Geometry H
8.3: Show that a Quadrilateral is a Parallelogram

Name: Key
Date: ______________ Period: ______

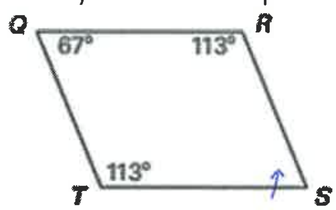
LEARNING TARGETS

- I can use properties to identify parallelograms.
- I can use coordinate geometry to identify parallelograms.

You can use the following conditions to determine whether a quadrilateral is a parallelogram.

Conditions for Parallelograms
A quadrilateral is a parallelogram if…. • Both pairs of opposite sides are parallel (definition) • Both pairs of opposite sides are congruent. • Both pairs of opposite angles are congruent. • The diagonals bisect each other. • One pair of opposite sides is congruent and parallel.

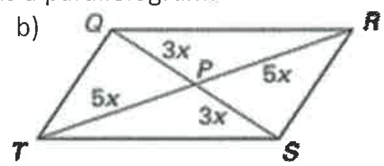
Example 1: Identify parallelograms
Explain how you know that quadrilateral *QRST* is a parallelogram.

a)


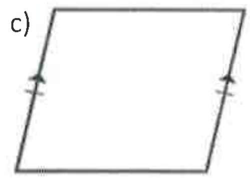
$360-113-113-67$
$=67°$

→ $m\angle S = 67°$

→ Since both pairs of opp angles are ≅, QRST is a parallelogram

b)


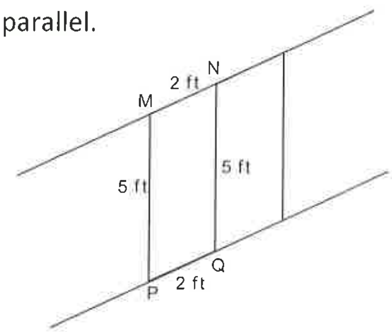
Since each half of $\overline{TR}$ and $\overline{QS}$ are the same, we can conclude that the diagonals bisect each other ∴ QRST is a parallelogram.

c)


Since one pair of sides is both congruent ∴ parallel, this quad is a parallelogram.

Example 2: Solve a real world problem

The figure shows part of a stair railing. Explain how you know that the support bars $\overline{MP}$ and $\overline{QN}$ are parallel.

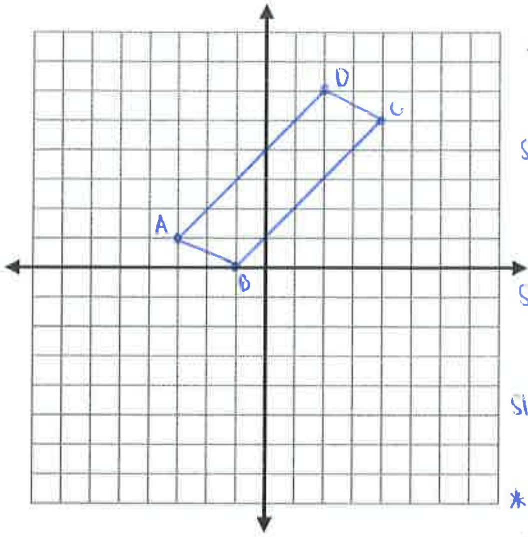


$\overline{MN} \cong \overline{PQ}$ and $\overline{MP} \cong \overline{QN}$

Since both pairs of opp sides are ≅, then MNQP is a parallelogram, and by definition, $\overline{MN} \parallel \overline{PQ}$ and $\overline{MP} \parallel \overline{QN}$

Example 3: Use coordinate Geometry to identify parallelograms

a) The vertices of *ABCD* are A(-3, 1), B(-1,0), C(4, 5), and D(2, 6). Show that *ABCD* is a parallelogram using the definition of parallelograms.



$\text{slope}_{\overline{AB}} = \frac{0-1}{-1+3} = \frac{-1}{2}$

$\text{slope}_{\overline{CD}} = \frac{6-5}{2-4} = \frac{1}{-2}$

$\Big\rangle\ \overline{AB} \parallel \overline{CD}$

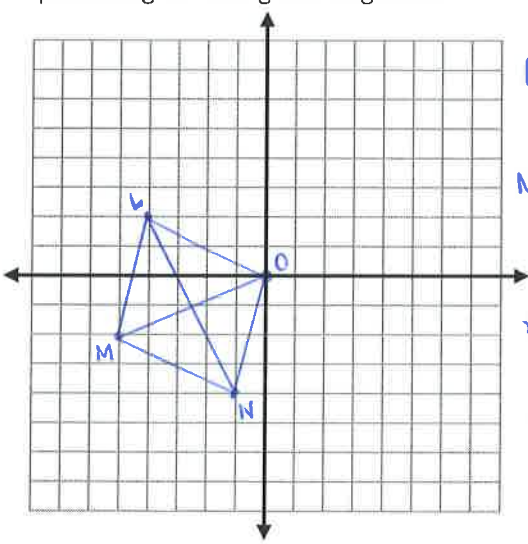
$\text{slope}_{\overline{AD}} = \frac{6-1}{2+3} = \frac{5}{5} = 1$

$\text{slope}_{\overline{BC}} = \frac{5-0}{4+1} = \frac{5}{5} = 1$

$\Big\rangle\ \overline{AD} \parallel \overline{BC}$

*Since both pairs of opp sides are parallel, then by definition, ABCD is a parallelogram

b) The vertices of *LMNO* are *L*(-4, 2), *M*(-5, -2), *N*(-1, -4) and *O*(0, 0). Show that LMNO is a parallelogram using the diagonals.



$\text{Midpoint}_{\overline{LN}} = \left(\frac{-4-1}{2}, \frac{2-4}{2}\right) = \left(-\frac{5}{2}, -\frac{2}{2}\right) = (-2.5, -1)$

$\text{Midpoint}_{\overline{MO}} = \left(\frac{0-5}{2}, \frac{0-2}{2}\right) = \left(-\frac{5}{2}, -\frac{2}{2}\right) = (-2.5, -1)$

*Since the midpoints of both diagonals are the same, we can conclude that diagonals bisect each other and LMNO is a parallelogram.

c) Use the *LMNO* from example b, prove that *LMNO* is a parallelogram using sides $\overline{LM}$ and $\overline{NO}$ only. → prove $\overline{LM}$ and $\overline{NO}$ are both congruent & parallel

$LM = \sqrt{(-5+4)^2+(-2-2)^2} = \sqrt{(-1)^2+(-4)^2} = \sqrt{1+16} = \sqrt{17}$

$NO = \sqrt{(0+1)^2+(0+4)^2} = \sqrt{(1)^2+(4)^2} = \sqrt{1+16} = \sqrt{17}$

$\Big\rangle\ \overline{LM} \cong \overline{NO}$

$m_{\overline{LM}} = \frac{-2-2}{-5+4} = \frac{-4}{-1} = 4$

$m_{\overline{NO}} = \frac{0+4}{0+1} = \frac{4}{1} = 4$

$\Big\rangle\ \overline{LM} \parallel \overline{NO}$

*Since one pair of opp sides is both parallel & congruent, LMNO is a parallelogram.

d) Could we prove a quadrilateral is a parallelogram using only side lengths?

Prove $\overline{LM} \cong \overline{NO}$: done in (c)

and

Prove $\overline{LO} \cong \overline{MN}$:

$LO = \sqrt{(0+4)^2+(0-2)^2} = \sqrt{(4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20}$

$MN = \sqrt{(-1+5)^2+(-4+2)^2} = \sqrt{(4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20}$

$\Big\rangle\ \overline{LO} \cong \overline{MN}$

* Since both pairs of opp sides are ≅, then LMNO is a parallelogram

Yes, by finding the distance of both pairs of opp sides to prove they are congruent