Geometry H
Extra Midterm Review

Name : Key
Date : ______ Period : ____

1. If M is the midpoint of segment DE, $DM = x^2 - 4x - 24$, and $EM = 2x + 3$, please find DE.

$x^2 - 4x - 24 = 2x + 3$

$x^2 - 6x - 27 = 0$

$(x-9)(x+3) = 0$

$\boxed{x = 9}$, $\cancel{x = -3}$

$DE = 2(9) + 3 = \boxed{21}$ ✓

$DE = 2(-3) + 3 = -3$ ✗

$\boxed{DE = 21}$

2. $\overline{PT}$ has endpoint $P(8, 0)$ and midpoint $M(6, -5)$. Find the coordinates for endpoint T.

x_1, y_1

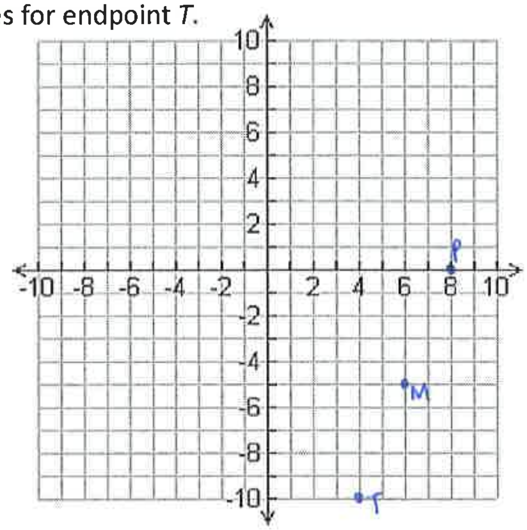
$M = \frac{x_1 + x_2}{2}$ $\quad$ $M = \frac{y_1 + y_2}{2}$

$\frac{6}{1} = \frac{8 + x_2}{2}$ $\quad$ $\frac{-5}{1} = \frac{0 + y_2}{2}$

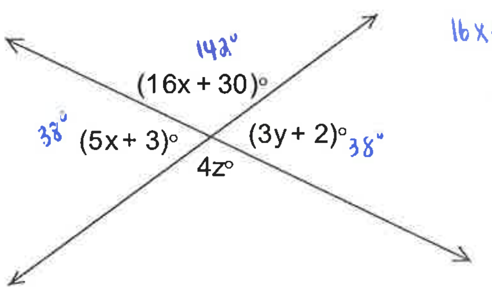
$12 = 8 + x_2$ $\quad$ $-10 = 0 + y_2$

$x_2 = 4$ $\quad$ $y_2 = -10$

$\boxed{T(4, -10)}$



3. Solve for x, y, and z.



$16x + 30 + 5x + 3 = 180$

$21x + 33 = 180$

$21x = 147$

$\boxed{x = 7}$

$3y + 2 = 38$

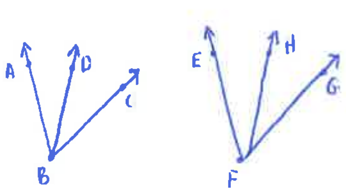
$3y = 36$

$\boxed{y = 12}$

$4z = 142$

$\boxed{z = 35.5}$

4. Given $\overrightarrow{BD}$ is an angle bisector for $\angle ABC$, and $\overrightarrow{FH}$ is an angle bisector for $\angle EFG$. Solve for x and y if $m\angle ABD = 0.2y - 0.5x$, $m\angle ABC = 116°$, $m\angle EFH = (172 + x)°$, and $m\angle HFG = 6y°$.



$2(0.2y - 0.5x) = 116$

$0.4y - x = 116$

$172 + x = 6y$

$-6y + x = -172$

$\Rightarrow$ $0.4y - \cancel{x} = 116$

$-6y + \cancel{x} = -172$

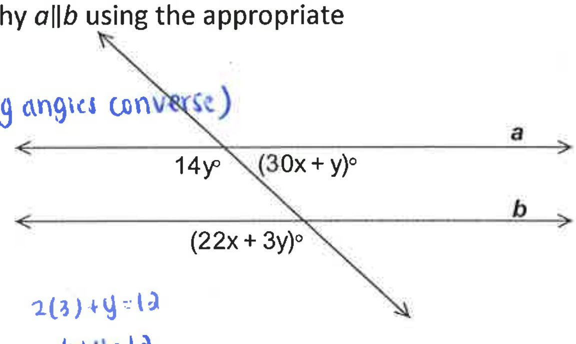
$-5.6y = -56$

$\boxed{y = 10}$

$-60 + x = -172$

$\boxed{x = -112}$

5. Please find the values of x and y that makes $a \| b$. Justify why $a \| b$ using the appropriate theorem/postulate.



$14y + 30x + y = 180$ (Linear Pair Postulate)

$\frac{30x + 15y}{15} = \frac{180}{15}$

$2x + y = 12$

$14y = 22x + 3y$ (corresponding angles converse)

$0 = 22x - 11y$

$11(2x + y = 12)$ $\Rightarrow$ $22x + \cancel{11y} = 132$

$22x - 11y = 0$ $\quad$ $22x - \cancel{11y} = 0$

$44x = 132$

$\boxed{x = 3}$

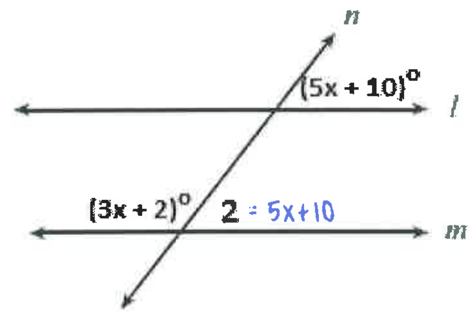
$2(3) + y = 12$

$6 + y = 12$

$\boxed{y = 6}$

6. Given: $l \| m$. Please solve for x.

Statements	Reasons
1. $l \| m$	1. Given
2. $m\angle 2 = 5x+10$	2. Corresponding Ang Postulate
3. $3x+2+5x+10=180$	3. Linear Pair Postulate
4. $8x+12=180$	4. Combine Like Terms
5. $8x=168$	5. Subtraction Prop
6. $x=21$	6. Division Prop



7. $\angle A$ and $\angle B$ are complementary angles. $\angle C$ and $\angle D$ are supplementary angles. Find the measures of the four angles, if $m\angle A = 2x^\circ$, $m\angle B = 6y^\circ$, $m\angle C = (6x+y)^\circ$, and $m\angle D = (4x+2y)^\circ$

$m\angle A + m\angle B = 90$

$\frac{2x+6y}{2} = \frac{90}{2}$

$\rightarrow x+3y=45$

$m\angle C + m\angle D = 180$

$6x+y+4x+2y=180$

$\rightarrow 10x+3y=180$

$-1(x+3y=45)$

$10x+3y=180$

$\Rightarrow$ $-x-\cancel{3y}=-45$

$10x+\cancel{3y}=180$

$9x=135$

$\boxed{x=15}$

$15+3y=45$

$3y=30 \Rightarrow \boxed{y=10}$

$\boxed{\begin{array}{l} m\angle A = 2(15) = 30^\circ \\ m\angle B = 6(10) = 60^\circ \\ m\angle C = 6(15)+10 = 100^\circ \\ m\angle D = 4(15)+2(10) = 80^\circ \end{array}}$

8. An angle is 275 less than four times its complement. Find the measure of the angle and its complement.

$x = 4(90-x)-275$

$x = 360-4x-275$

$5x = 85 \Rightarrow x=17$

$\boxed{\begin{array}{l} \text{The angle} = 17^\circ \\ \text{Its complement} = 73^\circ \end{array}}$

9. Two times the complement of an angle is 300 less than three times its supplement. What is the angle?

$2(90-x) = 3(180-x)-300$

$180-2x = 540-3x-300$

$180-2x = 240-3x$

$180+x = 240 \Rightarrow x=60$

$\boxed{\text{The angle is } 60^\circ}$

10. Point T is between points A and L. If $AT = x^2+2x-2$, $TL = x-2$, and $AL = 24$, find AT, and TL.

$x^2+2x-2+x-2=24$

$x^2+3x-4=24$

$x^2+3x-28=0$

$(x+7)(x-4)=0$

$\cancel{x=-7}, \boxed{x=4}$

$AT = (-7)^2+2(-7)-2 = 49-14-2 = 33$ ⊗

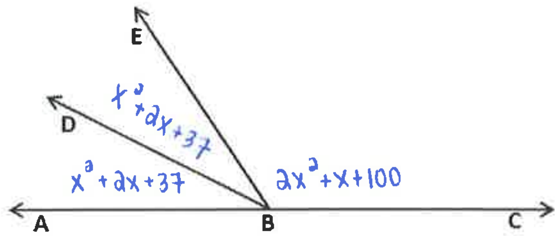
$(4)^2+2(4)-2 = 16+8-2 = 22$

$TL = -7-2=-9$ ⊗ ← can't have neg. lengths

$4-2=2$

$\boxed{\begin{array}{l} AT = 22 \\ TL = 2 \end{array}}$

11. $\overrightarrow{BD}$ bisects $\angle ABE$. $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. If $m\angle EBC = (2x^2 + x + 100)°$ and $m\angle ABD = (x^2 + 2x + 37)°$, please solve for x.



$4x^2+5x+174=180$

$4x^2+5x-6=0$

$4x^2+8x-3x-6=0$

$4x(x+2)-3(x+2)=0$

$(4x-3)(x+2)=0$

$\boxed{x=3/4,\ x=-2}$

12. Write the equation of the line that is perpendicular to the line $y = \frac{3}{2}x + 7$ that passes through the point (-3, 4).

$m=-\frac{2}{3}$, $x=-3$, $y=4$

$y=mx+b$

$4=-3(-\frac{2}{3})+b$

$4=2+b$

$b=2$

$\boxed{y=-\frac{2}{3}x+2}$

13. Three times an angle's complement is equal to half of its supplement. Find the angle.

$3(90-x)=\frac{1}{2}(180-x)$

$270-3x=90-0.5x$

$180=2.5x$

$x=72$

$\boxed{\text{The angle is } 72°}$

14. Write the equation of the line that passes through (3, 5) and is parallel to the line that passes through (3, 3) and (-3, -1).

$m=\frac{-1-3}{-3-3}=\frac{-4}{-6}=\frac{2}{3}$

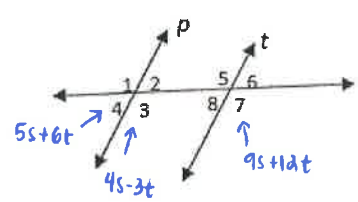
$m=\frac{2}{3}$, $x=3$, $y=5$ $\Rightarrow$ $y=mx+b$

$5=\frac{2}{3}(3)+b$

$5=2+b \Rightarrow b=3$

$\boxed{y=\frac{2}{3}x+3}$

15. Given $l \parallel m$, $m\angle 3 = (4s - 3t)°$, $m\angle 7 = (9s + 12t)°$, and $m\angle 4 = (5s + 6t)°$, please solve for s and t.



$4s-3t=9s+12t$ (corresponding)

$0=5s+15t$

$5s+6t+4s-3t=180$ (Linear Pair)

$9s+3t=180$

$5s+15t=0$

$-5(9s+3t=180) \Rightarrow$

$5s+\cancel{15t}=0$

$-45s-\cancel{15t}=-900$

$-40s=-900$

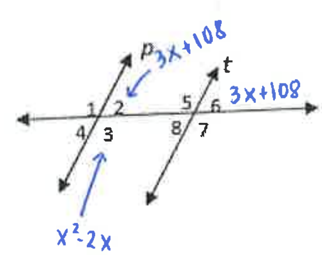
$\boxed{s=22.5}$

$0=5(22.5)+15t$

$0=112.5+15t$

$-112.5=15t \Rightarrow \boxed{t=-7.5}$

16. Given $p \parallel t$, $m\angle 3 = (x^2 - 2x)°$ and $m\angle 6 = (3x + 108)°$, please solve for x.



$m\angle 2=3x+108$ (corresponding angles)

$x^2-2x+3x+108=180$ (Linear Pair)

$x^2+x-72=0$

$(x+9)(x-8)=0$

$\boxed{x=-9,\ x=8}$ ← both values work

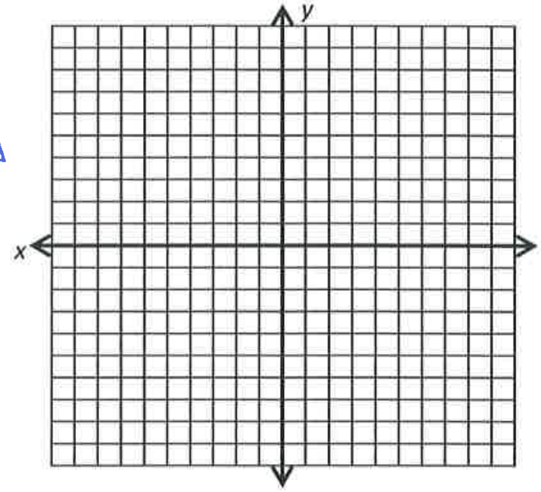
Check: $(-9)^2-2(-9)=99°$

$3(-9)+108=81°$

$(8)^2-2(8)=48°$

$3(8)+108=132°$

17. Please classify $\triangle ABC$ by its side lengths. Then determine if the triangle is a right triangle given coordinates A(2, 3), B(4, 7), C(6, 1).



$AB = \sqrt{(4-2)^2+(7-3)^2} = \sqrt{(2)^2+(4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(6-4)^2+(1-7)^2} = \sqrt{(2)^2+(-6)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

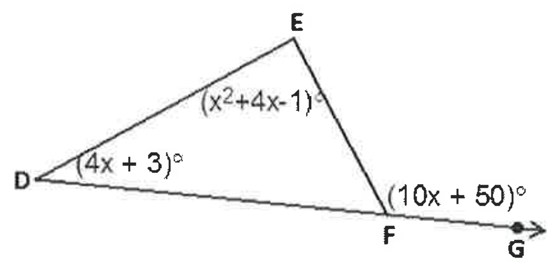
$AC = \sqrt{(6-2)^2+(1-3)^2} = \sqrt{(4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

isosceles $\triangle$

$m_{\overleftrightarrow{AB}} = \frac{7-3}{4-2} = \frac{4}{2} = \frac{2}{1}$ $m_{\overleftrightarrow{BC}} = \frac{1-7}{6-4} = \frac{-6}{2} = \frac{-3}{1}$ $m_{\overleftrightarrow{AC}} = \frac{1-3}{6-2} = \frac{-2}{4} = -\frac{1}{2}$

since $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ have opp. reciprocal slopes, $\overleftrightarrow{AB} \perp \overleftrightarrow{AC}$ and therefore there is a right angle so $\triangle ABC$ is a right triangle

18. Please find the measure of the exterior angle shown.



$10x+50 = x^2+4x-1+4x+3$

$10x+50 = x^2+8x+2$

$0 = x^2-2x-48$

$0 = (x-8)(x+6)$

$\boxed{x=8}$ $\cancel{x=-6}$

check:

$10(8)+50 = 130°$ ✓

$10(-6)+50 = -10°$ ✗

$\boxed{m\angle EFG = 130°}$

19. In $\triangle ABC$, $m\angle A$ is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

$m\angle B = x$

$m\angle A = 2x$

$m\angle C = x+8$

$m\angle A + m\angle B + m\angle C = 180$

$2x+x+x+8 = 180$

$4x+8 = 180$

$4x = 172$

$x = 43$

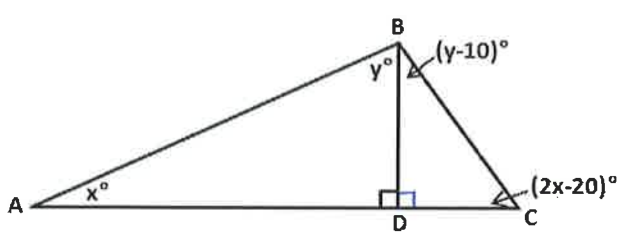
$m\angle A = 2(43) = 86°$

$m\angle B = 43°$

$m\angle C = 43+8 = 51°$

acute $\triangle$

20. Please solve for x and y.



Right $\triangle$:

$y-10+2x-20+90 = 180$

$y+2x+60 = 180$

$2x+y = 120$

Left $\triangle$:

$x+y+90 = 180$

$x+y = 90$

$2x+y = 120$

$-1(x+y = 90)$ $\Rightarrow$ $2x+y = 120$, $-x-y = -90$

$\boxed{x = 30}$

$30+y = 90$

$\boxed{y = 60}$

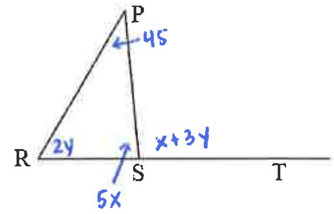
21. If $m\angle PST = (x+3y)°$, $m\angle RPS = 45°$, $m\angle PRS = 2y°$, and $m\angle PSR = 5x°$, find $m\angle PST$.

$x+3y = 2y+45$ (ext. angles thm)

$x + y = 45$

$2y+45+5x = 180$ (Δ sum thm)

$5x+2y = 135$



$-2(x+y=45) \Rightarrow$ $-2x - \cancel{2y} = -90$

$5x+2y = 135$ $\quad$ $5x + \cancel{2y} = 135$

$3x = 45$

$\boxed{x=15}$

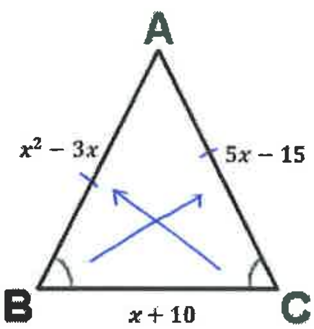
$15+y=45$

$\boxed{y=30}$

$m\angle PST = 15+3(30)$

$\boxed{m\angle PST = 105°}$

22. Using the diagram below, find the value of x.



$x^2 - 3x = 5x - 15$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$\cancel{x=3}$, $\boxed{x=5}$

Check:

$(3)^2 - 3(3) = 9 - 9 = 0$ ⊗ cant have zero side lengths

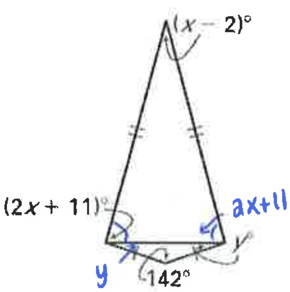
$(5)^2 - 3(5) = 10$

$5(5) - 15 = 10$

$5 + 10 = 15$

23. Solve for the indicated variable(s).

a.



Big Δ: $x - 2 + 2x + 11 + 2x + 11 = 180$

$5x + 20 = 180$

$5x = 160$

$\boxed{x=32}$

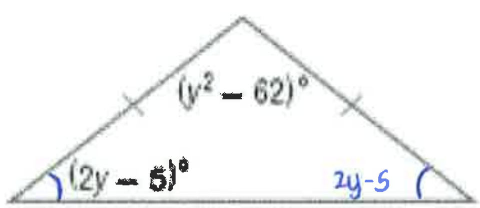
Small Δ: $y + y + 142 = 180$

$2y + 142 = 180$

$2y = 38$

$\boxed{y=19}$

b.



$y^2 - 62 + 2y - 5 + 2y - 5 = 180$

$y^2 + 4y - 72 = 180$

$y^2 + 4y - 252 = 0$

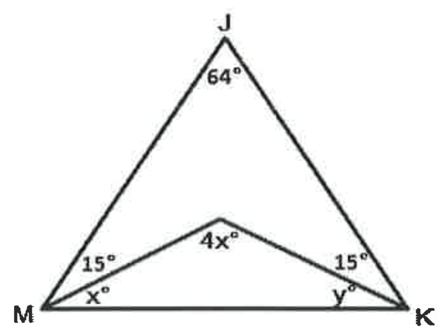
$(y+18)(y-14) = 0$

$\cancel{y=-18}$, $\boxed{y=14}$

Check: $2(-18) - 5 = -41$ ⊗

$2(14) - 5 = 23$ ✓

c.



small Δ: $x + 4x + y = 180$

$5x + y = 180$

Big Δ: $x + 15 + 64 + 15 + y = 180$

$x + y + 94 = 180$

$x + y = 86$

$5x + y = 180$

$-1(x + y = 86) \Rightarrow$

$5x + \cancel{y} = 180$

$-x - \cancel{y} = -86$

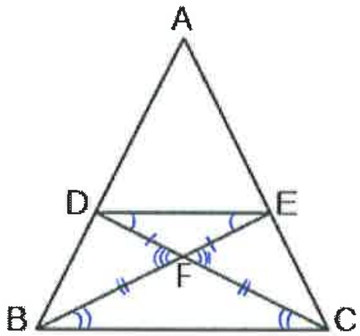
$4x = 94$

$\boxed{x = 23.5}$

$23.5 + y = 86$

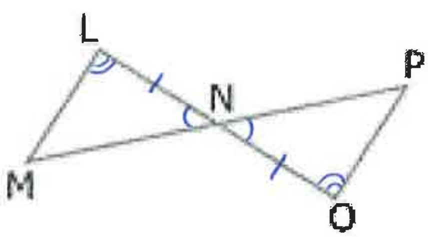
$\boxed{y = 62.5}$

24. Given: $\angle EDC \cong \angle DEF$
$\angle FBC \cong \angle FCB$
Prove: $\triangle DBF \cong \triangle ECF$



Statements	Reasons
1. <EDC ≅ <DEF	1. Given
2. $\overline{DF} \cong \overline{EF}$	2. Base Angles Converse
3. <FBC ≅ <FCB	3. Given
4. $\overline{BF} \cong \overline{CF}$	4. Base Angles Converse
5. <DFB ≅ <EFC	5. VAT
6. ΔDBF ≅ ΔECF	6. SAS

25. Given: N is the midpoint of $\overline{LO}$
$\overline{LM} \parallel \overline{OP}$
Prove: $\triangle LNM \cong \triangle ONP$

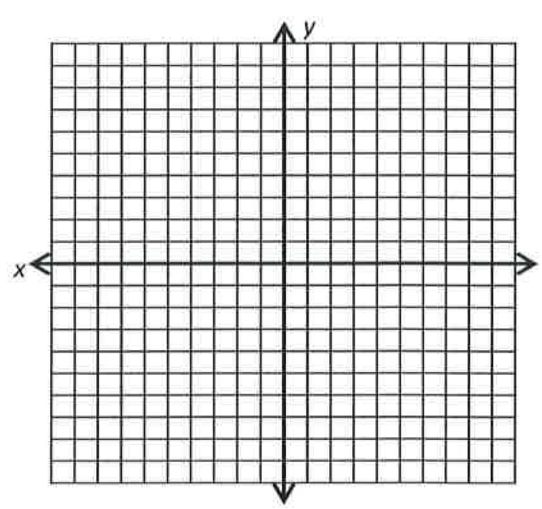


Statements	Reasons
1. N is the midpoint of $\overline{LO}$	1. Given
2. $\overline{LN} \cong \overline{ON}$	2. Def. of midpoint
3. <LNM ≅ <ONP	3. VAT
4. $\overline{LM} \parallel \overline{OP}$	4. Given
5. <MLN ≅ <PON	5. Alt. Int. Angles Thm
6. ΔLNM ≅ ΔONP	6. ASA

OR
<LMN ≅ <OPN →

OR AAS (if you used <LMN ≅ <OPN)

26. Dilate $\triangle ABC$ by a scale factor of ½ using (2, 4) as the center of dilation given points A(-8, 2), B(-2, 2), and C(-4, -4).



From center (2,4) to A (-8,2): (x -10, y -2) × 1/2

(x - 5, y - 1)

From center: (2-5, 4-1) ⇒ $\boxed{A'(-3,3)}$

From center (2,4) to B (-2,2) : (x -4, y-2) × 1/2

(x-2, y-1)

From center: (2-2, 4-1) ⇒ $\boxed{B'(0,3)}$

From center (2,4) to C (-4,-4): (x-6, y-8) × 1/2

(x-3, y-4)

From center: (2-3, 4-4) ⇒ $\boxed{C'(-1,0)}$