Decide whether you are given enough information to determine that the quadrilateral is a parallelogram.
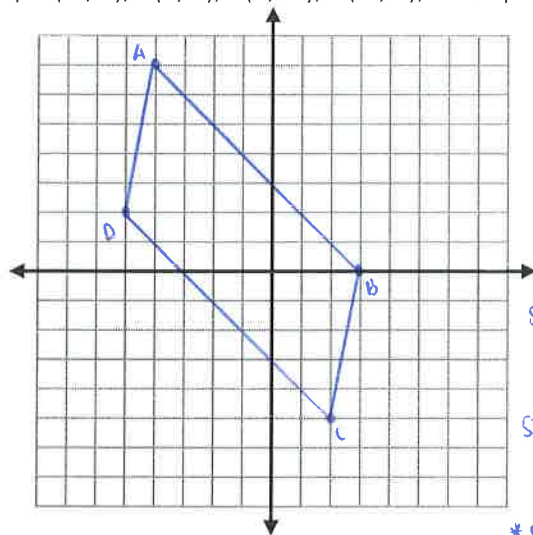
1) Opposite sides are parallel. Yes

2) Opposite sides are congruent Yes

3) Two pairs of consecutive sides are congruent. No

4) Two pairs of consecutive angles are congruent. No

5) Diagonals are congruent. No

6) Diagonals bisect each other. Yes

7) All four sides are congruent. Yes

8) Consecutive angles are supplementary. Yes

Prove that the points represent the vertices of a parallelogram. Use the method indicated.

9) A(-4, 7), B(3, 0), C(2, -5), D(-5, 2); Both pairs of opposite sides are parallel.

$$\text{Slope } \overline{AD} = \frac{2-7}{-5+4} = \frac{-5}{-1} = 5$$

$$\text{Slope } \overline{BC} = \frac{-5-0}{2-3} = \frac{-5}{-1} = 5$$

$$\Big\rangle \ \overline{AD} \ // \ \overline{BC}$$

$$\text{Slope } \overline{AB} = \frac{0-7}{3+4} = \frac{-7}{7} = -1$$

$$\text{Slope } \overline{CD} = \frac{2+5}{-5-2} = \frac{7}{-7} = -1$$

$$\Big\rangle \ \overline{AB} \ // \ \overline{CD}$$

*Since both pairs of opposite sides are parallel, ABCD is a parallelogram

10) A(-2, 8), B(2, 7), C(5, 1), D(1, 2); Both pairs of opposite sides are congruent.
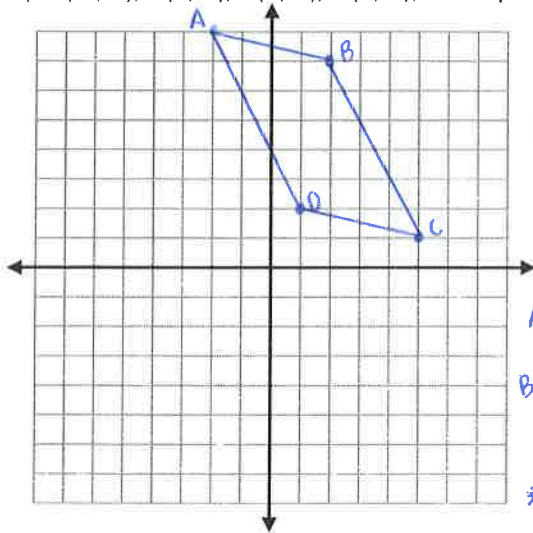
$$AB = \sqrt{(2+2)^2+(7-8)^2} = \sqrt{(4)^2+(-1)^2} = \sqrt{16+1} = \sqrt{17}$$

$$CD = \sqrt{(1-5)^2+(2-1)^2} = \sqrt{(-4)^2+(1)^2} = \sqrt{16+1} = \sqrt{17}$$

$$\Big\rangle \ \overline{AB} \cong \overline{CD}$$

$$AD = \sqrt{(1+2)^2+(2-8)^2} = \sqrt{(3)^2+(-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$$BC = \sqrt{(5-2)^2+(1-7)^2} = \sqrt{(3)^2+(-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$$\Big\rangle \ \overline{AD} \cong \overline{BC}$$

*Since both pairs of opposite sides are congruent, ABCD is a parallelogram
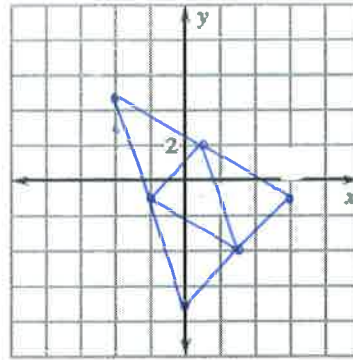
Find all possible coordinates for the fourth vertex of a parallelogram with the given vertices. Then draw the parallelogram on the graph.
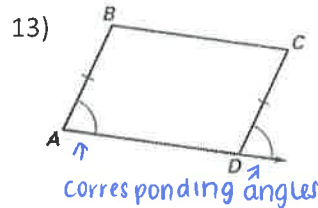
11) (4, -1), (-4, 1), (0, 8)

12) (3, -4), (-2, -1), (1, 2)

(8,6)
(-8,10)
(0,-8)
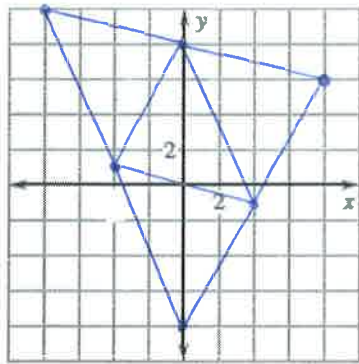
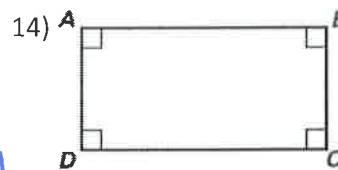(6,1)
(-4,5)
(0,-7)

**Describe how to prove that *ABCD* is a parallelogram.**

13)

corresponding angles

since a pair of corr. angles
is ≅, then $\overline{AB} // \overline{CD}$. since
$\overline{AB} // \overline{CD}$, and $\overline{AB} ≅ \overline{CD}$
then one pair of sides is
both congruent; parallel
ABCD is a parallelogram

14)

Both pairs of opp angles are ≅
(∠B ≅ ∠D, ∠A ≅ ∠C)
So ABCD is a parallelogram

15) The diagram shows a battering ram which was used in ancient times to break through walls. A log is suspended on ropes of equal length ($\overline{GF}$ and $\overline{HJ}$). The log swings, causing quadrilateral *FGHJ* to shift. In the diagram, $\overline{GH} ≅ \overline{FJ}$, and $\overline{GH}$ is parallel to the ground.
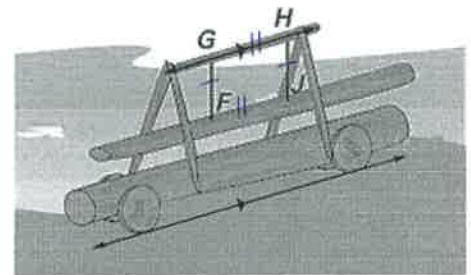
   a) Identify *FGHJ*. Explain.

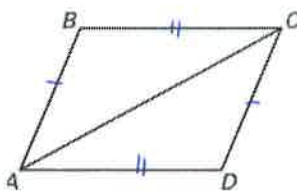   FGHJ is a parallelogram since both pairs of opposite sides
   are congruent

   b) *Explain* why the log is always parallel to the ground.

   Since we know FGHJ is a parallelogram (a) then $\overline{GH} // \overline{FJ}$. Since
   the ropes are equal length, $\overline{FJ}$ will always be parallel to the ground.

16) Complete the following proof.
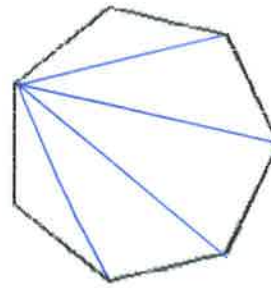   **Given:** $\triangle ABC ≅ \triangle CDA$
   Prove: *ABCD* is a parallelogram.

Statements	Reasons
1. $\triangle ABC ≅ \triangle CDA$	1. Given
2. $\overline{AB} ≅$ __CD__ $\overline{BC} ≅$ __AD__	2. CPCTC
3. *ABCD* is a parallelogram	3. both pairs of opp. sides are ≅

17) How many triangles are formed by drawing diagonals from one vertex in the figure? Find the sum of the measures of the interior angles in the figure.

A) 5, 900°

B) 5, 1080°

C) 6, 900°

D) 6, 1080°

$5 \cdot 180 = 900°$

18) The sum of the measures of the interior angles of a convex quadrilateral is _____.

A) 180°

B) 270°

C) 360°

D) 540°

19) The measure of each interior angle of a regular hexagon is _____.

A) 30°

B) 120°

C) 15°

D) 60°

$int = \dfrac{(6-2) \cdot 180}{6} = \dfrac{4 \cdot 180}{6} = \dfrac{720}{6} = 120°$

20) The measure of each exterior angle of a regular octagon is _____.

A) 22.5°

B) 67.5°

C) 45°

D) 135°

E)

$\dfrac{360}{8} = 45°$
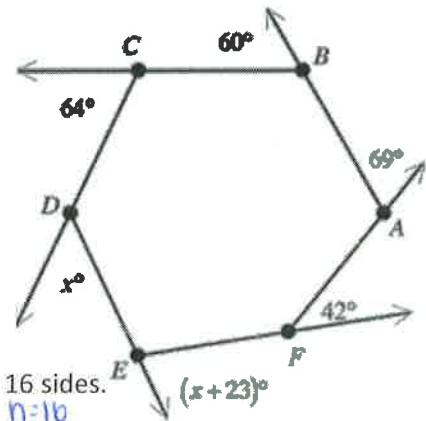
21) Find the value of x. (The figure may not be drawn to scale.)

A) 74

B) 108

C) 49

D) 51

$2x + 258 = 360$

$2x = 102$

$x = 51$

22) Find the measure of each exterior angle of a regular polygon with 16 sides.

A) 11.25°

B) 360°

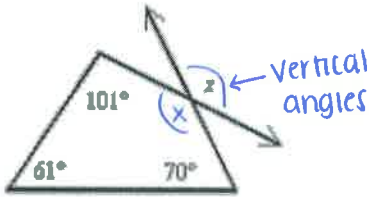C) 22.5°

D) 157.5°

$n = 16$

$\dfrac{360}{16} = 22.5°$

$60°$

C B

$64°$

$69°$

D

$x°$

A

$42°$

E $(x+23)°$ F

23) Find the measure of the missing angle.

$x + 246 = 360$

$\boxed{x = 114°}$

24) Find the value of x.

← vertical angles

$x + 232 = 360$

$\boxed{x = 128°}$

25) Find the values of x and y.
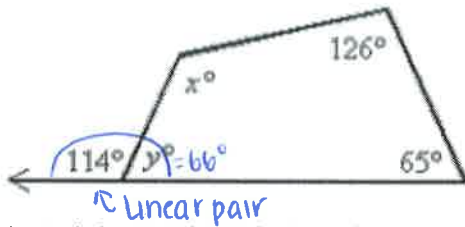
$114 + y = 180$

$\boxed{y = 66°}$

$x + 257 = 360$

$\boxed{x = 103}$

linear pair

26) Find the number of sides of a convex polygon if the measures of its interior angles have a sum of 2880°.

$2880 = (n-2) \cdot 180$

$16 = n - 2$

$\boxed{n = 18}$

27) Find the number of sides of a regular polygon with each interior angle equal to 171°.

$171 = \dfrac{(n-2) \cdot 180}{n}$

$171n = (n-2) \cdot 180$

→ $171n = 180n - 360$

$-9n = -360$

$\boxed{n = 40}$

28) Find the measure of an interior angle and an exterior angle of a regular polygon with 20 sides.

$\text{Int} = \dfrac{(20-2) \cdot 180}{20}$

$= \dfrac{18 \cdot 180}{20} = \dfrac{3240}{20} = \boxed{162°}$

$\text{Ext} = \dfrac{360}{20} = \boxed{18°}$
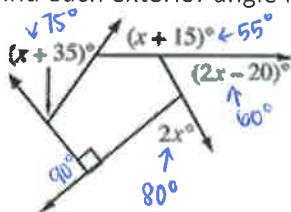
29) Find the measure of an interior angle and the measure of an exterior angle for a regular 32-gon.

$\text{Int} = \dfrac{(32-2) \cdot 180}{32}$

$= \dfrac{30 \cdot 180}{32} = \dfrac{5400}{32} = \boxed{168.75°}$

$\text{Ext} = \dfrac{360}{32} = \boxed{11.25°}$

30) Find each exterior angle measure in the diagram below
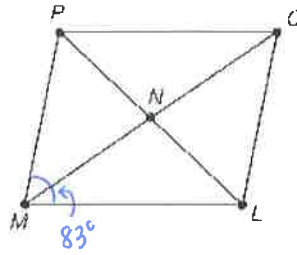

$6x + 120 = 360$

$6x = 240$

$\boxed{x = 40}$

31) For parallelogram PQLM, if $m\angle PML = 83°$, then $m\angle PQL = $ _____.

A) $m\angle PQM$

B) 83°

C) 97°

D) $m\angle QLM$

32) Consecutive angles in a parallelogram are always _____.

A) Congruent angles

B) Complementary angles

C) Supplementary angles

D) Vertical angles

33) Choose the statement that is NOT ALWAYS true.
    For any parallelogram _____.

A) The diagonals bisect each other

B) Opposite sides are congruent

C) The diagonals are perpendicular
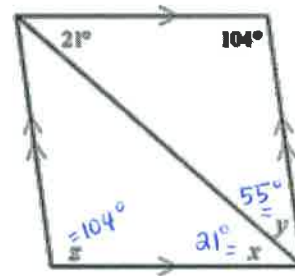
D) Opposite sides are congruent

34) Find the value of the variables in the parallelogram.

A) $x = 52°, y = 10.5°, z = 159°$

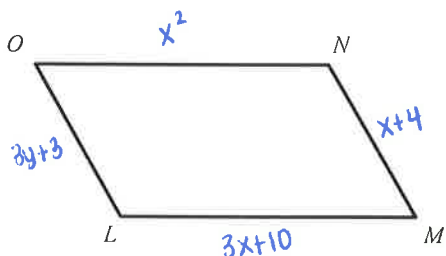B) $x = 21°, y = 44°, z = 104°$ (55 written above 44)

C) $x = 55°, y = 21°, z = 104°$

D) $x = 10.5°, y = 52°, z = 159°$

35) If $ON = x^2$, $LM = 3x + 10$, $NM = x + 4$, and $OL = 3y + 3$, find the values of x and y given that LMNO is a parallelogram.

$x^2 = 3x + 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\boxed{x=5}$ $\boxed{x=-2}$

If X = 5 : $3y + 3 = 9$

$3y = 6$

$\boxed{y=2}$

check: $3(2) + 3$
$= 6 + 3$
$= 9$ ✓

If x = -2 : $3y + 3 = 2$

$3y = -1$

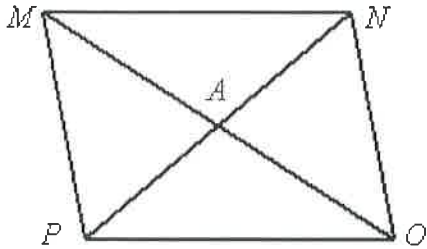$\boxed{y = -\frac{1}{3}}$

check: $3(-\frac{1}{3}) + 3$
$= -1 + 3$
$= 2$ ✓

36) Complete the statement for parallelogarm *ABCD*. Justify your answer.

$\overline{AD} \cong$ _BC_

opposite sides of a pgram are ≅
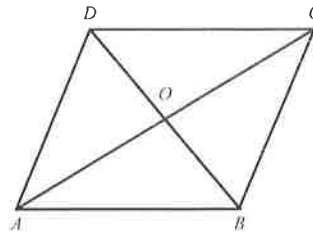
37) Find *AM* in the parallelogram if *PN* = 10 and *MO* = 19.

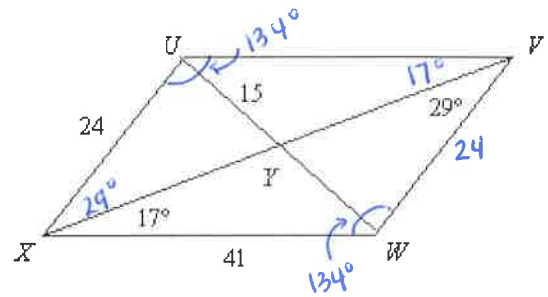

$\frac{1}{2} MO = AM$

$\frac{1}{2}(19) = AM$

$AM = 9.5$

38) *UVWX* is a parallelogram, $m\angle WXV = 17°$, $m\angle WVX = 29°$, $XW = 41$, $UX = 24$, $UY = 15$

a) Find $m\angle WVU$. = 17+29 = 46°

b) Find *WV*. = 24
(opp sides are ≅)

c) Find $m\angle XUV$. = 134°
(consecutive angles are supplementary)
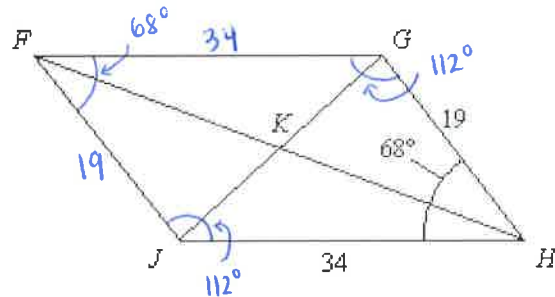
d) Find *UW*. 15+15 = 30
(diagonals bisect each other)

39) *FGHJ* is a parallelogram, $m\angle JHG = 68°$, $JH = 34$, $GH = 19$

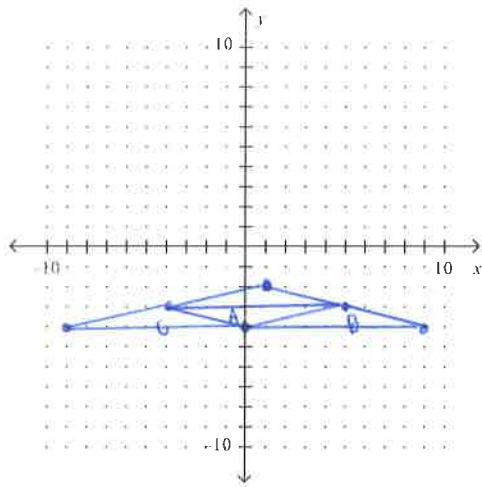a) Find $m\angle FJH$. = 112°
(consecutive angles are supplementary)

b) Find *JF*. = 19
(opp sides are ≅)

c) Find $m\angle GFJ$. = 68°
(opp angles are ≅)

d) Find *FG*. = 34
(opp sides are ≅)

40) Find a fourth point, D, so that a parallelogram is formed using the vertices $A(0, -4)$, $B(5, -3)$, $C(-4, -3)$, and D in any order. Plot your point and draw the parallelogram in the coordinate plane.
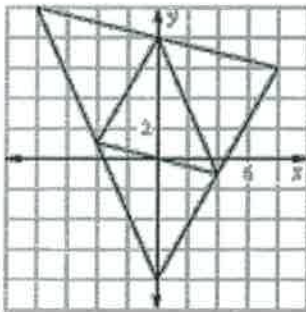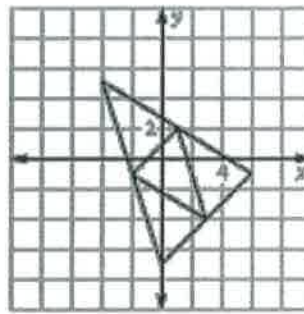
$(1, -2)$

$(9, -4)$

$(-9, -4)$

Answer Key

1) Yes ✓
2) Yes ✓
3) No ✓
4) No ✓
5) No ✓
6) Yes ✓
7) Yes ✓
8) Yes ✓
9) Slope of $\overline{AB}$ = slope of $\overline{CD}$ = -1; ✓
   Slope of $\overline{BC}$ = slope of $\overline{DA}$ = 5,
   Since both pairs of opposite sides are parallel, $ABCD$ is a parallelogram.
10) $AB = CD = \sqrt{17}$; ✓
    $BC = DA = 3\sqrt{5}$.
    Since both pairs of opposite sides are congruent, $ABCD$ is a parallelogram.
11) (8, 6), (0, -8), and (-8, 10) ✓

12) (6,-1), (0,-7) and (-4, 5) ✓

13) $\overline{AB} \parallel \overline{CD}$ by Corr. Angles ✓ Converse. Since one pair of opposite sides is both parallel and congruent, $ABCD$ is a parallelogram.
14) Since both pairs of opposite angles are congruent, $ABCD$ is a parallelogram. ✓
15) a) $\overline{GF} \cong \overline{HJ}$ and $\overline{GH} \cong \overline{FJ}$, so $FGHJ$ is a parallelogram since both pairs of opposite sides are congruent. ✓
    b)$FGHJ$ is always a □, so $\overline{GH} \parallel \overline{FJ}$. Because $\overline{GH}$ is parallel to the ground, then $\overline{FJ}$ is also parallel to the ✓ ground. The moving log is always parallel to the ground.

16) Given; $\overline{CD}$; $\overline{AD}$; CPCTC; In a quadrilateral, if both pair of opp. Sides are congruent, ✓ then it is a parallelogram.
17) A ✓
18) C ✓
19) B ✓
20) C ✓
21) D ✓
22) C ✓
23) 114° ✓
24) 128° ✓
25) X=103, y = 66 ✓
26) 18 ✓
27) 40 ✓
28) Int angle: 162°, ext angle: 18° ✓
29) ≈ 168.8°, ≈ 11.2° ✓
30) 55°, 60°, 75°, 80° ✓
31) B ✓
32) C ✓
33) C ✓
34) B ✓
35) X = 5, y = 2 ✓ and X= -2 and y= -1/3
36) $\overline{BC}$, opp. Sides of parallelogram are congruent ✓
37) 9.5 ✓
38) a) 46° b)24 c) 134° d) 30 ✓
39) a) 112° b)19 c) 68° d) 34 ✓
40) (1, -2), (9, -4), or (-9, -4) ✓