

For each statement below, determine if it is always true, sometimes true or never true.

- The legs of an isosceles triangle are perpendicular to the base **never**
- The acute angles of a right triangle are congruent. **sometimes**
- An equiangular triangle is also equilateral. **always**
- If one of the exterior angles of an isosceles triangle is 120° , then the triangle is equilateral. **always**

5. TRUE OR FALSE:

If in $\triangle ABC$, $m\angle A$ is 30° more than $m\angle B$, and $m\angle B$ is 24 less than $m\angle C$, then $\triangle ABC$ is acute.

$$\begin{array}{l} m\angle A = x - 24 + 30 \\ m\angle B = x - 24 \\ m\angle C = x \end{array} \quad \left\{ \begin{array}{l} x + 6 + x - 24 + x = 180 \\ 3x - 18 = 180 \\ 3x = 198 \Rightarrow \boxed{x = 66} \end{array} \right.$$

$$\begin{array}{l} m\angle A = 66 + 6 = 72^\circ \\ m\angle B = 66 - 24 = 42^\circ \\ m\angle C = 66^\circ \end{array} \quad \left\{ \begin{array}{l} \text{acute } \triangle \end{array} \right.$$

6. MULTIPLE CHOICE:

Which of the following triangles does not exist?

- I. acute isosceles II. right scalene
 III. obtuse equilateral IV. obtuse scalene

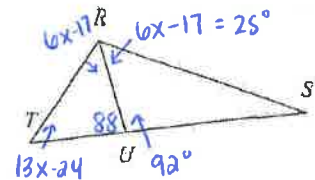
- a. I only b. II only **c. III only** d. II and III e. II, III, IV

7. \overline{RU} bisects $\angle TRS$. If $m\angle RTU = (13x - 24)^\circ$, $m\angle TRS = (12x - 34)^\circ$, and $m\angle RUS = 92^\circ$, find $m\angle RSU$.

\downarrow
each half must be $(6x - 17)^\circ$

$$\begin{array}{l} \triangle RTU: 13x - 24 + 88 + 6x - 17 = 180 \\ 19x + 47 = 180 \\ 19x = 133 \\ \boxed{x = 7} \end{array}$$

$$\begin{array}{l} \triangle RSU: 26 + 92 + m\angle RSU = 180 \\ 117 + m\angle RSU = 180 \\ \boxed{m\angle RSU = 63^\circ} \end{array}$$

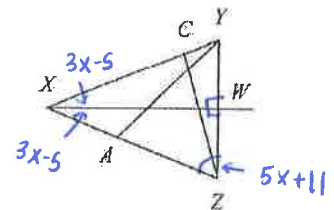


8. \overline{XW} bisects $\angle ZXY$. If $m\angle WZX = (5x + 11)^\circ$, $m\angle ZXY = (6x - 10)^\circ$, and $\overline{XW} \perp \overline{ZY}$, find $m\angle WXY$.

\downarrow
each half is $(3x - 5)^\circ$

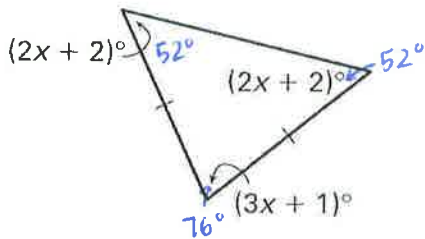
$$\begin{array}{l} \triangle XZW: 3x - 5 + 5x + 11 + 90 = 180 \\ 8x + 96 = 180 \\ 8x = 84 \\ \boxed{x = 10.5} \end{array}$$

$$\begin{array}{l} m\angle WXY = 3(10.5) - 5 \\ \boxed{m\angle WXY = 26.5^\circ} \end{array}$$



Find the value of x . Then classify the triangle by its angles.

9.



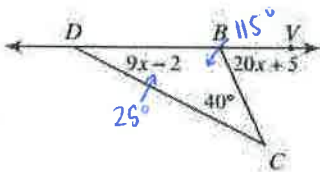
$$7x + 5 = 180$$

$$7x = 175$$

$$x = 25$$

The Δ is acute since all \angle 's are less than 90° .

11.



$$20x + 5 = 9x - 2 + 40$$

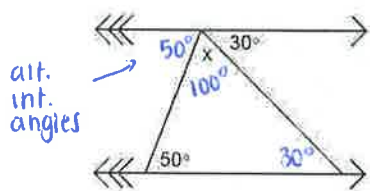
$$20x + 5 = 9x + 38$$

$$11x + 5 = 38$$

$$11x = 33 \Rightarrow x = 3$$

since $m\angle DBC = 115^\circ$, the Δ is obtuse

13. Note: Triangle not drawn to scale!



alt. int. angles

$$50 + x + 30 = 180$$

$$x + 80 = 180$$

$$x = 100$$

The Δ is obtuse

14. If $m\angle DCA = [(x-2)^2]^\circ$, $m\angle A = (5x)^\circ$, and $m\angle B = (3x+4)^\circ$, please find the value of x and classify ΔABC by its sides and its angles.

$$(x-2)^2 = 5x + 3x + 4$$

$$x^2 - 4x + 4 = 8x + 4$$

$$x^2 - 12x = 0$$

$$x(x-12) = 0 \Rightarrow x \neq 0, x = 12$$

$$\text{check: } x=0: m\angle A = 5(0) = 0$$

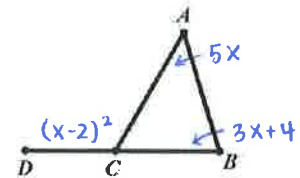
↑ cant have 0° angle measures

$$x=12: m\angle A = 60^\circ$$

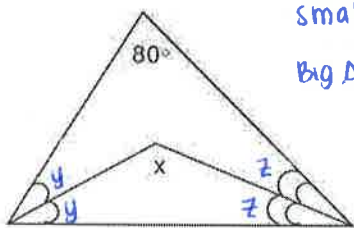
$$m\angle B = 40^\circ$$

$$m\angle ACB = 80^\circ$$

acute scalene



15. Solve for x .



$$\text{small } \Delta: x + y + z = 180 \Rightarrow y = 180 - x - z$$

$$\text{big } \Delta: 80 + 2y + 2z = 180 \Rightarrow 80 + 2(180 - x - z) + 2z = 180$$

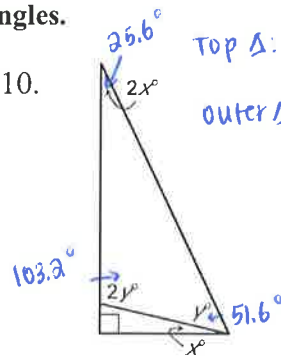
$$80 + 360 - 2x - 2z + 2z = 180$$

$$440 - 2x = 180$$

$$-2x = -260$$

$$x = 130$$

10.



$$\text{Top } \Delta: 3y + 2x = 180$$

$$\text{outer } \Delta: 3x + y + 90 = 180$$

$$3x + y = 90$$

$$2x + 3y = 180$$

$$-3(3x + y = 90)$$

$$2x + 3y = 180$$

$$-9x - 3y = -270$$

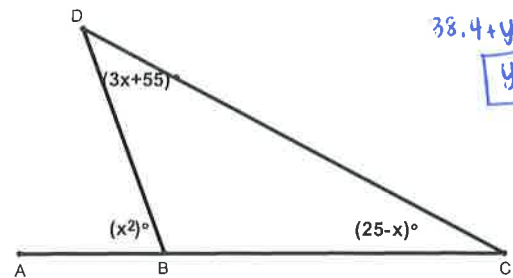
$$-7x = -90$$

$$x = 12.8$$

Top Δ : obtuse

Sm Δ : Right

12.



$$x^2 = 3x + 55 + 25 - x$$

$$x^2 = 2x + 80$$

$$x^2 - 2x - 80 = 0$$

$$(x-10)(x+8) = 0$$

$$x = 10, x = -8$$

When $x = 10$:

$$m\angle C = 15^\circ$$

$$m\angle D = 85^\circ$$

$$m\angle DBC = 80^\circ$$

acute

When $x = -8$:

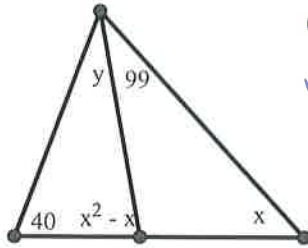
$$m\angle C = 33^\circ$$

$$m\angle D = 31^\circ$$

$$m\angle PBC = 116^\circ$$

obtuse

16. Find the values of x and y .



Left Δ : $x^2 - x + y + 40 = 180 \Rightarrow x^2 + x + 41 - x + 40 = 180$

Whole Δ : $y + 99 + x + 40 = 180$

$x + y = 41$

$y = 41 - x$

$y = 41 - 11$

$y = 30$

$x^2 - 2x + 81 = 180$

$x^2 - 2x - 99 = 0$

$(x - 11)(x + 9) = 0$

$x = 11$ (crossed out)

can't have lower right \angle as negative

17. Given: $\overline{AB} \perp \overline{BC}$, \overline{BD} bisects $\angle ABC$, $m\angle ABD = (x + 5y)^\circ$, $m\angle DBC = (2x + 2y + 3)^\circ$. Find the values of x and y .

each half must be 45°

$x + 5y = 45$

$2x + 2y + 3 = 45$

$2x + 2y = 42$

$-2(x + 5y = 45)$

$2x + 2y = 42 \Rightarrow$

$-2x - 10y = -90$

$2x + 2y = 42$

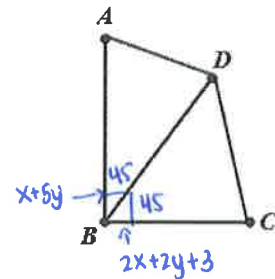
$-8y = -48$

$y = 6$

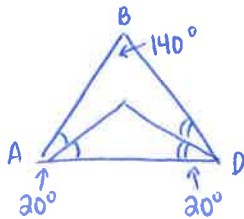
$x + 5(6) = 45$

$x + 30 = 45$

$x = 15$



18. \overline{AO} and \overline{DO} are the angle bisectors of $\angle DAB$ and $\angle BDA$, respectively. $\overline{CD} \cong \overline{BD} \cong \overline{AB}$, and $m\angle C = 40^\circ$. Find $m\angle BAO$.



$m\angle A = 180 - 140$

$= 40 \div 2$

$= 20^\circ$

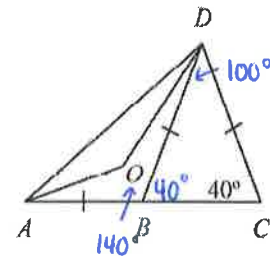
$m\angle D = 180 - 140$

$= 40 \div 2$

$= 20^\circ$

Both angles are bisected

so each half is 10°



19. A triangle has vertices $A(1, 1)$, $B(3, 0)$ and $C(2, 3)$. Graph the triangle and classify it by its sides and determine if it is a right triangle. Show all work. Justify your answer.

$AB = \sqrt{(3-1)^2 + (0-1)^2} = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

$BC = \sqrt{(2-3)^2 + (3-0)^2} = \sqrt{(-1)^2 + (3)^2} = \sqrt{1+9} = \sqrt{10}$

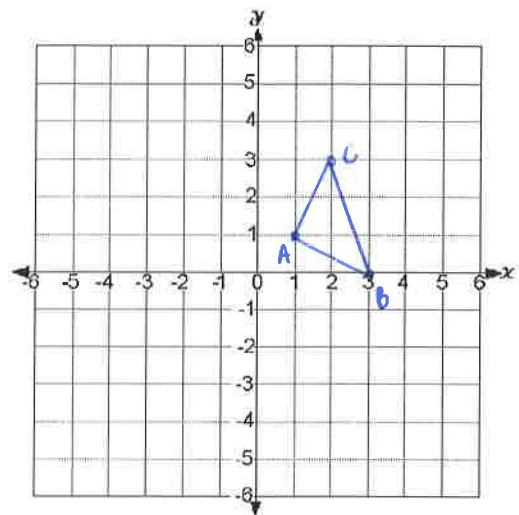
$AC = \sqrt{(2-1)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$

Isosceles

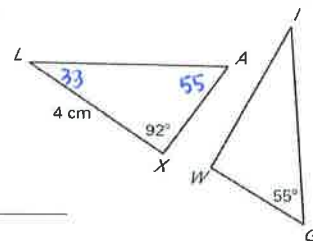
$m_{\overline{AB}} = \frac{0-1}{3-1} = \frac{-1}{2}$

$m_{\overline{AC}} = \frac{3-1}{2-1} = \frac{2}{1}$

since \overline{AB} and \overline{AC} have opp. reciprocal slopes, $\triangle ABC$ is a right Δ



20. In the diagram, $\triangle ALX \cong \triangle GIW$. Complete the following.

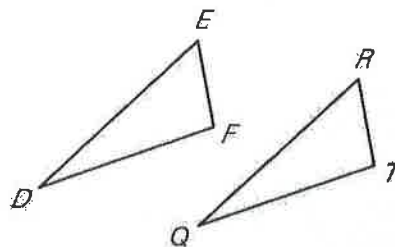


- a. $\overline{LX} \cong \underline{\overline{IW}}$ b. $\angle I \cong \underline{\angle L}$
- c. $\angle A \cong \underline{\angle G}$ d. $\overline{WG} \cong \underline{\overline{XA}}$
- e. $m\angle A = \underline{55^\circ}$ f. $m\angle W = \underline{92^\circ}$
- g. $m\angle I = \underline{33^\circ}$ h. $m\angle L = \underline{33^\circ}$
- i. $IW = \underline{4\text{cm}}$ j. $\triangle LAX \cong \underline{\triangle IGW}$

State the congruence that is needed to prove $\triangle DEF \cong \triangle QRT$ using the given postulate or theorem.

21. Given: $\angle D \cong \angle Q$, $\angle F \cong \angle T$ using AAS

Either $\overline{EF} \cong \overline{RT}$ OR $\overline{DE} \cong \overline{QR}$



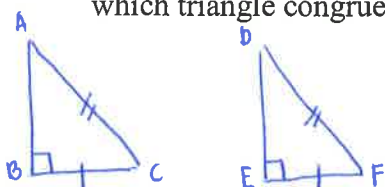
22. Given: $\angle E \cong \angle R$, $\overline{EF} \cong \overline{RT}$ using ASA

$\angle F \cong \angle T$

23. Given: $\overline{DE} \cong \overline{QR}$, $\angle D \cong \angle Q$ using SAS

$\overline{DF} \cong \overline{QT}$

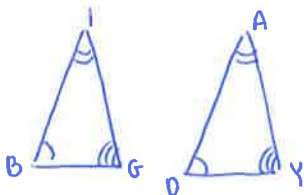
24. In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{CB} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. $\triangle ABC \cong \triangle DEF$ by which triangle congruency postulate? (HINT: It may be helpful to draw a picture!)



By H-L

25. If $\triangle BIG \cong \triangle DAY$, all of the following are true EXCEPT:

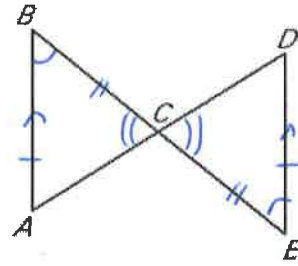
- ~~A.~~ $\triangle DYA \cong \triangle BGI$ ~~B.~~ $\triangle GIB \cong \triangle YAD$ ~~C.~~ $\overline{YD} \cong \overline{GB}$ **(D)** $\angle IGB \cong \angle YAD$ ~~E.~~ $\overline{GI} \cong \overline{YA}$



Complete the following proofs.

26. Given: $\overline{AB} \parallel \overline{DE}$, $\overline{AB} \cong \overline{DE}$

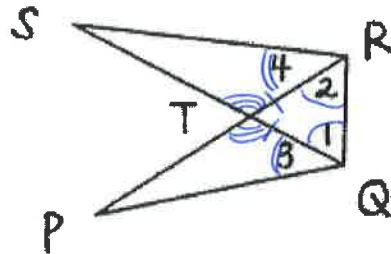
Prove: C is midpoint of \overline{BE}



Statements	Reasons
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. $\angle B \cong \angle E$	2. Alt. Int. Angles Thm
3. $\overline{AB} \cong \overline{DE}$	3. Given
4. $\angle BCA \cong \angle ECD$	4. VAT
5. $\triangle BCA \cong \triangle ECD$	5. AAS
6. $\overline{BC} \cong \overline{EC}$	6. CPCTC
7. C is the midpoint of \overline{BE}	7. Def. of midpoint

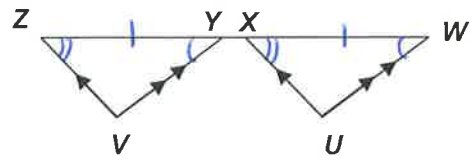
27. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\overline{ST} \cong \overline{PT}$



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overline{RT} \cong \overline{QT}$	2. Base Angles Converse
3. $\angle 3 \cong \angle 4$	3. Given
4. $\angle QTP \cong \angle RTS$	4. VAT
5. $\triangle QTP \cong \triangle RTS$	5. ASA
6. $\overline{ST} \cong \overline{PT}$	6. CPCTC

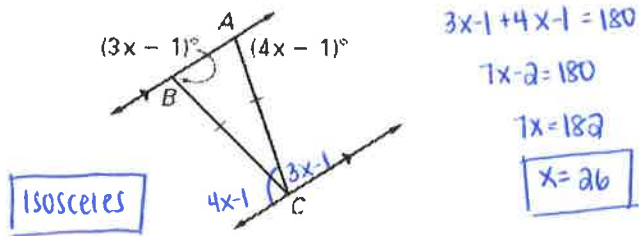
28. Given : $\overline{WU} \parallel \overline{YV}$, $\overline{XU} \parallel \overline{ZV}$, $\overline{WX} \cong \overline{YZ}$
 Prove : $\triangle WXU \cong \triangle YZV$



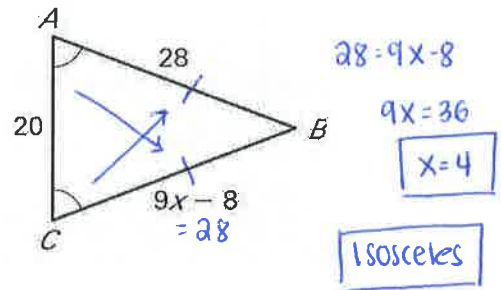
Statements	Reasons
1. $\overline{WU} \parallel \overline{YV}$	1. Given
2. $\angle ZYV \cong \angle XWU$	2. Corresponding Angles Postulate
3. $\overline{XU} \parallel \overline{ZV}$	3. Given
4. $\angle YZV \cong \angle WXU$	4. Corresponding Angles Postulate
5. $\overline{WX} \cong \overline{YZ}$	5. Given
6. $\triangle WXU \cong \triangle YZV$	6. ASA
7.	7.

Find the value of x and classify the triangle by its sides.

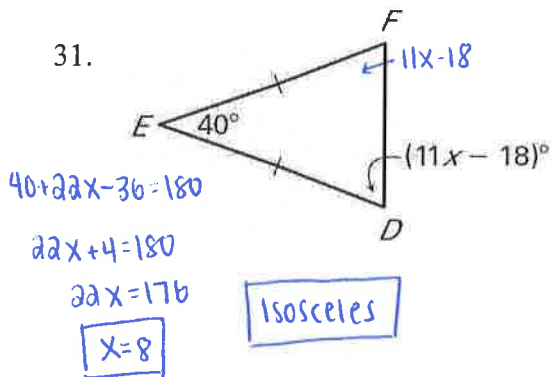
29.



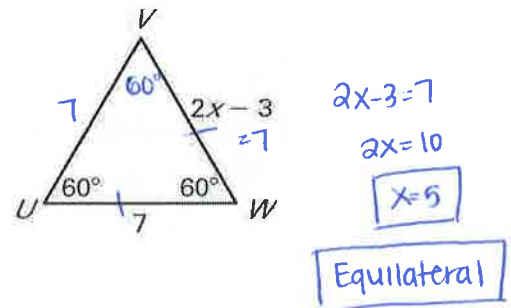
30.



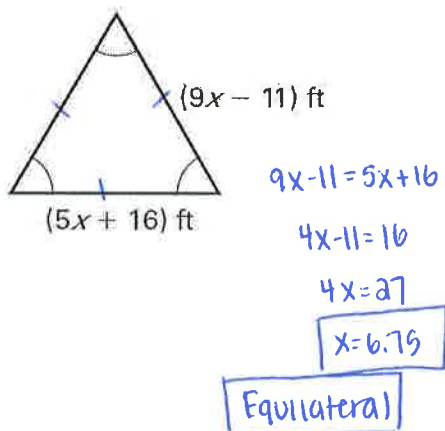
31.



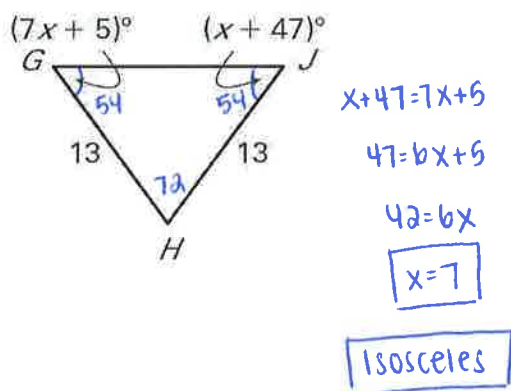
32.



33.



34.



35. In triangle DEF , $m\angle D = (4x + 2)^\circ$, $m\angle E = (6x - 30)^\circ$, and $m\angle F = 3x^\circ$. Classify the triangle by angles and sides. Explain your reasoning.

$$4x + 2 + 6x - 30 + 3x = 180$$

$$13x - 28 = 180$$

$$13x = 208$$

$$x = 16$$

$$m\angle D = 66^\circ$$

$$m\angle E = 66^\circ$$

$$m\angle F = 48^\circ$$

Isosceles Acute

36. Find the measures of the sides of isosceles $\triangle KLM$ with base \overline{KL} .

$$12 - d = 4d - 13$$

$$12 = 5d - 13$$

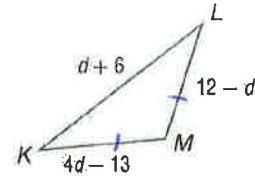
$$25 = 5d$$

$$d = 5$$

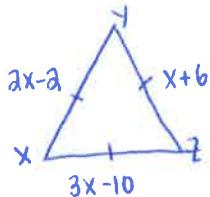
$$KL = 5 + 6 = 11$$

$$LM = 12 - 5 = 7$$

$$KM = 4(5) - 13 = 20 - 13 = 7$$



37. $\triangle XYZ$ is equilateral. \overline{XY} is two less than two times a number, \overline{YZ} is six more than the number, and \overline{XZ} is ten less than three times the number. Find the measure of all sides of $\triangle XYZ$.



$$x + 6 = 3x - 10$$

$$6 = 2x - 10$$

$$16 = 2x$$

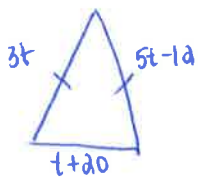
$$x = 8$$

$$XY = 14$$

$$YZ = 14$$

$$XZ = 14$$

38. The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles (there are three different answers). Find the perimeter of the triangle using the smallest value of t .

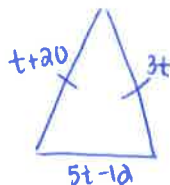


$$3t = 5t - 12$$

$$-2t = -12$$

$$t = 6$$

OR

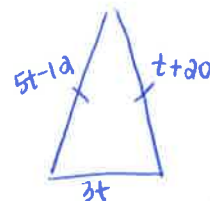


$$t + 20 = 3t$$

$$20 = 2t$$

$$t = 10$$

OR



$$5t - 12 = t + 20$$

$$4t = 32$$

$$t = 8$$

When $t = 6$:

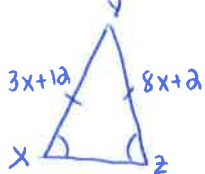
$$3(6) = 18$$

$$5(6) - 12 = 18$$

$$6 + 20 = 26$$

P = 62 units

39. In $\triangle XYZ$, $\angle X \cong \angle Z$. If $XY = 3x + 12$ and $YZ = 8x + 2$, find the value of x .



$$3x + 12 = 8x + 2$$

$$12 = 5x + 2$$

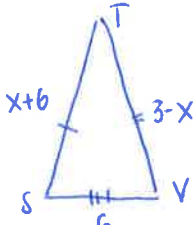
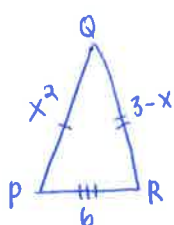
$$10 = 5x$$

$$x = 2$$

40. $\triangle PQR \cong \triangle STV$

$$PQ = x^2, SV = 6, ST = x + 6, TV = 3 - x$$

Find all possible values for x . Then find the perimeter of $\triangle PQR$.



$$x^2 = x + 6$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3, x = -2$$

$$x = 3: PQ = 9$$

QR = 0 ← cant have zero side lengths

$$PR = 6$$

$$x = -2: PQ = 4$$

$$QR = 5$$

$$PR = 6$$

P = 15 units