Name : **Key**

Date : _____ Period : ____

1. In what polygon is the sum of the measures of the interior angles of the polygon equal to twice the sum of the measures of the exterior angles, one per vertex?

twice exterior $= 2 \cdot 360 = 720°$

$n = 6$:   Sum $= (6-2) \cdot 180$ $\Rightarrow$ $\boxed{\text{hexagon}}$

$720 = $ sum

2. In quadrilateral ABCD, the measures of $\angle A, \angle B, \angle C$, and $\angle D$ are in the ratio of 1:2:3:4, respectively. Please find the measures of the four angles.

$1x + 2x + 3x + 4x = 360$ Angles: $36°, 72°, 108°, 144°$

$10x = 360$

$\boxed{x = 36}$

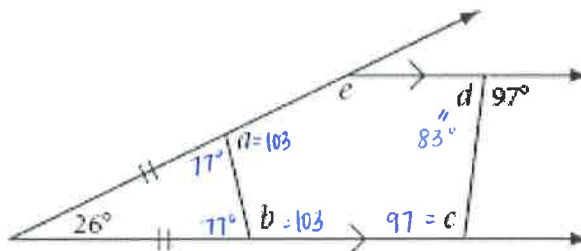3. In the following diagram, please find each lettered angle measure.

$a = 180 - 77 = 103°$

$b = 180 - 77 = 103°$

$c = 97°$

$d = 180 - 97 = 83°$

$e = 540 - 386 = 154°$

4. The figure at the right is called a regular semi-octagon. If $m\angle E = (3x+3y+9)°$ and $m\angle A = (2x+y-4.5)°$, what are the values of x and y?

each int angle $= \dfrac{(8-2) \cdot 180}{8} = 135°$ $2x+y-4.5 = 67.5$

$3x+3y+9 = 135$ $2x+y = 72$
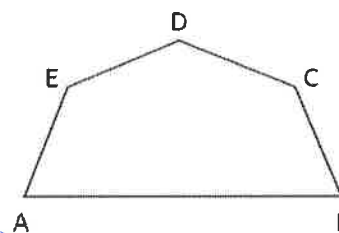
$3x+3y = 126$ $y = 72-2x$

$3(x+y) = 126$

$42-x = 72-2x$ $y = 72-2(30)$

$x+y = 42 \Rightarrow y = 42-x$ $\boxed{x = 30}$ $\boxed{y = 12}$

5. Two exterior angles of a pentagon total 200°. Of the remaining three angles, the second angle is 20° less than twice the first angle. The third angle is 20° more than the first angle. Please find the measure of each remaining exterior angle.

1st: $x = 40°$ $x + 2x-20 + x + 20 = 160$

2nd: $2x-20 = 60°$ $4x = 160$

3rd: $x+20 = 60°$ $\boxed{x = 40}$

6. What is the name of a regular polygon if the ratio of the measure of an interior angle to the measure of an exterior angle is 7:2?

$\frac{2x = 360}{n}$

$360 = 2xn$

$180 = xn$

$n = \frac{180}{x}$

$7x = \frac{(n-2)\cdot 180}{n}$

$7xn = 180n - 360$

$7xn - 180n = -360$

$n(7x-180) = -360$

$n = \frac{-360}{7x-180}$

$\frac{180}{x} = \frac{-360}{7x-180}$
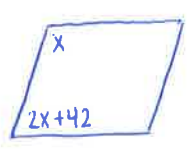
$-360x = 1260x - 32400$

$-1620x = -32400$

$x = 20$

$n = \frac{180}{20}$

$\boxed{n = 9}$

nonagon

7. The measure of one interior angle of a parallelogram is 42 degrees more than twice the measure of another angle. Find the measure of each angle of the parallellogram.
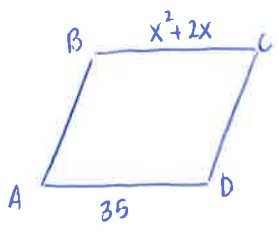

$x + 2x + 42 = 180$

$3x = 138$

$x = 46$

Angles: $46°, 46°, 134°, 134°$

8. Sketch parallelogram ABCD. Given that $BC = (x^2 + 2x)\,cm$ and $AD = 35\,cm$, please find the value(s) of x.

$x^2 + 2x = 35$

$x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$\boxed{x = -7, \; x = 5}$

Check: $BC = (7)^2 + 2(-7)$
$= 49 - 14$
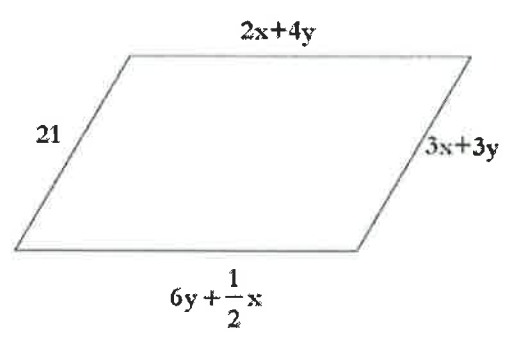$= 35$

$BC = (5)^2 + 2(5)$
$= 25 + 10$
$= 35$

9. If the given quadrilateral is a parallelogram, find the value of x and y.

$2x + 4y = 6y + \frac{1}{2}x$

$1.5x = 2y$

$\frac{3}{2}x = \frac{2}{1}y$

$\frac{3}{4}x = y$

$y = \frac{3}{4}(4)$
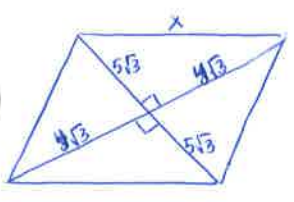
$\boxed{y = 3}$

$3x + 3y = 21$

$3(x+y) = 21$

$x + y = 7$

$x + \frac{3}{4}x = 7$

$\frac{7}{4}x = \frac{7}{1} \cdot \frac{4}{7}$

$\boxed{x = 4}$

10. Find the perimeter of a parallelogram with diagonals of lengths $10\sqrt{3}$ inches and $8\sqrt{3}$ inches given the diagonals are perpendicular. Please write your answer in simplest radical form.

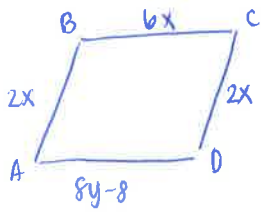$(5\sqrt{3})^2 + (4\sqrt{3})^2 = x^2$

$(25)(3) + (16)(3) = x^2$

$75 + 48 = x^2$

$123 = x^2$

$x = \sqrt{123}$

$\boxed{P = 4\sqrt{123}\ in}$

11. ABCD is a parallelogram. AB = 2x, AD = 8y – 8, and BC = 6x. If the perimeter of $\square ABCD$ is 160 meters, please find the values of x and y.

$\dfrac{6x=8y-8}{2}$

$3x=4y-4$

$4y=3x+4$

$10x+8y-8=160$

$\dfrac{10x+8y=168}{2}$

$5x+4y=84$

$4y=84-5x$

$3x+4=84-5x$

$8x=80$

$\boxed{x=10}$

$4y=30+4$

$4y=34$

$\boxed{y=8.5}$

12. MERY is a parallelogram. Please find the values of x and y.

$4x-8=8y-12$

$4x-8y=-4$

$4x=8y-4$

$x=2y-1$

$y-8=\frac{1}{4}x$

$x=4y-32$

$4y-32=2y-1$

$2y-32=-1$

$2y=31$

$\boxed{y=15.5}$
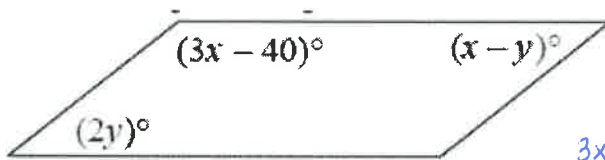
$x=2(15.5)-1$

$\boxed{x=30}$
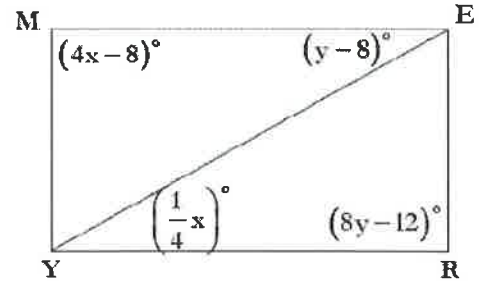
13. Determine the values of x and y for which ABCD is a parallelogram.

$x-y=2y$

$x=3y$

$3x-40+x-y=180$

$4x-y=220$

$-y=-4x+220$

$y=4x-220$

$y=4(3y)-220$

$y=12y-220$

$-11y=-220$

$\boxed{y=20}$

$x=3(20)$

$\boxed{x=60}$

14. Given the parallelogram below, please find the values of x and y.
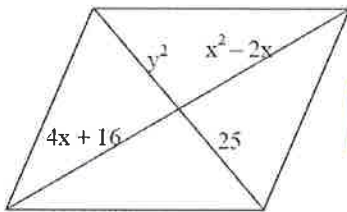
$y^2=25$

$y=\sqrt{25}$

$\boxed{y=-5, y=5}$

$x^2-2x=4x+16$

$x^2-6x-16=0$

$(x-8)(x+2)=0$

$\boxed{x=8, x=-2}$

15. QUAD is a parallelogram. Please find the values of x and y.

$2x-y=18$

$2x=18+y$

$y=2x-18$

$x+y=6+2y$

$x-6=y$

$2x-18=x-6$

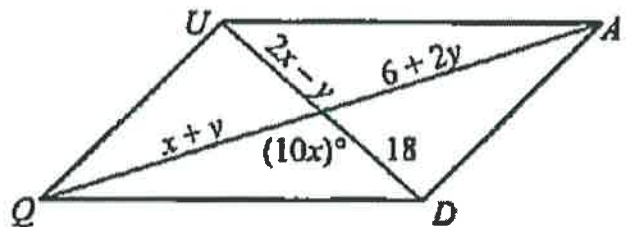$\boxed{x=12}$

$y=24-18$

$\boxed{y=6}$

16. The vertices of a quadrilateral are $J(-6,2)$, $K(-1,3)$, $L(2,-3)$, and $M(-3, -4)$. Use one of the methods we talked about to prove that $JKLM$ is a parallelogram.
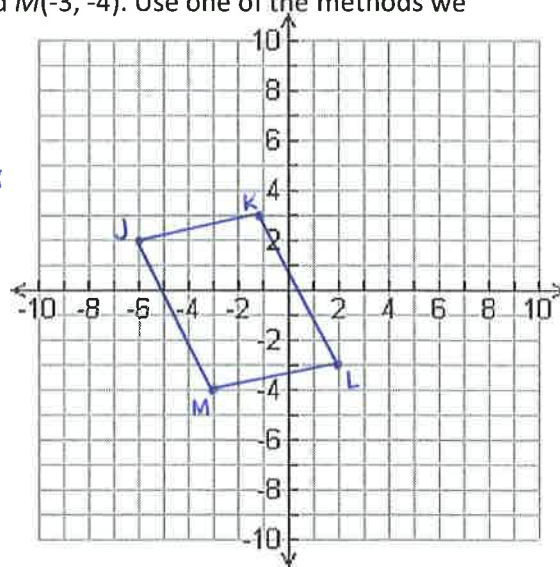
Slopes: slope of $\overline{JM} = \frac{-4-2}{-3+6} = \frac{-6}{3} = -2$

$\left.\begin{array}{c}\end{array}\right\}$ $\overline{JM} \parallel \overline{KL}$

slope of $\overline{KL} = \frac{-3-3}{2+1} = \frac{-6}{3} = -2$

slope of $\overline{JK} = \frac{3-2}{-1+6} = \frac{1}{5}$

$\left.\begin{array}{c}\end{array}\right\}$ $\overline{JK} \parallel \overline{ML}$

slope of $\overline{ML} = \frac{-3+4}{2+3} = \frac{1}{5}$

* Both pairs of opp. sides are parallel, so by definition, JKLM is a parallelogram

Distances : $JK = \sqrt{(-1+6)^2 + (3-2)^2} = \sqrt{(5)^2 + (1)^2} = \sqrt{25+1} = \sqrt{26}$

$ML = \sqrt{(-3-2)^2 + (-4+3)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$

$JM = \sqrt{(-3+6)^2 + (-4-2)^2} = \sqrt{(3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$KL = \sqrt{(2+1)^2 + (-3-3)^2} = \sqrt{(3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

* Both pairs of opp. sides are $\cong$, so JKLM is a parallelogram

Answer Key :

1. Hexagon

2. 36°, 72°, 108°, 144°

3. a=103°, b=103°, c=97°, d=83°, e=154°

4. x=30, y=12

5. 40°, 60°, 60°

6. Nonagon

7. 46°, 46°, 134°, 134°

8. -7, 5

9. x=4, y=3

10. $4\sqrt{123}$ inches

11. x=10, y=8.5

12. x=30, y=15.5

13. x=60, y=20

14. x = -2, 8 ; y = -5, 5

15. x=12, y=6

16. Slopes : JK = $\frac{1}{5}$, LM = $\frac{1}{5}$, JM = -2, KL = -2. So $\overline{JK} \parallel \overline{LM}$ and $\overline{JM} \parallel \overline{KL}$. Both pairs of opp. sides are parallel so it's a parallelogram.

OR

Distances : JK = $\sqrt{26}$, LM = $\sqrt{26}$, JM = $3\sqrt{5}$, KL = $3\sqrt{5}$. So $\overline{JK} \cong \overline{LM}$ and $\overline{JM} \cong \overline{KL}$, both pairs of opp. sides congruent so it's a parallelogram.

OR

Slopes and Distances: JK = $\frac{1}{5}$, LM = $\frac{1}{5}$ and JK = $\sqrt{26}$, LM = $\sqrt{26}$ so one pair of opposite sides is both parallel and congruent.

OR

Slopes and Distances: JM = -2, KL = -2 and JM = $3\sqrt{5}$, KL = $3\sqrt{5}$ so one pair of opposite sides is both parallel and congruent.