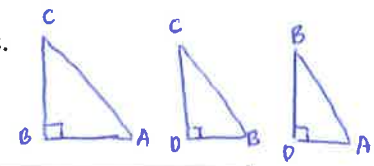
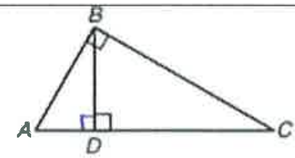
Geometry H
7.3 Notes: Similarity in Right Triangles

Name: Key
Date: ______ Period: ______

LEARNING TARGETS

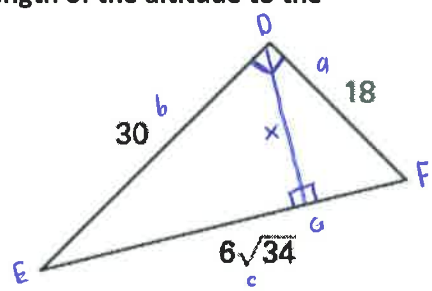
- I can apply similarity relationships in right triangles to solve problems.
- I can use geometric mean to find segment lengths in right triangles.



Altitudes and Similar Triangles	
Theorem: The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.	**Example:**  $\overline{BD}$ is the altitude to the hypotenuse of $\triangle ABC$, so $\triangle ABC \sim$ $\triangle DBC$ $\sim$ $\triangle ADB$

Example 1: Find the length of the altitude to the hypotenuse.

Solution: Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.



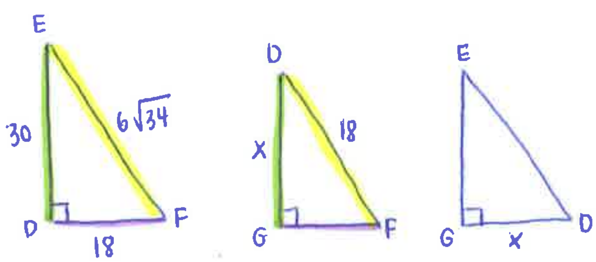
Step 1 Use Converse of Pythagorean Theorem to determine if the triangle is a right triangle.

$(6\sqrt{34})^2 \overset{?}{=} 30^2+18^2$

$1224 \overset{?}{=} 1224$

$\boxed{\text{Right } \triangle \text{ since } c^2=a^2+b^2}$

Step 2 Draw in the altitude to the hypotenuse. Identify the similar triangles and sketch them so that the corresponding angles and sides have the same orientation.



Step 3 Find the length of the altitude. Use your diagrams from step 1 to set up and solve a proportion.

$\frac{x}{30} = \frac{18}{6\sqrt{34}}$

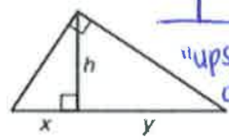
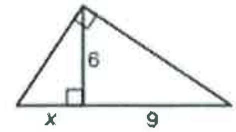
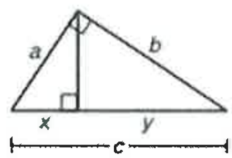
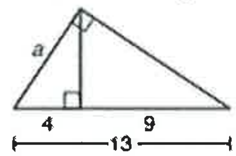
$\frac{540}{6\sqrt{34}} = \frac{6\sqrt{34}\,x}{6\sqrt{34}}$

Like terms so divide!

$x = \frac{90}{\sqrt{34}}$ ← cant keep √ in denominator so rationalize!

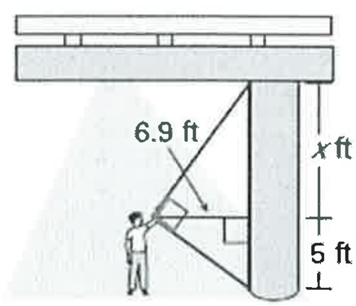
$x = \frac{90 \cdot \sqrt{34}}{\sqrt{34} \cdot \sqrt{34}} = \frac{90\sqrt{34}}{\sqrt{1156}} = \frac{90\sqrt{34}}{34} = \frac{45\sqrt{34}}{17}$

simplify!

Geometric Means		
Words	**Symbols**	**Example**
Geometric Mean (Altitude) Theorem The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments of the hypotenuse.	$\perp$ "upside down T"  $\frac{x}{h}=\frac{h}{y}$ * use when given the length of the altitude *	Solve for x.  $\frac{x}{6}=\frac{6}{9} \Rightarrow 9x=36$ $\boxed{x=4}$
Geometric Mean (Leg) Theorem The length of a leg of a right triangle is the geometric mean of the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.	 $\frac{x}{a}=\frac{a}{c}$ **or** $\frac{y}{b}=\frac{b}{c}$ "pinball" or "slingshot"	Solve for a.  $\frac{4}{a}=\frac{a}{13} \Rightarrow a^2=52$ $a=\sqrt{52}$ $\boxed{a=2\sqrt{13}}$

Example 2: Find a height using indirect measurement.

To find the clearance under an overpass, you need to find the height of a concrete support beam. You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.



use upside down T b/c altitude is given

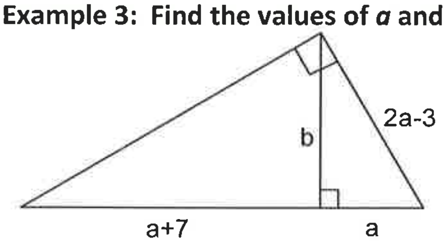
$\frac{5}{6.9}=\frac{6.9}{x} \Rightarrow 5x=47.61$

$x=9.5$

The beam is about 14.5 ft tall

Height $= 9.5+5=14.5$

Example 3: Find the values of *a* and *b*.



Check $a=\frac{1}{2}$: $2(\frac{1}{2})-3=1-3=-2$

Given altitude, try upside down T:

$\frac{a+7}{b}=\frac{b}{a}$

$a^2+7a=b^2$

$(9)^2+7(9)=b^2$

$81+63=b^2$

$144=b^2$

$\boxed{b=12}$

Try pinball to solve for a first:

$\frac{a}{2a-3}=\frac{2a-3}{a+a+7} \Rightarrow \frac{a}{2a-3}=\frac{2a-3}{2a+7}$

$2a^2+7a=4a^2-12a+9$

$0=2a^2-19a+9$

$0=(2a-1)(a-9)$

$a\cancel{=}\frac{1}{2}$, $\boxed{a=9}$