Decide whether you are given enough information to determine that the quadrilateral is a parallelogram.

Opposite sides are parallel.
 Opposite sides are congruent
 Two pairs of consecutive sides are congruent.
 Two pairs of consecutive angles are congruent.
 Diagonals are congruent.
 Diagonals are congruent.
 Diagonals are congruent.
 Consecutive angles are supplementary.

Prove that the points represent the vertices of a parallelogram. Use the method indicated.

9) A(-4, 7), B(3, 0), C(2, -5), D(-5, 2); Both pairs of opposite sides are parallel.



10) A(-2, 8), B(2, 7), C(5, 1), D(1, 2); Both pairs of opposite sides are congruent.



Find all possible coordinates for the fourth vertex of a parallelogram with the given vertices. Then draw the parallelogram on the graph.

12) (3, -4), (-2, -1), (1, 2)





Describe how to prove that ABCD is a parallelogram.





- 15) The diagram shows a battering ram which was used in ancient times to break through walls. A log is suspended on ropes of equal length (\overline{GF} and \overline{HJ}). The log swings, causing quadrilateral *FGHJ* to shift. In the diagram, $\overline{GH} \cong \overline{FJ}$, and \overline{GH} is parallel to the ground.
 - a) Identify FGHJ. Explain.
 - b) *Explain* why the log is always parallel to the ground.

16) Complete the following proof. **Given:** $\triangle ABC \cong \triangle CDA$ **Prove:** *ABCD* is a parallelogram.



Statements	Reasons
1. $\triangle ABC \cong \triangle CDA$	1.
2. <i>AB</i> ≅	2.
<i>BC</i> ≃	
3. <i>ABCD</i> is a parallelogram	3.



- 17) How many triangles are formed by drawing diagonals from one vertex in the figure? Find the sum of the measures of the interior angles in the figure.
 - A) 5, 900°
 - B) 5,1080°
 - C) 6,900°
 - D) 6,1080°

18) The sum of the measures of the interior angles of a convex quadrilateral is ______.

- A) 180°
- B) 270°
- C) 360°
- D) 540°

19) The measure of each interior angle of a regular hexagon is ______.

- A) 30°
- B) 120°
- C) 15°
- D) 60°

20) The measure of each exterior angle of a regular octagon is ______.

- A) 22.5°
- B) 67.5°
- C) 45°
- D) 135°
- E)

21) Find the value of x. (The figure may not be drawn to scale.)

- A) 74
- B) 108
- C) 49
- D) 51

22) Find the measure of each exterior angle of a regular polygon with 16 sides. E

- A) 11.25°
- B) 360°
- C) 22.5°
- D) 157.5°



23) Find the measure of the missing angle.



24) Find the value of x.



25) Find the values of x and y.



26) Find the number of sides of a convex polygon if the measures of its interior angles have a sum of 2880°.

27) Find the number of sides of a regular polygon with each interior angle equal to 171°.

28) Find the measure of an interior angle and an exterior angle of a regular polygon with 20 sides.

- 29) Find the measure of an interior angle and the measure of an exterior angle for a regular 32-gon.
- 30) Find each exterior angle measure in the diagram below



31) For parallelogram PQLM, if $m \angle PML = 83^\circ$, then $m \angle PQL = _$.

- A) $m \angle PQM$
- B) 83°
- C) 97°
- D) $m \angle QLM$

32) Consecutive angles in a parallelogram are always ______.

- A) Congruent angles
- B) Complementary angles
- C) Supplementary angles
- D) Vertical angles
- Choose the statement that is NOT ALWAYS true.
 For any parallelogram ______.
 - A) The diagonals bisect each other
 - B) Opposite sides are congruent
 - C) The diagonals are perpendicular
 - D) Opposite sides are congruent

34) Find the value of the variables in the parallelogram.

- A) $x = 52^{\circ}, y = 10.5^{\circ}, z = 159^{\circ}$
- B) $x = 21^{\circ}, y = 55^{\circ}, z = 104^{\circ}$
- C) $x = 55^{\circ}, y = 21^{\circ}, z = 104^{\circ}$
- D) $x = 10.5^{\circ}, y = 52^{\circ}, z = 159^{\circ}$



35) If $ON = x^2$, LM = 3x + 10, NM = x + 4, and OL = 3y + 3, find the values of x and y given that LMNO is a parallelogram.





36) Complete the statement for parallelogarm *ABCD*. Justify your answer.

 $\overline{AD} \cong$



37) Find AM in the parallelogram if PN = 10 and MO = 19.



- 38) UVWX is a parallelogram, $m \angle WXV = 17^\circ$, $m \angle WVX = 29^\circ$, XW = 41, UX = 24, UY = 15
 - a) Find $m \angle WVU$.
 - b) Find WV.
 - c) Find $m \angle XUV$.
 - d) Find UW.
- 39) FGHJ is a parallelogram, $m \angle JHG = 68^\circ$, JH = 34, GH = 19
 - a) Find $m \angle FJH$.
 - b) Find JF.

F G G 19 68° 19 H

- c) Find $m \angle GFJ$.
- d) Find FG.



40) Find a fourth point, *D*, so that a parallelogram is formed using the vertices *A*(0, -4), *B*(5, -3), *C*(-4, -3), and *D* in any order. Plot your point and draw the parallelogram in the coordinate plane.



Answer Key

- 1) Yes
- 2) Yes
- 3) No
- 4) No
- 5) No
- 6) Yes
- 7) Yes
- 8) Yes
- Slope of AB = slope of CD = -1;
 Slope of BC = slope of DA = 5,
 Since both pairs of opposite
 sides are parallel, ABCD is a
 parallelogram.
- 10) $AB = CD = \sqrt{17}$; $BC = DA = 3\sqrt{5}$. Since both pairs of opposite sides are congruent, *ABCD* is a parallelogram.
- 11) (8, 6), (0, -8), and (-8, 10)



12) (6,-1), (0,-7) and (-4, 5)



- AB || CD by Corr. Angles Converse. Since one pair of opposite sides is both parallel and congruent, ABCD is a parallelogram.
- 14) Since both pairs of opposite angles are congruent, *ABCD* is a parallelogram.
- 15) a) $\overline{GF} \cong \overline{HJ}$ and $\overline{GH} \cong \overline{FJ}$, so *FGHJ* is a parallelogram since both pairs of opposite sides are congruent. b)*FGHJ* is always a Y, so $\overline{GH} \parallel \overline{FJ}$. Because \overline{GH} is parallel to the ground, then \overline{FJ} is also parallel to the ground. The moving log is always parallel to the ground.

- 16) Given; CD; AD; CPCTC; In a quadrilateral, if both pair of opp. Sides are congruent, then it is a parallelogram.
- 17) A
- 18) C 19) B
- 20) C
- 21) D
- 22) C
- 23) 114°
- 24) 128°
- 25) X=103, y = 66
- 26) 18
- 27) 40
 - 28) Int angle: 162°, ext angle: 18°
 - 29) ≈ 168.8°, ≈ 11.2°
- 30) 55°, 60°, 75°, 80°
- 31) B
- 32) C
- 33) C
- 34) B
- 35) X = -2 or 5, y = -1/3 or 2
- 36) \overline{BC} , opp. Sides of
- parallelogram are congruent 37) 9.5
- 38) a) 46° b)24 c) 134° d) 30
- 39) a) 112° b)19 c) 68° d) 34
- 40) (1, -2), (9, -4), or (-9, -4)