Geometry H
6.1 Notes: Proportions

Name: _____
Date: _____ Period: _____

- ✓ I can solve proportions
- ✓ I can find a geometric mean

Vocabulary:

- A **ratio** is a comparison of two quantities using division. The ratio *a* to *b*, where *b* is not zero, can be written as $\frac{a}{b}$ or $a:b$.
- A **proportion** is an equation in which two ratios are equal. For example, $\frac{a}{b}=\frac{c}{d}$ is an example of a proportion. The numbers *b* and *c* are called the **means** and the numbers *a* and *d* are called the **extremes.**
  - ➢ The truth of a proportion can be tested by using the **Cross Products Property**, which states:
    If $\frac{a}{b}=\frac{c}{d}$, where $b\neq 0$ and $d\neq 0$, then $ad=bc$ (also known as product of means = product of extremes).
- The **geometric mean** of two positive numbers *a* and *b* is the positive number *x* that satisfies $\frac{a}{x}=\frac{x}{b}$. So, $x^2=ab$ and $x=\sqrt{ab}$

Example 1: Solve a proportion.
→To solve a proportion, you use the Cross Products Property to write an equivalent equation.

a) Solve the proportion: $\frac{x}{8}=\frac{6}{9}$

First, cross multiply: $9\cdot x=8\cdot 6$
Now, simplilfy: $9x=48$
To solve for x, divide both sides by 9: $\frac{9x}{9}=\frac{48}{9}$
Solution: $x=\frac{48}{9}$ (Since the fraction cannot be reduced, just leave it ☺)

b) Solve the proportion: $\frac{x}{5}=\frac{x+3}{7}$

First, cross multiply: $7\cdot x=5(x+3)$
Now, simplilfy (don't forget to distribute!!!): $7x=5x+15$
To solve for x, subtract 5x from each side: $2x=15$
Finally, divide both sides by 2: $\frac{2x}{2}=\frac{15}{2}$

Solution: $x = \frac{15}{2}$ (Since the fraction cannot be reduced, just leave it ☺)

c) Solve the proportion: $\frac{x}{5} = \frac{45}{x}$

First, cross multiply: $x \cdot x = 45 \cdot 5$

Now, simplilfy: $x^2 = 225$

To solve for x, take the positive and negative square root of both sides: $\sqrt{x^2} = \pm\sqrt{225}$

Simplify the radicals: $x = \pm 15$

Solution: $x = \pm 15$ (If this were an applied problem, we might only need the positive solution. Since it's not, we'll keep both values ☺)

d) Solve the proportion: $\frac{x+1}{4} = \frac{12}{x+3}$

First, cross multiply: $(x+1)(x+3) = 4 \cdot 12$

Now, simplilfy (don't forget to "FOIL" the left hand side: $x^2 + 3x + x + 3 = 48$

Keep simplifying by combining like terms: $x^2 + 4x + 3 = 48$

Solve the quadratic equation by factoring: $x^2 + 4x - 45 = 0$

$$(x+9)(x-5) = 0$$

$$x = -9 \text{ } OR \text{ } x = 5$$

Solution: $x = -9$ *or* $x = 5$ (we could check to verify that both solutions produce equivalent fractions ☺)

Example 2: Find a geometric mean.

To find the geometric mean of two numbers, we use the definition of geometric mean: $x = \sqrt{ab}$. We will always use the positive square root and we must write the answer in simplest radical form.

a) Find the geometric mean of 3 and 27.

First, substitute 3 and 27 in for a and b into the definition: $x = \sqrt{3 \cdot 27}$

Simplify the statement: $x = \sqrt{81}$

Since 81 is a perfect square, x = 9.

Solution: The geometric mean of 3 and 27 is 9.

b) Find the geometric mean of 6 and 15.

First, substitute 6 and 15 in for a and b into the definition: $x = \sqrt{6 \cdot 15}$

Simplify the statement: $x = \sqrt{90}$

Since 90 is not a perfect square, we must simplify the radical. Find the largest perfect square that divides into 90 and write $\sqrt{90}$ as a product.: $x = \sqrt{9} \cdot \sqrt{10}$ (The largest perfect square that divides into 90 is 9)

Simplify the statement: $x = 3\sqrt{10}$

Solution: The geometric mean of 3 and 27 is $3\sqrt{10}$.

Geometry H Name: _____

Homework: Solving Proportions and Geometric Mean Date: _____ Period: _____

Solve the proportion.

1) $\frac{7}{12}=\frac{x}{48}$

2) $\frac{11}{a}=\frac{55}{75}$

3) $\frac{14}{y-5}=\frac{2}{3}$

4) $\frac{2z}{27}=\frac{3z+9}{81}$

5) $\frac{48}{68}=\frac{b+2}{b+7}$

6) $\frac{9}{s}=\frac{s}{16}$

7) $\frac{19}{32}=\frac{7d+3}{15d-11}$

8) $\frac{x}{111}=\frac{5x-28}{333}$

9) $\frac{4x}{6x+4}=\frac{x}{25}$

10) Use the proportions $\frac{a+b}{2a-b}=\frac{5}{4}$ and $\frac{b}{a+9}=\frac{5}{9}$ to find the values of a and b.

Find the geometric mean of the two numbers. Leave answers in simplest radical form.

10) 6 and 24

11) 7 and 28

12) 4 and 12

13) 9 and 12

14) 15 and 45

15) 12 and 48

Why Did the Envelope Lose the Race with the Cardboard Box?

Cross out the letters above each correct answer. When you finish, write the remaining letters in the spaces at the bottom of the page.

1. Water was leaking from a faucet at a rate of 1.5 gal every 5 min. If it took 18 min to stop the leak, how much water was wasted?

2. The ratio of mango juice to guava juice in Paradise Punch is 5 to 3. Leilani has 32 fl oz of mango juice. How much guava juice does she need?

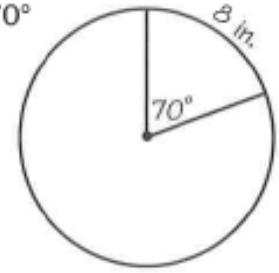
3. A locomotive is 58 ft long and 11 ft wide. A special effects designer makes a model that is 20 in. long. How wide should it be?

4. A marathon runner ran the first 4 mi in 27.8 min. If she continues running at this pace, how long will it take her to run the entire marathon of 26.2 mi?

5. A map of Yosemite National Park is drawn to a scale of 1 in. = 1.65 mi. On the map, Tioga Pass is 13.8 in. from Yosemite Falls. What is the actual distance?

6. If it took 1.5 qt of paint to paint the wall on the left, how many quarts will be needed to paint the wall on the right?

7. For this circle, a 70° central angle cuts off an arc of 8 in. What is the circumference of the circle? (There are 360° in a circle.)

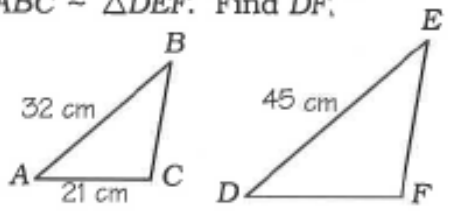


8. Red brass is an alloy made by combining copper with zinc in a ratio of 16 to 3. How much zinc should be combined with 40 kg of copper to make red brass?

9. A photograph is 7 in. long and 4.5 in. wide. A yearbook editor has the photo reduced to fit a space 1.8 in. wide. How long is the reduced photograph?

10. An ant that weighs 0.004 oz can lift a bread crumb that weighs 0.2 oz. If a 120-pound person were proportionally as strong as the ant, how much could the person lift?

11. In similar figures, corresponding angles are congruent and corresponding sides are in proportion. In the figure below, $\triangle ABC \sim \triangle DEF$. Find DF.



TH 22.8 mi	**EB** 3.8 in.	**I** 28.7 cm	**OX** 182.1 min	**S** 7.5 kg	**TW** 3.2 in.	**IN** 6000 lb	**A** 23.5 mi	**SS** 2.8 qt	**RE** 41.1 in.
TA 42.5 in.	**S** 5.4 gal	**TI** 5200 lb	**ME** 29.5 cm	**ON** 18.5 fl oz	**TO** 3.2 qt	**E** 9.2 kg	**VE** 2.8 in.	**UP** 19.2 fl oz	**RY** 175.1 min

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Answer Key:

1) X = 28	2) a = 15	3) y = 26
4) z = 3	5) b = 10	6) s = 12
7) d = 5	8) x = 14	9) x = 16
10) 45 and 30		
11) 12	12) 14	13) $6\sqrt{3}$
14) $6\sqrt{3}$	15) $15\sqrt{3}$	16) 24

1) 5.4 gal	2) 19.2 oz	3) 3.8 in
4) 182.1 min	5) 22.8 mi	6) 3.2 qt
7) 41.1 in.	8) 7.5 kg	9) 2.8 in
10) 6,000 lb	11) 29.5 cm	