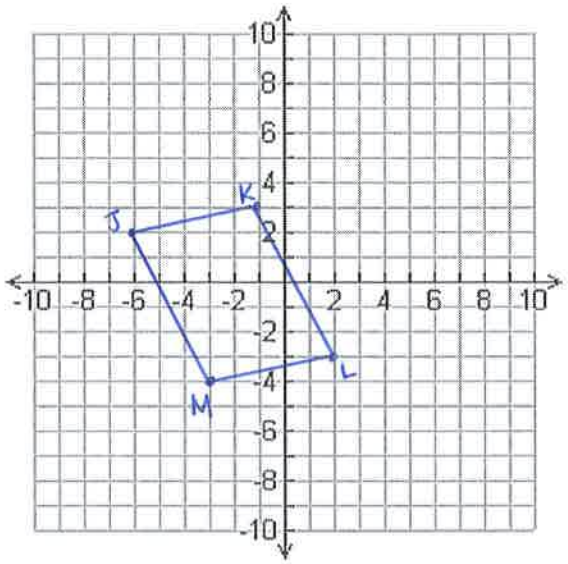
$\rightarrow \overline{AD} \cong \overline{BC}$

ABCD is an isosceles trapezoid since $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$.

61. The vertices of a quadrilateral are $J(-6,2)$, $K(-1,3)$, $L(2,-3)$, and $M(-3,-4)$. Use one of the methods we talked about to prove that $JKLM$ is a parallelogram.

→ See 8.6 notes Ex 1 for possibilities:

I'll use slope to prove both pairs of opp sides are //:



Slope $\overline{JK} = \frac{3-2}{-1+6} = \frac{1}{5}$

Slope $\overline{ML} = \frac{-4+3}{-3-2} = \frac{-1}{-5} = \frac{1}{5}$ $\Rightarrow \overline{JK} \parallel \overline{ML}$

Slope $\overline{JM} = \frac{-4-2}{-3+6} = \frac{-6}{3} = -2$

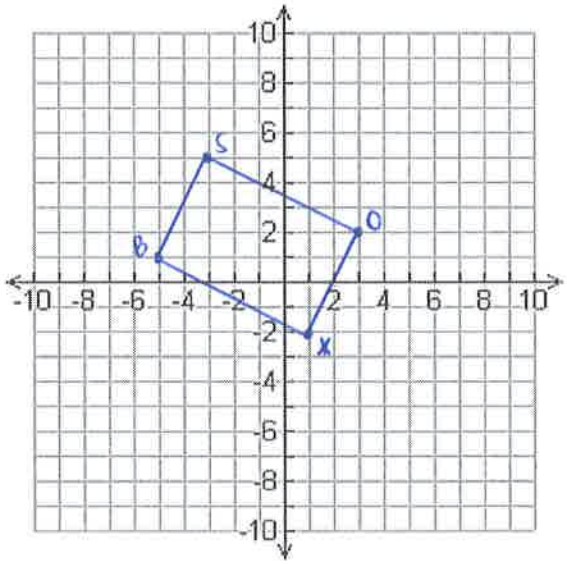
Slope $\overline{KL} = \frac{-3-3}{2+1} = \frac{-6}{3} = -2$ $\Rightarrow \overline{JM} \parallel \overline{KL}$

- Since both pairs of opp. sides are //, by definition, JKLM is a parallelogram

62. Quadrilateral $BSOX$ has vertices $B(-5, 1)$, $S(-3, 5)$, $O(3, 2)$, and $X(1, -2)$.

a. Please prove that $BSOX$ is a rectangle.

b. Find the area and perimeter of $BSOX$.



a.) Both pairs of opp sides are $\cong$:

$BS = \sqrt{(-3+5)^2+(5-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$OX = \sqrt{(1-3)^2+(-2-2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ $\Rightarrow \overline{BS} \cong \overline{OX}$

$SO = \sqrt{(3+3)^2+(2-5)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$BX = \sqrt{(1+5)^2+(-2-1)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$ $\Rightarrow \overline{SO} \cong \overline{BX}$

one angle is a right angle:

Slope $\overline{BS} = \frac{5-1}{-3+5} = \frac{4}{2} = \frac{2}{1}$

Slope $\overline{SO} = \frac{2-5}{3+3} = \frac{-3}{6} = -\frac{1}{2}$ $\Rightarrow \overline{BS} \perp \overline{SO}$

Since both pairs of opp sides are $\cong$ and the quad contains all right $\angle$'s, then BSOX is a rectangle.

↳ both pairs of opp sides have diff lengths, otherwise this might be a square!

b) Perimeter $= 2\sqrt{5} + 2\sqrt{5} + 3\sqrt{5} + 3\sqrt{5}$

$P = 10\sqrt{5}$ units

Area $= l \times w$

$= (2\sqrt{5})(3\sqrt{5})$

$= 6\sqrt{25}$

$= 6(5)$

$A = 30 \text{ units}^2$