

This packet contains a review of some arithmetic and Algebraic skills that are necessary for success in this class. These skills include manipulating fractions, simplifying expressions, solving equations, writing equations of lines, factoring, using the quadratic formula, solving systems of linear equations, and simplifying radicals. Throughout the year, these skills will be embedded in problems that we will solve making geometric and algebraic connections. If you are having difficulty remembering any of these skills, you should set up an appointment for extra help.

Part I: PREALGEBRA SKILLS

➤ Manipulating Fractions

You should be able to add, subtract, multiply and divide fractions with like and unlike denominators. If you need a refresher on these skills, you can view the following videos:

- Go to www.khanacademy.org
- Search "Adding and subtracting fractions", or "multiplying two fractions: example", or "dividing fractions example"
- Watch the video that appears in the list (there are several results that will appear, look for the picture of the play button to choose the video 😊)

TRY THESE:

1) $\frac{1}{2} + \frac{3}{8}$

$$\frac{4}{8} + \frac{3}{8} = \boxed{\frac{7}{8}}$$

2) $\frac{2}{3} - \frac{5}{4}$

$$\frac{8}{12} - \frac{15}{12} = \boxed{\frac{-7}{12}}$$

3) $\frac{7}{9} \cdot \frac{4}{10}$

$$\frac{28}{90} = \boxed{\frac{14}{45}}$$

4) $\frac{1}{5} \div \frac{12}{7}$

$$\frac{1}{5} \cdot \frac{7}{12} = \boxed{\frac{7}{60}}$$

➤ Simplifying expressions

You should be able to simplify expressions by combining like terms and using the distributive property. If you need a refresher on these skills, you can view the following videos:

- Go to www.khanacademy.org
- Search "How to simplify an expression by combining like terms and the distributive property"
- Watch the video that appears in the list 😊

TRY THESE:

Simplify the expressions.

5) $2(x + 4) - 7$

$$2x + 8 - 7 = \boxed{2x + 1}$$

6) $4 - 3(-3x + 5)$

$$4 + 9x - 15 = \boxed{9x - 11}$$

➤ **Solving equations**

You should be able to solve multi-step linear equations with variables on one or both sides of the equation. If you need a refresher on solving equations, you can view the following videos:

- Go to www.khanacademy.org
- Search "how to solve one -step equations with fractions and decimals" or "solving a more complicated equation", or "example: two-step equation with numerator x", or "solving a proportion with an unknown variable (example)", or "proportions 2 exercise examples"
- Watch the video that appears in the list ☺

TRY THESE:

Solve the equations.

7) $4.6 = 4m - 3.4$

$8 = 4m$

$m = 2$

8) $\frac{p}{6} + 9 = 14$

~~$\frac{p}{6} = 5$~~

$p = 30$

9) $6w + 5(w - 2) = 23$

$6w + 6w - 10 = 23$

$11w - 10 = 23$

$11w = 33$

$w = 3$

10) $14 - 4z = 2(17 - z)$

$14 - 4z = 34 - 2z$

$14 = 34 + 2z$

$-20 = 2z \Rightarrow z = -10$

11) $\frac{x}{12} = \frac{8}{48}$

$48x = 96$

$x = 2$

12) $\frac{2x-1}{15} = \frac{18}{54}$

$270 = 54(2x-1)$

$270 = 108x - 54$

$324 = 108x \Rightarrow x = 3$

PART II: EQUATIONS OF LINES

➤ **Writing equations in slope -intercept form**

You should be able to write equations of lines in slope-intercept form ($y = mx + b$) given either slope and y-intercept, slope and a point, or two points. If you need a refresher on these skills, you can view the following videos:

- Go to www.khanacademy.org
- Search "finding a linear equation given a point and slope", or "slope-intercept equation from two solutions example"
- Watch the video that appears in the list ☺

TRY THESE:

$y = mx + b$

Write the equation of the line in slope-intercept form given the following information:

13) slope = 2

y-intercept = -6

$y = 2x - 6$

14) slope = $-\frac{5}{6}$
point (6, -1)

$y = mx + b$

$-1 = -\frac{5}{6}(6) + b$

$-1 = -5 + b$

$b = 4$

$y = -\frac{5}{6}x + 4$

15) two points: $(-6, -10)$ and $(9, 10)$

$m = \frac{10 - (-10)}{9 - (-6)} = \frac{10 + 10}{15} = \frac{20}{15} = \frac{4}{3}$

$y = mx + b$

$-10 = \frac{4}{3}(-6) + b$

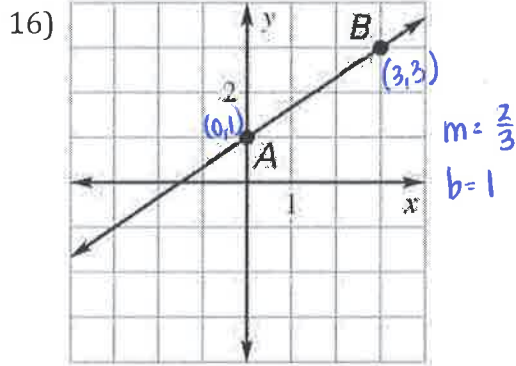
$-10 = -8 + b$

$-10 = -8 + b$

$b = -2$

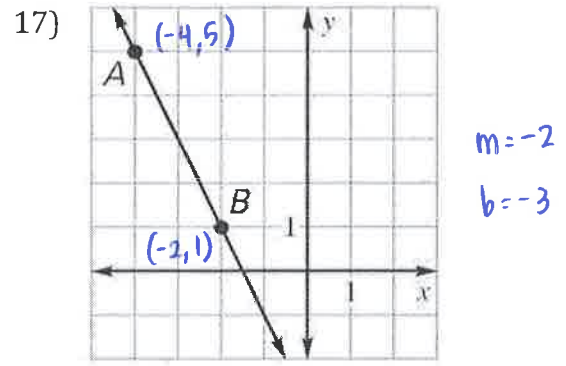
$y = \frac{4}{3}x - 2$

Write the equation of the line in slope-intercept form given the graph of the line.



$$m = \frac{3-1}{3-0} = \frac{2}{3}$$

$$y = \frac{2}{3}x + 1$$



$$m = \frac{5-1}{-4-2} = \frac{5-1}{-4-2} = \frac{4}{-2} = -2$$

$$\begin{aligned} y &= mx + b \\ 1 &= -2(-2) + b \\ 1 &= 4 + b \\ -3 &= b \end{aligned}$$

$$y = -2x - 3$$

➤ **Interpreting slope**

You should be able to use slope to describe whether a line is rising, falling, horizontal, or vertical. If you've forgotten how, visit the following website for a review of what slopes really mean and how to use them to predict which direction a line will go:

<http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/01-graphing-slopes-of-lines-01>

TRY THESE:

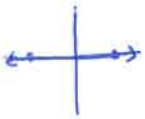
For the slopes provided below, tell whether the line is rising, falling, vertical, or horizontal.

18) $m = 0$

19) $m = -4$

20) m is undefined

21) $m = 1$



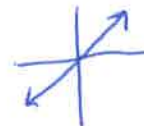
horizontal



falling



vertical



rising

Part III: SOLVING A SYSTEM OF EQUATIONS

In Algebra I, you were shown three techniques for solving a system of linear equations: graphing method, substitution method, and the elimination (or linear combination) method. We are going to review the substitution and elimination methods in this packet. You should be familiar with solving systems using both of these methods.

➤ **Solve a system using substitution method.**

This method consists of solving one of the equations for any variable and then substituting the resulting equation into the other equation, leaving a single variable equation.

Example:

Solve the system
$$\begin{aligned} 3x - y &= 4 \\ -4x + 2y &= 2 \end{aligned}$$

Step 1: Choose one equation and solve for one variable.

$$\begin{aligned} 3x - y &= 4 \\ -y &= -3x + 4 \\ y &= 3x - 4 \end{aligned}$$

Step 2: Substitute the value (expression) for that variable into the other equation in the system. Solve for the remaining variable.

$$\begin{aligned} -4x + 2(3x - 4) &= 2 \\ -4x + 6x - 8 &= 2 \\ 2x - 8 &= 2 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Step 3: Substitute the value you found in step 2 into either equation and solve for the other variable.

$$\begin{aligned} 3(5) - y &= 4 \\ 15 - y &= 4 \\ -y &= -11 \\ y &= 11 \end{aligned}$$

The solution is (5, 11)

For more help reviewing solving linear systems using substitution method, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: solving linear systems by substitution
- Watch the video that appears in the list 😊

TRY THESE:

Solve the system using the substitution method.

22)
$$\begin{aligned} 2x + y &= 5 \Rightarrow y = 5 - 2x \\ -6x - 3y &= -15 \end{aligned}$$

$$\begin{aligned} -6x - 3(5 - 2x) &= -15 \\ -6x - 15 + 6x &= -15 \\ -15 &= -15 \end{aligned}$$

IR or infinitely many solutions

23)
$$\begin{aligned} 2x + 3y &= 3 \\ x - 6y &= -6 \Rightarrow x = -6 + 6y \end{aligned}$$

$$\begin{aligned} 2(-6 + 6y) + 3y &= 3 & x &= -6 + 6(1) \\ -12 + 12y + 3y &= 3 & x &= -6 + 6 \\ -12 + 15y &= 3 & x &= 0 \\ 15y &= 15 \\ y &= 1 \end{aligned}$$

(0, 1)

Part IV: FACTORING

➤ Factoring using greatest common factor (GCF):

To factor using a GCF, take the greatest common factor for the numerical coefficients. When choosing the GCF for the variables, if all terms have a common variable, take the ones with the lowest exponent.

Example: $6x^5 + 18x^4 + 9x^3$

GCF: coefficients: 3, variables: x^3

GCF = $3x^3$ → Divide each term by the GCF to find terms to put in parentheses

Answer: $3x^3(2x^2 + 6x + 3)$

For more help reviewing GCF, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: Finding the greatest common factor of two monomials
- Watch the video that appears in the list ☺

TRY THESE:

Factor each of the following by finding the GCF.

26) $3b^2 - 6b$

$3b(b-2)$

27) $10xy - 15x^2y^2$

$5xy(2-3xy)$

28) $2x^2 + 8x + 4$

$2(x^2 + 4x + 2)$

29) $2ma + 4mb + 2mc$

$2m(a+2b+c)$

30) $9ab^2 - 6ab - 3a$

$3a(3b^2 - 2b - 1)$

31) $26x^4y - 39x^3y^2 + 52x^2y^3 - 13xy^4$

$13xy(2x^3 - 3x^2y + 4xy^2 - y^3)$

➤ Factoring Quadratics (Case I trinomials):

Case I is when there is a coefficient of 1 in front of your variable² term (x^2).

You have two hints that will help you:

- 1) When the last sign is addition, both signs are the same and match the middle term.
- 2) When the last sign is subtraction, both signs are different and the larger number goes with the sign of the middle term.

Examples:

Hint #1:

$x^2 - 5x + 6$

$(x -)(x -)$

Find factors of 6, w/ sum of -5.

$(x - 3)(x - 2)$

CHECK USING FOIL

Hint #2:

$x^2 + 5x - 36$

$(x -)(x +)$

Find factors of -36 w/ difference of 5.

$(x - 4)(x + 9)$

CHECK USING FOIL

For more help reviewing factoring trinomials with leading coefficient of 1, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: More examples of factoring quadratics with a leading coefficient of 1
- Watch the video that appears in the list ☺

TRY THESE:

Factor each of the following quadratic equations

32) $x^2 - 16x + 48$

$(x-12)(x+4)$

33) $x^2 + 18x + 72$

$(x+12)(x+6)$

34) $x^2 - 3x - 28$

$(x-7)(x+4)$

35) $z^2 + 9z - 36$

$(z+12)(z-3)$

➤ Factoring Quadratics (Case 2: Difference of two squares – D.O.T.S.)

Case two is when you have two perfect square terms separated by a subtraction sign, also known as a difference of two squares.

Here are some hints to help you determine if you should factor D.O.T.S.:

- 1) You have a binomial (an expression with two terms) that contains two perfect squares.
- 2) The sign in between the two terms is a subtraction sign.
(You cannot factor a sum of two squares!!!)

To factor, find the square root of each term. Make two sets of parentheses, and put the square root of the first term at the beginning of each set of parentheses, put the square root of the second term at the end of each set of parentheses, and alternate signs, one group gets + and the other group gets -.

Examples:

Factor: $9x^2 - 25$

$\sqrt{9x^2} = 3x$

$\sqrt{25} = 5$

Solution: $(3x - 5)(3x + 5)$

Factor: $36x^4 + 81$

Although both terms are perfect squares, it is not a difference of two squares, so it is not factorable!

For more help reviewing factoring trinomials with leading coefficient of 1, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: factoring a difference of two squares
- Watch the video that appears in the list (there are actually 2, you can watch the one with Example 1) 😊

TRY THESE:

Factor the following, if possible. If not possible, write "not factorable."

36) $x^2 - 36$

$(x-6)(x+6)$

37) $144x^2 - 121$

$(12x-11)(12x+11)$

38) $x^2 + 81$

Not factorable

$(x-9)(x+9) \neq x^2+81$

➤ Factoring Quadratics (Case 3: trinomials)

Case 3 is factoring trinomials in which the leading coefficient is not 1. There are several different ways to do this type of factoring – finding GCF, guess and check, or factoring by grouping are probably the ways that you were shown in Algebra I.

Method 1 – Guess and Check

For this method, you guess at the factors that produce the first and last terms of the trinomial. Then you check the outer and inner terms to see if they have a sum of the middle term of the trinomial. Don't forget, you should always look for a GCF first!

Examples:

Factor completely: $6x^2 + 5x - 4$

There's no GCF, so let's use Guess & Check

$$6x^2 = 3x \cdot 2x \text{ or } 6x \cdot x$$

$$-4 = -1 \cdot 4 \text{ or } -2 \cdot 2 \text{ or } -4 \cdot 1$$

$$\text{First Guess: } (3x-1)(2x+4)$$

$$\text{Let's check by FOIL: } 6x^2 + 12x - 2x - 4 \neq 6x^2 + 5x - 4$$

$$\text{Second guess: } (3x+4)(2x-1)$$

$$\text{Let's check by FOIL: } 6x^2 - 3x + 8x - 4 = 6x^2 + 5x - 4$$

It checks, we found our answer!

Factor completely: $4x^2 + 30x + 14$

There's a GCF, let's factor it out

$$2(2x^2 + 15x + 7)$$

Let's factor that new trinomial

$$2x^2 = 2x \cdot x$$

$$7 = 7 \cdot 1$$

$$\text{First Guess: } 2(2x+1)(x+7)$$

$$\text{Let's Check: } 2(2x^2 + 14x + x + 7) =$$

$$4x^2 + 30x + 14$$

It checks, we found our answer!

HINTS: Don't forget to keep the GCF throughout the problem if there is one!

Method 2 – Factor by Grouping

This method only works if you take the GCF out first, if there is one. You then find the product of the first and last terms. Then you use that product to find two factors that have the sum of the middle term of the trinomial. You replace the middle term with the two factors, break the new expression into two groups and find the GCF of each group. You know you're doing it correctly if you get the same binomial in each set of terms!

Example:

Factor completely: $6x^2 + 5x - 4$

The product of the first and last terms: $(6x^2)(-4) = -24x^2$

We want to find factors of $-24x^2$ that will have a sum of $5x$: $8x$ and $-3x$

Replace the middle term with these two terms: $6x^2 + 8x - 3x - 4$

Factor out the GCF from each group: $2x(3x + 4) - 1(3x + 4)$

Factor out the binomial GCF: $(3x+4)(2x - 1)$

Check by FOIL: $6x^2 - 3x + 8x - 4 = 6x^2 + 5x - 4$

As you can see, it doesn't really matter which method you choose! Choose which method works best for you 😊

For more help reviewing factoring trinomials with leading coefficient other than 1, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: factoring by grouping
- Watch the video that appears in the list 😊

TRY THESE:

Factor the following completely!

$2 \cdot -18 = -36 < \frac{-12}{3}$

39) $2x^2 - 9x - 18$

$2x^2 - 12x + 3x - 18$

$2x(x-6) + 3(x-6)$

$(x-6)(2x+3)$

$12 \cdot -7 = -84 < \frac{21}{-4}$

40) $12x^2 + 17x - 7$

$12x^2 - 4x + 21x - 7$

$4x(3x-1) + 7(3x-1)$

$(4x+7)(3x-1)$

$2 \cdot -15 = -30 < \frac{6}{-5}$

41) $2x^2 + x - 15$

$2x^2 + 6x - 5x - 15$

$2x(x+3) - 5(x+3)$

$(2x-5)(x+3)$

$2 \cdot 4 = 8 < \frac{8}{1}$

42) $2x^2 + 9x + 4$

$2x^2 + 8x + 1x + 4$

$2x(x+4) + 1(x+4) \Rightarrow (2x+1)(x+4)$

Part V: SOLVING QUADRATIC EQUATIONS**➤ Solve a quadratic equation by factoring.**

For this method, you use the techniques we learned in part 1 to factor the quadratic. You then use the zero product property to find possible solutions for the equation. You must remember that in order to use this technique, you must make sure that your equation is equal to zero!

Example:

Solve: $x^2 + 5x = 6$

Let's make this equation equal zero.

$x^2 + 5x - 6 = 0$

No GCF, let's factor!

$(x+6)(x-1) = 0$

Use zero product property.

$x+6=0 \quad x-1=0$

$x=-6 \quad x=1$

Solution set: $\{-6, 1\}$

Solve: $3x^2 - 4x - 4 = 0$

No GCF, let's factor!

$(3x+2)(x-2) = 0$

Use zero product property.

$3x+2=0 \quad x-2=0$

$3x=-2 \quad x=2$

$x=-2/3$

Solution set: $\{-2/3, 2\}$

For more help reviewing solving quadratic equations by factoring, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: solving a quadratic by factoring
- Watch the video that appears in the list 😊

TRY THESE:

Solve the following by factoring.

43) $x^2 - 11x + 19 = -5$

$x^2 - 11x + 24 = 0$

$(x-8)(x-3) = 0$

$x=8, x=3$

44) $6n^2 - 18n - 18 = 6$

$6n^2 - 18n - 24 = 0$

$\frac{6(n^2 - 3n - 4)}{6} = \frac{0}{6}$

$n^2 - 3n - 4 = 0$

$(n-4)(n+1) = 0$

$n=4, n=-1$

➤ **Solve a quadratic equation by using the quadratic formula.**

This method works for any quadratic equation, as long as it's in the standard form of $ax^2 + bx + c = 0$.

The quadratic formula is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Examples:

Solve: $2x^2 + 3x - 20 = 0$

It's in standard form already,

so, $a = 2$, $b = 3$, and $c = -20$

Substitute the values into the formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-20)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 160}}{4}$$

$$x = \frac{-3 \pm \sqrt{169}}{4}$$

$$x = \frac{-3 \pm 13}{4}$$

Solution set: $\{-4, 5/2\}$

Solve: $2m^2 - 7m - 13 = -10$

Rewrite in standard form:

$$2m^2 - 7m - 3 = 0$$

Now, $a = 2$, $b = -7$, and $c = -3$

Substitute the values into the formula:

$$m = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$m = \frac{7 \pm \sqrt{49 + 24}}{4}$$

$$m = \frac{7 \pm \sqrt{73}}{4}$$

Solution set: $\left\{ \frac{7 + \sqrt{73}}{4}, \frac{7 - \sqrt{73}}{4} \right\}$

For more help reviewing solving quadratic equations using the quadratic formula, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: Example 1: Using the quadratic formula
- Watch the video that appears in the list 😊

TRY THESE:

Solve each of the following using the quadratic formula.

45) $3x^2 - x - 16 = 0$

$$\frac{1 \pm \sqrt{(-1)^2 - 4(3)(-16)}}{2(3)} = \frac{1 \pm \sqrt{1 + 192}}{6}$$

$$= \frac{1 \pm \sqrt{193}}{6}$$

$$\left\{ \frac{1 - \sqrt{193}}{6}, \frac{1 + \sqrt{193}}{6} \right\}$$

46) $6x^2 + 7x - 3 = 0$

$$\frac{-7 \pm \sqrt{(7)^2 - 4(6)(-3)}}{2(6)} = \frac{-7 \pm \sqrt{49 + 72}}{12}$$

$$= \frac{-7 \pm \sqrt{121}}{12} = \frac{-7 \pm 11}{12} \begin{cases} \frac{-7 + 11}{12} = \frac{4}{12} = \frac{1}{3} \\ \frac{-7 - 11}{12} = \frac{-18}{12} = -\frac{3}{2} \end{cases}$$

$$\left\{ -\frac{3}{2}, \frac{1}{3} \right\}$$

Part VI: Simplifying Radicals

You should be able to write a radical in simplest radical form. This basically means that the number under the radical in your answer should not have any perfect square factors.

There are two methods for simplifying radicals. The perfect square method asks for you to find the largest perfect square that can divide into the number under the radical, and then use the product property for radicals to simplify. The prime factorization method has you break the number under the radical down into its prime factors and then look for perfect square partners. Let's look at both methods in the examples below.

Examples:

Write the following in simplest radical form.

$$\sqrt{18}$$

Option 1: Perfect square method.

The largest perfect square that divides into 18 is 9.

$$\sqrt{18} = \sqrt{9 \cdot 2}$$

Now we simplify the perfect square radical.

$$\sqrt{18} = 3\sqrt{2}$$

Option 2: Prime Factorization

$$\sqrt{18} = \sqrt{2 \cdot 3 \cdot 3}$$

Look for pairs of matching factors.

For each pair, place one factor outside the radical.

$$\sqrt{18} = \sqrt{2 \cdot \underbrace{3 \cdot 3}}$$

$$\sqrt{18} = 3\sqrt{2}$$

$$\sqrt{108}$$

Option 1: Perfect square method

The largest perfect square that divides into 108 is 36.

$$\sqrt{108} = \sqrt{36 \cdot 3}$$

Now we simplify the perfect square radical.

$$\sqrt{108} = 6\sqrt{3}$$

Option 2: Prime factorization

$$\sqrt{108} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$$

Look for pairs of matching factors.

For each pair, place one factor outside the radical

$$\sqrt{108} = \sqrt{\underbrace{2 \cdot 2} \cdot \underbrace{3 \cdot 3} \cdot 3}$$

$$\sqrt{108} = 2 \cdot 3\sqrt{3}$$

$$\sqrt{108} = 6\sqrt{3}$$

For more help with simplifying radicals, you can view this video on Khan Academy:

- Go to www.khanacademy.org
- Search: "simplifying radicals"
- Watch the video that appears in the list ☺

TRY THESE:

Put the following expressions in simplest radical form.

47) $\sqrt{81}$

$$= \boxed{9}$$

48) $\sqrt{50}$

$$= \sqrt{25} \sqrt{2}$$

$$= \boxed{5\sqrt{2}}$$

49) $\sqrt{48}$

$$= \sqrt{16} \sqrt{3}$$

$$= \boxed{4\sqrt{3}}$$

50) $\sqrt{28}$

$$= \sqrt{4} \sqrt{7}$$

$$= \boxed{2\sqrt{7}}$$