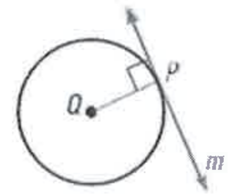
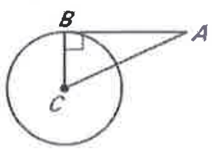
Geometry H
10.1 (cont.): Use Properties of Tangents

Name: Key
Date: ______ Period: ______

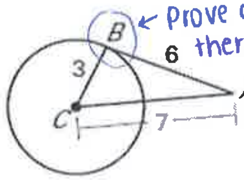
- I can use properties of tangents to verify a tangent to a circle.
- I can use properties of tangents to find segment lengths in circles.

Theorem	Example
In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.	• If line m is tangent to ⊙Q, then $m \perp \overline{QP}$. • If $m \perp \overline{QP}$, then line m is tangent to ⊙Q. 

Example 1: Verify a tangent to a circle.
In the diagram, $\overline{BC}$ is a radius of $\odot C$. Determine whether $\overleftrightarrow{AB}$ is tangent to $\odot C$. Explain your reasoning.

a) 

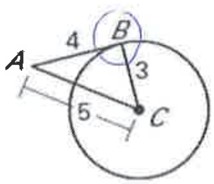
Yes, since $\overline{BC}$ is perpendicular to $\overline{AB}$, $\overline{AB}$ is tangent to ⊙C

b) 

← Prove or disprove there is a 90° angle by using converse of Pythag. Thm

$c^2 \_ a^2 + b^2$
$7^2 \_ 3^2 + 6^2$
$49 \geq 45$

Since this Δ is obtuse, $\overline{BC}$ is not ⊥ to $\overline{AB}$ so $\overline{AB}$ is not tangent to ⊙C

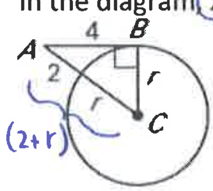
c) 

$c^2 \_ a^2 + b^2$
$5^2 \_ 3^2 + 4^2$
$25 = 25$

Since this Δ is right, $\overline{BC} \perp \overline{AB}$ so $\overline{AB}$ is tangent to ⊙C

Example 2: Find length of radius of circle. we know there is a right angle already

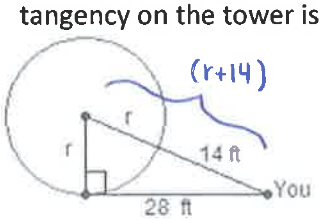
a) In the diagram, $\overline{AB}$ is tangent to $\odot C$ at point B. Find the radius r of $\odot C$.



$a^2 + b^2 = c^2$
$4^2 + r^2 = (2+r)^2$
$16 + r^2 = (2+r)(2+r)$
$16 + \cancel{r^2} = 4 + 4r + \cancel{r^2}$
$-\cancel{r^2} \quad -\cancel{r^2}$

$16 = 4 + 4r$
$12 = 4r$
$\boxed{r = 3}$

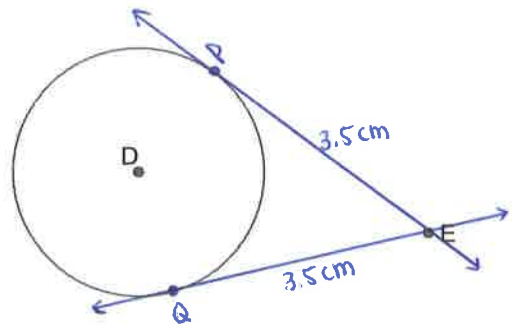
b) You are standing 14 feet from a circular water tower. The distance from you to the point of tangency on the tower is 28 feet. What is the radius of the tower?

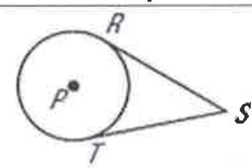


$r^2 + 28^2 = (r+14)^2$
$r^2 + 784 = (r+14)(r+14)$
$\cancel{r^2} + 784 = \cancel{r^2} + 28r + 196$
$-\cancel{r^2} \quad -\cancel{r^2}$
$784 = 28r + 196$
$588 = 28r \Rightarrow \boxed{r = 21 \text{ ft}}$

Explore:
From a point in a circle's exterior, you can draw exactly two different tangents. Use the diagram below to complete the following:

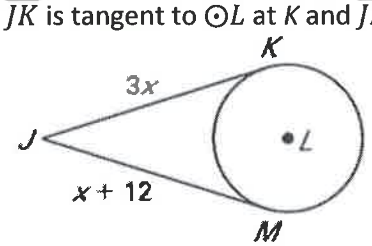
- Draw the two tangents $\overline{EP}$ and $\overline{EQ}$ to $\odot D$.
- Connect $\overline{ED}, \overline{DQ}$, and $\overline{DP}$.
- What types of segments are $\overline{DQ}$ and $\overline{DP}$?
  What can we conclude about $\overline{DQ}$ and $\overline{DP}$.
- Is $\Delta DPE \cong \Delta DQE$? Explain.
- Is $\overline{PE} \cong \overline{QE}$? Yes, both segments have lengths of about 3.5 cm



Theorem	Example
If two segments are tangent to a circle from the same external point, then the segments are congruent.	 If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

Example 3: Find lengths of tangent segments.

a) $\overline{JK}$ is tangent to $\odot L$ at *K* and $\overline{JM}$ is tangent to $\odot L$ at *M*. Find the value of *x*.



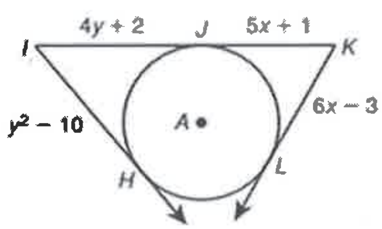
$\overline{JK} \cong \overline{JM}$

$3x = x + 12$

$2x = 12$

$\boxed{x = 6}$

b) $\overline{IH}, \overline{IK}$, and $\overline{IL}$ are tangent to $\odot A$. What is *IK*?



$\overline{IH} \cong \overline{IJ}$

$y^2 - 10 = 4y + 2$

$y^2 - 4y - 12 = 0$

$(y-6)(y+2) = 0$

$\boxed{y=6}$ $y \cancel{=} {-2}$

$4(-2)+2$

$= -6$

can't have neg. lengths

$\overline{JK} \cong \overline{KL}$

$5x + 1 = 6x - 3$

$1 = x - 3$

$\boxed{4 = x}$

$IK = 4(6) + 2 + 5(4) + 1$

$IK = 26 + 21$

$\boxed{IK = 47}$