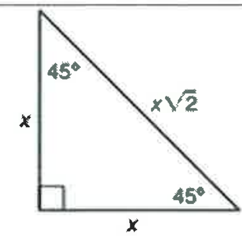
Geometry – H
7.4 – Special Right Triangles

Name: Key
Date: ______________ Period: ________

LEARNING TARGETS

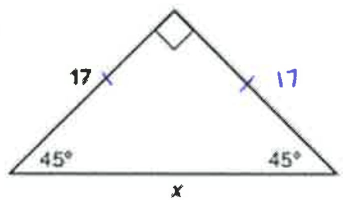
- I can apply special right triangle ratios to find unknown side lengths.
- I can use special right triangles in real world situations.

Theorem	Diagram
45° – 45°– 90° Triangle Theorem In a 45° – 45°– 90°, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg	 hyp = leg $\cdot \sqrt{2}$

Example 1: Find lengths in a 45° – 45°– 90° triangle

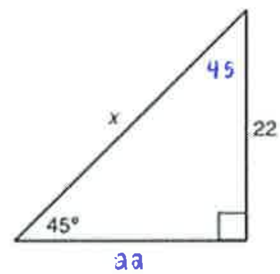
Find the value of x. Leave answer in simplest radical form.

a.



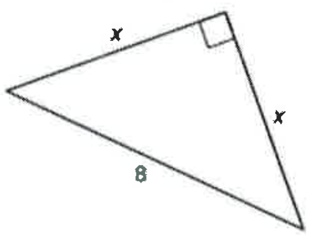
$\boxed{x = 17\sqrt{2}}$

b.



$\boxed{x = 22\sqrt{2}}$

c.

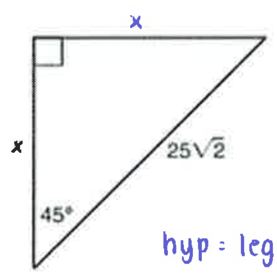


hyp = leg $\cdot \sqrt{2}$

$\frac{8}{\sqrt{2}} = \frac{x\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$x = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{\sqrt{4}} = \frac{8\sqrt{2}}{2} = \boxed{4\sqrt{2} = x}$

d.



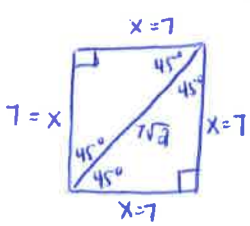
hyp = leg $\cdot \sqrt{2}$

$\frac{25\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = \frac{x\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$\boxed{x = 25}$

Example 2: Apply 45° – 45°– 90° Triangle Theorem

Find the area of the square whose diagonal is $7\sqrt{2}$ inches long.



hyp = leg $\cdot \sqrt{2}$

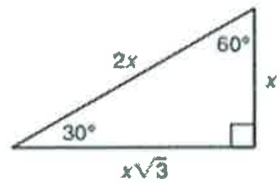
$\frac{7\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = \frac{x\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

x = 7

Area = l × w

A = (7)(7)

$\boxed{A = 49\text{ in}^2}$

Theorem	Diagram
30° – 60°– 90° Triangle Theorem In a 30° – 60°– 90°, the length of the hypotenuse is twice the length of shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.	 hyp = ~~hm~~ short · 2 long = short · $\sqrt{3}$

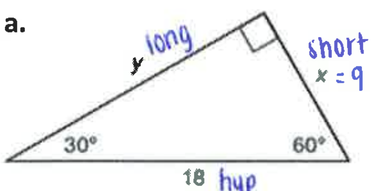
**Note – The short leg is always opposite the 30° angle! long leg always opposite the 60° angle!

- It is best to find the length of the short leg first (if it is not already given)

<u>Example 3: Find lengths in a 30° – 60°– 90° triangle</u>

Find the values of x and y. Leave answer in simplest radical form.

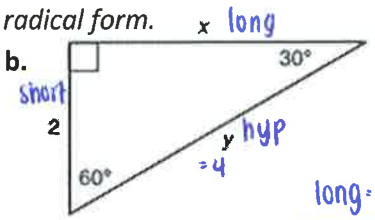
a.



hyp = short · 2
$18 = x \cdot 2$
$\boxed{x = 9}$

long = short · $\sqrt{3}$
$\boxed{y = 9\sqrt{3}}$

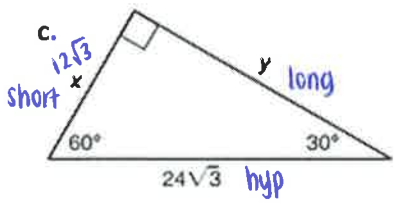
b.



hyp = short · 2
$y = 2 \cdot 2$
$\boxed{y = 4}$

long = short · $\sqrt{3}$
$\boxed{x = 2\sqrt{3}}$

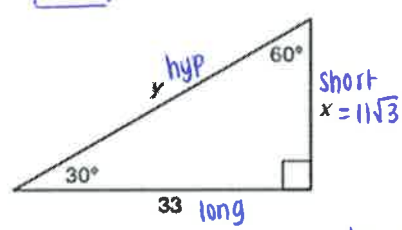
c.



hyp = short · 2
$\frac{24\sqrt{3}}{2} = \frac{x \cdot \cancel{2}}{\cancel{2}} \Rightarrow \boxed{x = 12\sqrt{3}}$

long = short · $\sqrt{3}$
$y = 12\sqrt{3}\sqrt{3}$
$y = 12\sqrt{9}$
$y = 12 \cdot 3$
$\boxed{y = 36}$

d.

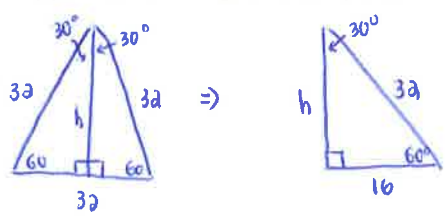


long = short · $\sqrt{3}$
$\frac{33}{\sqrt{3}} = \frac{x\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$
$x = \frac{33}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{33\sqrt{3}}{3} = \boxed{11\sqrt{3}}$

hyp = short · 2
$y = 11\sqrt{3} \cdot 2$
$\boxed{y = 22\sqrt{3}}$

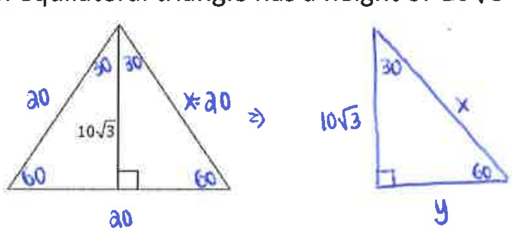
<u>Example 4: Apply 30° – 60°– 90° Triangle Theorem</u>

a. You make a guitar pick that resembles and equilateral triangle with side lengths of 32 mm. What is the approximate height of the pick?



long = short · $\sqrt{3}$
$h = 16\sqrt{3}$ mm
$\boxed{h \approx 27.7\text{mm}}$

b. An equilateral triangle has a height of $10\sqrt{3}$. What is the length of a side of the triangle?



long = short · $\sqrt{3}$
$\frac{10\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = \frac{y\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$
$y = 10$

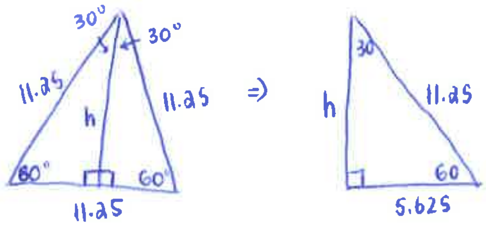
hyp = short · 2
$x = 2 \cdot 10$
$x = 20$

$\boxed{\text{Each side is 20 units}}$

Example 5: Applications of Special Right Triangles

a. Regulation billiard balls are 2.25 inches in diameter. The rack used to group 15 billiard balls is in the shape of an equilateral triangle. What is the approximate height of the triangle formed by the rack? Leave answer in simplest radical form and round to the nearest quarter of an inch.

each side $= 2.25$ in $\times$ 5 balls $= 11.25$

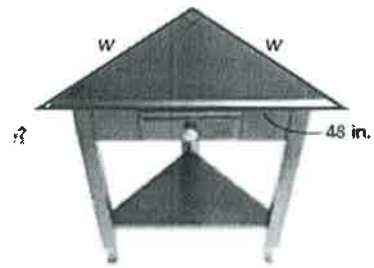


long = short $\cdot \sqrt{3}$

$h = 5.625\sqrt{3}$ or $\boxed{\dfrac{45\sqrt{3}}{8} \text{ inches}}$

$\approx \boxed{9.74 \text{ inches}}$

b. This tabletop is an isosceles right triangle. The length of the front edge of the table is 48 inches. What is the length w of each side edge? Round your answer to the nearest tenth of an inch.

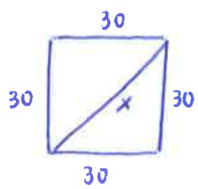


hyp = leg $\cdot \sqrt{2}$

$\dfrac{48}{\sqrt{2}} = \dfrac{w \cdot \cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

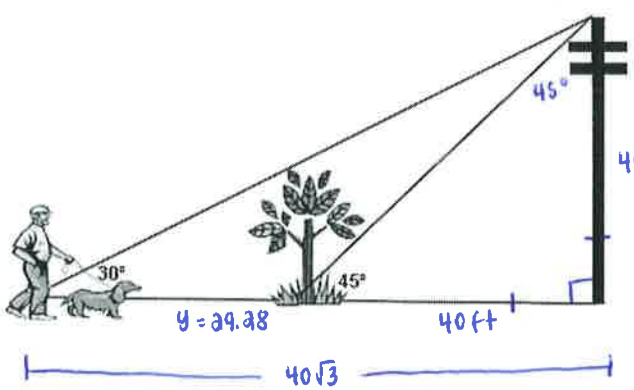
$w = \dfrac{48 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \dfrac{48\sqrt{2}}{2} = 24\sqrt{2} \approx \boxed{33.9 \text{ inches}}$

c. The perimeter of a square is 120 cm. Find the length of the diagonal of the square. Give your answer in simplest radical form.



$\boxed{\text{diagonal} = 30\sqrt{2} \text{ cm}}$

d. A man is walking his dog on level ground in a straight line with the dog's favorite tree. The angle of elevation from the man's present position to the top of a nearby telephone pole is 30°. The angle of elevation from the tree to the top of the telephone pole is 45°. If the telephone pole is 40 feet tall, how far is the man with the dog from the tree? Express your answer to the nearest tenth of a foot.

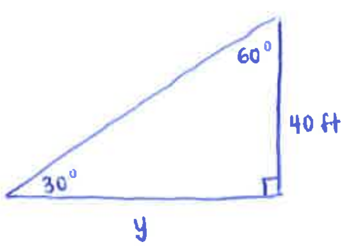


$y = 40\sqrt{3} - 40$

$y = 29.28$

The dog → tree $= 29.28 + 40$

$= \boxed{69.3 \text{ ft}}$



long = short $\cdot \sqrt{3}$

$y = 40\sqrt{3}$