Unit 4

50. If $\frac{3}{x-4} = \frac{x}{7}$, find the possible values of x.

$21 = x^2 - 4x$

$0 = x^2 - 4x - 21$

$0 = (x-7)(x+3)$

$\boxed{x = 7,\ x = -3}$

51. Find the geometric mean of 15 and 9 in simplest radical form.

$\sqrt{15 \cdot 9} = \sqrt{135} = \sqrt{9}\sqrt{15} = \boxed{3\sqrt{15}}$

52. The measures of the angles of a triangle are in the extended ratio of 5:9:10. Find the measures of the angles of the triangle.

$5x + 9x + 10x = 180$

$24x = 180$

$\boxed{x = 7.5}$

Angles: $5(7.5) = 37.5°$

$9(7.5) = 67.5°$

$10(7.5) = 75°$

53. The area of a rectangle is 294 yards^2. The length and width are in the ratio of 3 : 2. Please find the length, width, and perimeter of the rectangle.

3x 2x

$A = lw$

$294 = (3x)(2x)$

$294 = 6x^2$

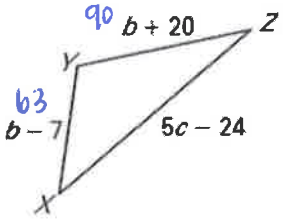
$49 = x^2 \Rightarrow \boxed{x = 7}$

$L = 3(7) = 21$ yd

$W = 2(7) = 14$ yd

$P = 70$ yd

54. The side lengths in ΔXYZ are related in an extended ratio of XY:YZ:XZ : 7:10:14. Please solve for b and c.



$\frac{b-7}{b+20} = \frac{7}{10}$

$7b + 140 = 10b - 70$

$140 = 3b - 70$

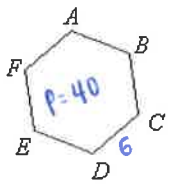
$210 = 3b \Rightarrow \boxed{b = 70}$

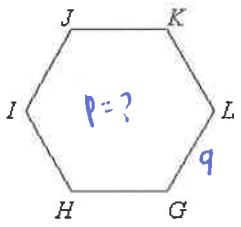
$\frac{90}{5c-24} = \frac{10}{14}$

$50c - 240 = 1260$

$50c = 1500 \Rightarrow \boxed{c = 30}$

55. The hexagons below, ABCDEF and JKLGHI, are similar. If CD = 6, LG = 9, and the perimeter of ABCDEF is 40, what is the perimeter of JKLGHI?





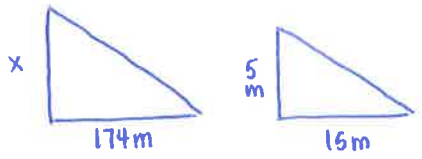
$\frac{ABC}{JKL}: \frac{6}{9} = \frac{40}{x}$

$6x = 360$

$x = 60$

$\boxed{P = 60 \text{ units}}$

56. A building casts a shadow 174 meters long. At the same time, a pole 5 meters high casts a shadow 15 meters long. What is the height of the building?

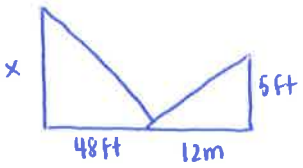


$\frac{x}{5} = \frac{174}{15}$

$15x = 870 \Rightarrow x = 58$

58 m tall

57. Michelle wanted to measure the height of her school's flagpole. She placed a mirror on the ground 48 feet from the flagpole, then walked backwards until she was able to see the top of the pole in the mirror. Her eyes were 5 feet above the ground and she was 12 feet from the mirror. Please find the height of the flagpole.



$\frac{x}{5} = \frac{48}{12}$

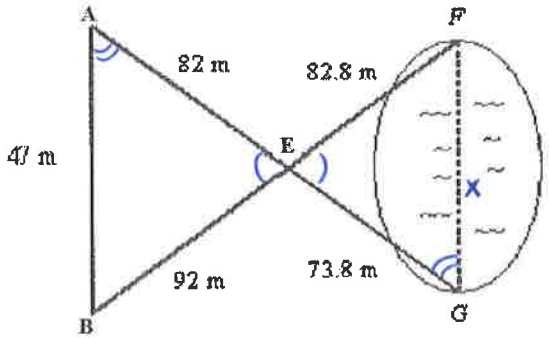
$12x = 240$

$x = 20$

20 ft tall

58. Campsites F and G are on opposite sides of a lake. A survey crew made the measurements shown on the diagram. Assuming that the segments formed by $\overline{AB}$ and $\overline{FG}$ are parallel, please explain why the triangles are similar and find the distance between the two campsites?

Note: The diagram is not to scale.



$\angle A \cong \angle G$ or $\angle B \cong \angle F$ (alt int) and $\angle AEB \cong \angle GEF$ (VAT)

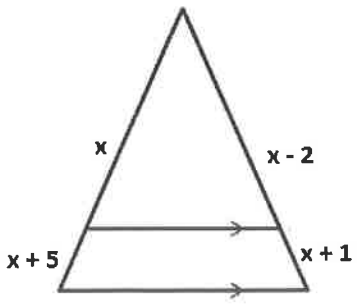
$\triangle AEB \sim \triangle GEF$ by AA~

$\frac{47}{x} = \frac{82}{73.8} \Rightarrow 82x = 3468.6$

$x = 42.3$m

59. Please use the diagrams below to solve for x.

a.



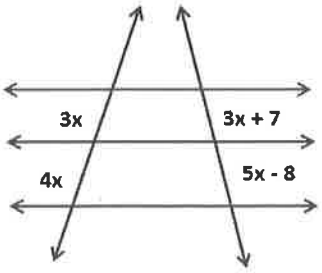
$\frac{x}{x+5} = \frac{x-2}{x+1} \Rightarrow x^2 + x = x^2 - 2x + 5x - 10$

$x = 3x - 10$

$-2x = -10$

$x = 5$

b.



$\frac{3x}{4x} = \frac{3x+7}{5x-8} \Rightarrow 12x^2 + 28x = 15x^2 - 24x$

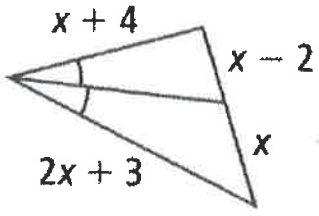
$28x = 3x^2 - 24x$

$0 = 3x^2 - 52x$

$0 = x(3x - 52)$

$x \neq 0, \; x = 52/3$

c.



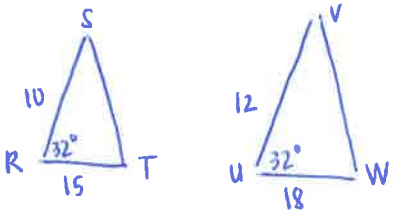
$\frac{x}{x-2} = \frac{2x+3}{x+4} \Rightarrow x^2 + 4x = 2x^2 - 4x + 3x - 6$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x = 6, \; x = -1$

60. In ΔRST, RS = 10, RT = 15, and $m\angle R = 32°$. In ΔUVW, UV = 12, UW = 18, and $m\angle U = 32°$. Are these triangles similar? If so, explain why and write a similarity statement.



$\frac{\Delta RST}{\Delta UVW}$: $\frac{10}{12}, \frac{15}{18}$

$\downarrow \quad \downarrow$

$\frac{5}{6}, \frac{5}{6}$

Yes, Similar by SAS~

$\Delta RST \sim \Delta UVW$

61. Is a dilation by a scale factor of $\frac{2}{3}$ an enlargement or a reduction? How can you tell?

Reduction, $0 < k < 1$

62. Is a dilation by a scale factor of 5 an enlargement or a reduction? How can you tell?

Enlargement, $k > 1$

63. What are the coordinates of the polygon A(1,5) B(3,3) C(2, −6) D(−4, −2) after it is dilated by a scale factor of 4?

$A(1,5) \to A'(4,20)$ $\quad C(2,-6) \to C'(8,-24)$

$B(3,3) \to B'(12,12)$ $\quad D(-4,-2) \to D'(-16,-8)$

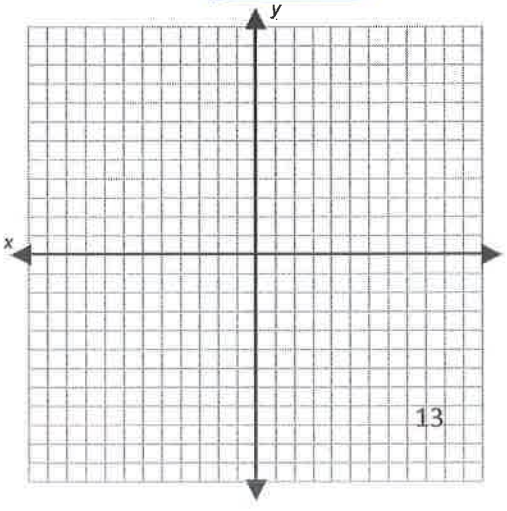
64. What are the coordinates of the polygon A(−4,8) B(2,4) C(0,2) D(−4, 6) after it is dilated by a scale factor of $\frac{1}{2}$?

$A(-4,8) \to A'(-2,4)$ $\quad D(-4,6) \to D'(-2,3)$

$B(2,4) \to B'(1,2)$

$C(0,2) \to C'(0,1)$

65. ΔDEF has coordinates D(0, 5), E(4, 1) and F(2, 1). Please dilate the triangle using center (-1, 2) and a scale factor of 2.



From center (-1,2) to D(0,5): $(x+1, y+3) \times 2$

$(x+2, y+6)$

From (-1,2): $(-1+2, 2+6) \Rightarrow$ D'(1,8)

From center (-1,2) to E(4,1): $(x+5, y-1) \times 2$

$(x+10, y-2)$

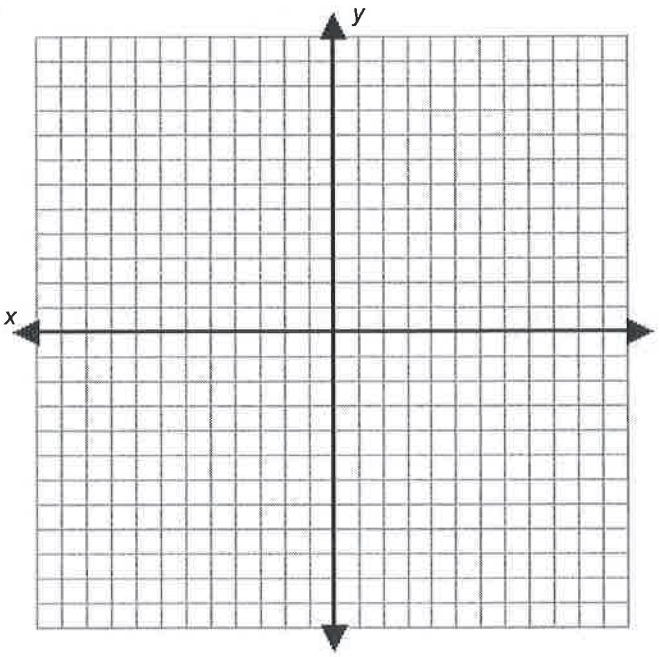
From (-1,2): $(-1+10, 2-2) \Rightarrow$ E'(9,0)

From center (-1,2) to F(2,1): $(x+3, y-1) \times 2$

$(x+6, y-2)$

From (-1,2): $(-1+6, 2-2) \Rightarrow$ F'(5,0)

66. Find the coordinates of the dilation image of ΔGHJ centered at the point (2, 4) with a scale factor of ½ given coordinates G(-8, 2), H(-2, 2), and J(-4, -4).



From center (2,4) to G(-8,2): $(x-10, y-2) \times \frac{1}{2}$

$(x-5, y-1)$

From (2,4): $(2-5, 4-1) \Rightarrow$ G'(-3,3)

From center (2,4) to H(-2,2): $(x-4, y-2) \times \frac{1}{2}$

$(x-2, y-1)$

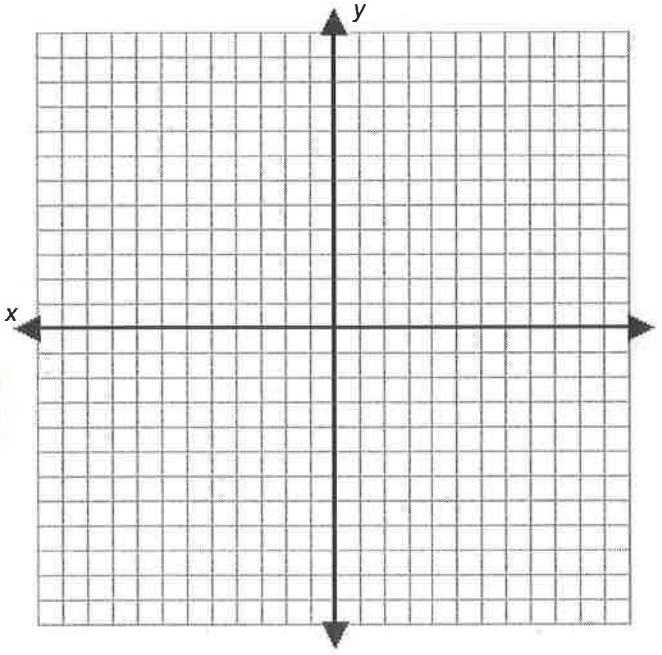
From (2,4): $(2-2, 4-1) \Rightarrow$ H'(0,3)

From center (2,4) to J(4,-4): $(x-6, y-8) \times \frac{1}{2}$

$(x-3, y-4)$

From (2,4): $(2-3, 4-4) \Rightarrow$ J'(-1,0)

67. Find the coordinates of the dilation image of $\overline{ST}$ centered at point (-3, 4) with a scale factor of 2 given coordinates S(1, 3) and T(-1, 4).



From center (-3,4) to S(1,3): $(x+4, y-1) \times 2$

$(x+8, y-2)$

From (-3,4): $(-3+8, 4-2) \Rightarrow$ S'(5,2)

From center (-3,4) to T(-1,4): $(x+2, y+0) \times 2$

$(x+4, y+0)$

From (-3,4): $(-3+4, 4+0) \Rightarrow$ T'(1,4)