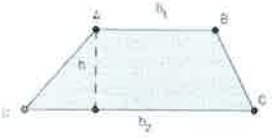
Geometry H
11.2/11.5 Notes: Areas of Trapezoids, Rhombuses, Kites, and Sectors of Circles

Name: Key
Date: _____ Period: ____

- I can find areas of special quadrilaterals.
- I can find areas of sectors in circles.

Investigation 1: Use the diagram of trapezoid *ABCD* below to complete the following.



1) What type of quadrilateral is formed when *ABCD* is rotated 180° ? Explain.

Parallelogram

2) What is the relationship between the area of trapezoid *ABCD* and the quadrilateral formed in step 1?

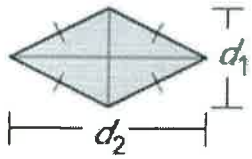
The area of the trapezoid will be half of the area of the parallelogram

3) Using the quadrilateral formed in Step 1, write an equation to find the area of trapezoid *ABCD* using h, b_1 and b_2

$A_{pgram} = (b_1+b_2)(h)$

$A_{trap} = \frac{1}{2}(b_1+b_2)(h)$ or $\frac{1}{2}(h)(b_1+b_2)$

Investigation 2: Use the diagram of the rhombus below to complete the following.



1) What do you know about the diagonals of a rhombus?

They are perpendicular and bisect each other

2) What type of triangles are formed by the diagonals? What do we know about all four triangles?

Right triangles; they are congruent

3) What are the side lengths of one triangle?

height = $\frac{d_1}{2}$; base = $\frac{d_2}{2}$

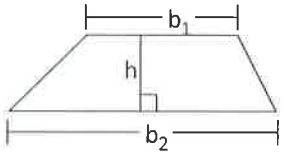
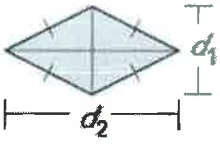
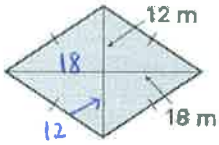
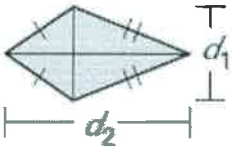
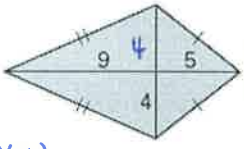
4) What is the area of one triangle?

$A = \frac{1}{2}\left(\frac{d_2}{2}\right)\left(\frac{d_1}{2}\right) = \frac{d_1 d_2}{8}$

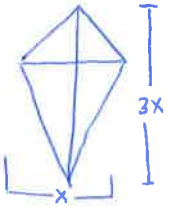
5) Using your answer from question 4, write an equation to find the area of a rhombus in terms of d_1 and d_2.

$A = 4\left(\frac{d_1 d_2}{8}\right) = \frac{d_1 d_2}{2}$ or $\frac{1}{2} d_1 d_2$

Note: We could do a similar proof to develop the area of a kite ☺

Shape	Formula	Example
Trapezoid	$A=\frac{1}{2}h(b_1+b_2)$ 	Find the area:  $A=\frac{1}{2}(15)(18+40)$ $=\frac{1}{2}(15)(58)$ $A=435\,m^2$
Rhombus	$A=\frac{1}{2}d_1d_2$ 	Find the area:  $d_1=24$ $d_2=36$ $A=\frac{1}{2}(24)(36)$ $A=432\,m^2$
Kite	$A=\frac{1}{2}d_1d_2$ 	Find the area:  $d_1=8$ $d_2=14$ $A=\frac{1}{2}(8)(14)$ $A=56\,units^2$

Example 1: One diagonal of a rhombus is three times as long as the other diagonal. The area of the rhombus is 24 square feet. What are the lengths of the diagonals?



$A=\frac{1}{2}d_1d_2$

$24=\frac{1}{2}(x)(3x)$

$24=\frac{1}{2}(3x^2)$

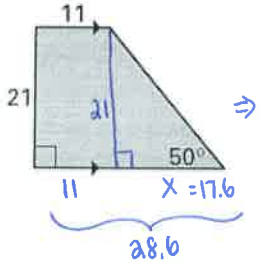
$24=1.5x^2$

$16=x^2$

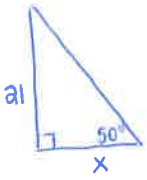
$x=4$

diagonals: $x=4$ ft, $3x=12$ ft

Example 2: Please find the area of the polygon below.



⇒



$\frac{\tan 50°}{1}=\frac{21}{X}$

$21=X\tan 50°$

$X=\frac{21}{\tan 50°}$

$X=17.6$

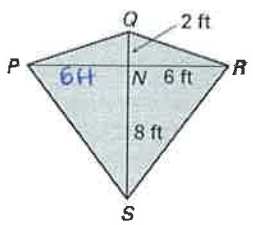
$A=\frac{1}{2}h(b_1+b_2)$

$A=\frac{1}{2}(21)(11+28.6)$

$A=\frac{1}{2}(21)(39.6)$

$A=415.8\,units^2$

Example 3: Please find the area of the polygon below.



$d_1=10$

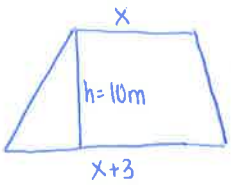
$d_2=12$

$A=\frac{1}{2}(10)(12)$

$A=60\,ft^2$

Example 4: Solve for an unknown measure

The height of a trapezoid is 10 meters and the area is 95 square meters. One base is 3 meters longer than the other base. What are the lengths of the bases?



$A = \frac{1}{2}h(b_1+b_2)$

$95 = \frac{1}{2}(10)(x+x+3)$

$95 = 5(2x+3)$

$95 = 10x+15$

$80 = 10x$

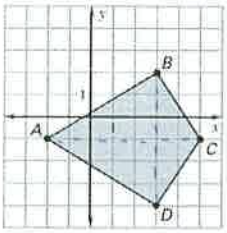
$x = 8$

bases: $x = 8m$

$x+3 = 11m$

Example 5: Find area in the coordinate plane

Find the area of the kite with vertices $A(-2, -1)$, $B(3, 2)$, $C(5, -1)$, and $D(3, -4)$.



$d_1 = 6$

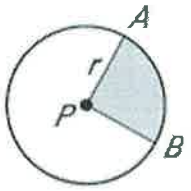
$d_2 = 7$

$A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(6)(7)$

$A = 21 \text{ units}^2$

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram to the right, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$.



We can find the area of the sector two ways:

$$\frac{\text{Area of sector } APB}{\text{Area of circle}} = \frac{m\overset{\frown}{AB}}{360^\circ} \rightarrow \frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360^\circ}$$

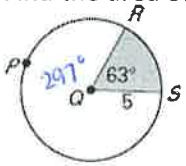
(part of area; part of degrees; whole area; total degrees)

OR

$$\text{Area of sector } APB = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$$

Example 6: Find area of a sector

Find the area of sectors formed by $\angle RQS$.



shaded: $\frac{A}{\pi(5)^2} = \frac{63}{360}$

$360A = 63\pi \cdot 25$

$360A = 4948.$

$A = 13.7 \text{ units}^2$

non: $\frac{A}{\pi(5)^2} = \frac{297}{360}$

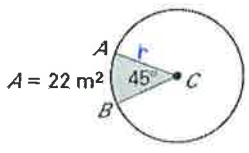
$360A = 297 \cdot \pi \cdot 25$

$360A = 23326.3$

$A \approx 64.8 \text{ units}^2$

Example 7: Use Area of a Sector Theorem

Use the diagram to find the area of $\odot C$.



$\frac{22}{\pi r^2} = \frac{45}{360}$

$7920 = 45\pi r^2$

$7920 = 141.4r^2$

$56 \approx r^2$

$r \approx 7.5$

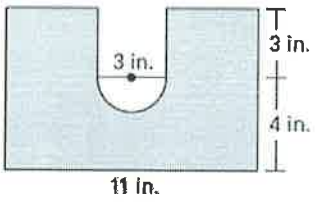
$A = \pi r^2$

$A = \pi(7.5)^2$

$A \approx 176.7 \text{ m}^2$

Example 8: Find an area

A contractor needs to cut a section out of a rectangular piece of wood as shown. To the nearest square inch, what is the area of the remaining wood?



▭: $A = bh$
$= (11)(7)$
$= 77 in^2$

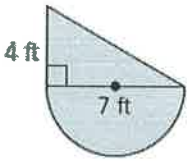
□: $A = bh$
$= (3)(3)$
$= 9 in^2$

○: $A = \frac{1}{2}\pi r^2$
$= \frac{1}{2}\pi(1.5)^2$
$= 1.125\pi$
$= 3.5 in^2$

$Area = 77 - 9 - 3.5$
$= 64.5$
$A \approx 65 in^2$

Example 9: Area of composite figures

a) Find the area of the figure.

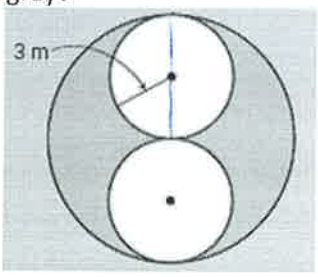


△: $A = \frac{1}{2}(7)(4)$
$A = 14 ft^2$

○: $A = \frac{1}{2}\pi r^2$
$A = \frac{1}{2}\pi(3.5)^2$
$A = 6.125\pi$
$A \approx 19.2 ft^2$

$Total = 33.2 ft^2$

b) The two white congruent circles just fit into the gray circle below. What is the area that appears gray?



$Area\ white = \pi(3)^2$
$= 9\pi \times 2 = 18\pi \approx 56.5 m^2$

$Area\ gray = \pi(6)^2$
$= 36\pi \approx 113.1 m^2$

$Gray - white = 113.1 - 56.5$
$= 56.6 m^2$