Geometry H
Pythagorean Theorem and Its Converse Review (7.1 – 7.2)

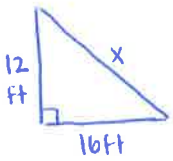
Name: Key
Date: _____ Period: ____

Part I: Multiple Choice Practice

1. A radio station is going to construct a 12-foot tower on top of a building. The tower will be supported by three cables, each attached to the top of the tower and to points on the roof of the building that are 16 feet from the base of the tower. Find the total length of the three cables.

A) 80 ft
B) 60 ft
C) 100 ft
D) 20 ft

One cable:

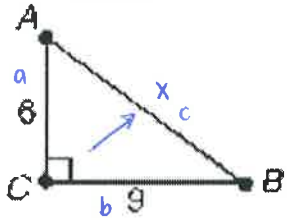


$12^2+16^2=x^2$
$144+256=x^2$
$400=x^2$
$x=\sqrt{400}$
$x=20$

Three cables:
$20 \times 3 = 60$ ft total

2. $\triangle ABC$ is a right triangle. $AB =$ _____.

A) $3\sqrt{13}$
B) $3\sqrt{6}$
C) $3\sqrt{5}$
D) 117



$a^2+b^2=c^2$
$6^2+9^2=c^2$
$36+81=c^2$
$117=c^2$
$c=\sqrt{117}$
$c=\sqrt{9}\sqrt{13} = 3\sqrt{13}$

3. If a, b, and c are sides of a right triangle, which of the following are also sides of a right triangle?

A) The square root of each length. $\left(\sqrt{a},\sqrt{b},\sqrt{c}\right)$
B) Twice the length of each side. $(2a, 2b, 2c)$
C) Four more than the length of each side. $(a + 4, b + 4, c + 4)$
D) The square of each length. (a^2, b^2, c^2)

4. Which of the following sets of a numbers is a Pythagorean Triple?

A) $\sqrt{3},\sqrt{4},\sqrt{5}$
B) $\frac{1}{3},\frac{1}{4},\frac{1}{5}$
C) 12, 16, 20
D) $3^2, 4^2, 5^2$

$4(3)\ 4(4)\ 4(5)$
3, 4, 5

5. A set of Pythagorean Triples is _____.

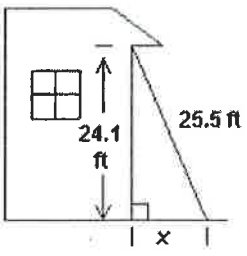
A) 3, 5, 9
B) 6, 9, 12
C) 1, 1, 2
D) 5, 12, 13

$5^2+12^2=13^2$
$25+144=169$
$169=169$ ✓

6. A 25.5 foot ladder rests against the side of a house at a point 24.1 feet above the ground. The foot of the ladder is x feet from the house. Find the value of x to one decimal place.

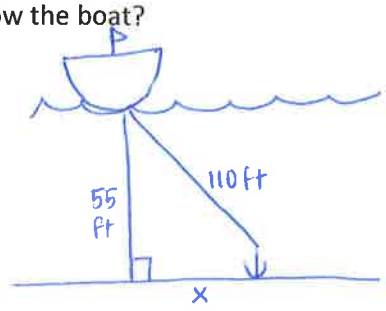
A) 1.9
B) 7.0
C) 8.3
D) 10.1

$x^2+(24.1)^2=(25.5)^2$
$x^2+580.81=650.25$
$x^2=69.44$
$x=\sqrt{69.44}$
$x=8.3$



7. A scuba diver has a taut rope connecting the dive boat to an anchor on the ocean floor. The rope is 110 feet long. The water is 55 feet deep. To the nearest tenth of a foot, how far is the anchor from a point directly below the boat?

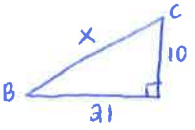
A) 95.3 ft
B) 123.0 ft
C) 81.4 ft
D) 89.8 ft



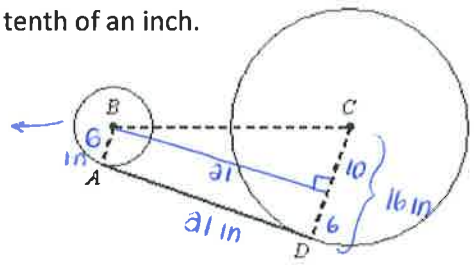
$X^2+55^2=110^2$
$X^2+3025=12100$
$X^2=9075$
$X=\sqrt{9075}$
$X=95.3$ ft

8. $\overline{AD}$ is perpendicular to the radius of each circle in the figure below. If $BA = 6$ inches, $AD = 21$ inches, and $CD = 16$ inches, find the length of $\overline{BC}$ to the nearest tenth of an inch.

A) 18.9 inches
B) 29.7 inches
C) 21.8 inches
D) 23.3 inches



$10^2+21^2=X^2$
$100+441=X^2$
$541=X^2$
$X=\sqrt{541} \Rightarrow X=23.3$ in



9. Which set of lengths cannot form a right triangle?

A) 6mm, 12mm, 13 mm $6^2+12^2=13^2 \Rightarrow 180=169$ (X)
B) 5 mm, 12mm, 13 mm $5^2+12^2=13^2 \Rightarrow 25+144=169 \Rightarrow 169=169$
C) 2.5 mm, 6 mm, 6.5 mm
D) 10mm, 24mm, 26 mm

10. Given lengths of 7, 6, and 9, then lengths _____.

A) form an obtuse triangle
B) form a right triangle
C) form an acute triangle
D) cannot form a triangle.

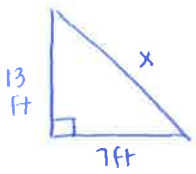
a b c
$c^2=a^2+b^2$
$9^2=7^2+6^2$
$81=49+36$
$81<85$
$c^2<a^2+b^2$

11. Choose the set that is the possible side lengths of a right triangle.

A) 4, 9, 13 $4^2+9^2=13^2 \Rightarrow 97=169$ (X)
B) 1, 1, 2 $1^2+1^2=2^2 \Rightarrow 1+1=4$ (X)
C) $\sqrt{2},\sqrt{2},2$ $(\sqrt{2})^2+(\sqrt{2})^2=2^2 \Rightarrow 2+2=4 \Rightarrow 4=4$
D) 8, 15, 25

Part II: Open Ended

12. How long is a string reaching from the top of a 13-foot pole to a point on the ground that is 7 feet from the base of the pole?



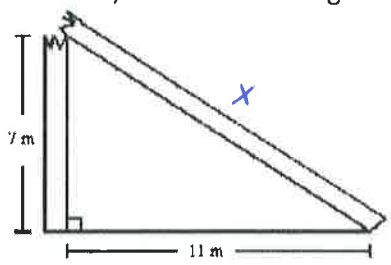
$13^2+7^2=x^2$

$169+49=x^2$

$218=x^2$

$x=\sqrt{218} \Rightarrow$ **14.8 ft**

13. A utility pole broke and fell as shown. To the nearest tenth of a meter, what was the original height of the pole?



$7^2+x^2=11^2$

$49+x^2=121$

$x^2=72$

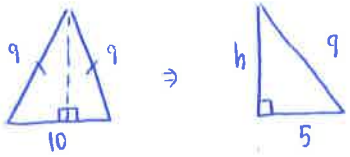
$x=\sqrt{72}$

$x=8.48$

orig. height $= 8.5+7$

$= 15.5$ m

14. Find the altitude (height) of an isosceles triangle with base 10 and congruent sides of length 9.



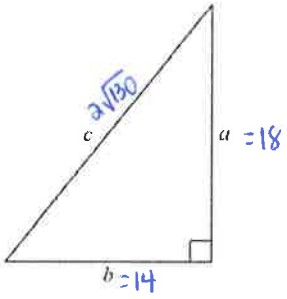
$h^2+5^2=9^2$

$h^2+25=81$

$h^2=56$

$h=\sqrt{56}=\sqrt{4}\sqrt{14}=$ **$2\sqrt{14}$**

15. Find the area of this triangle if $b = 14$ and $c = 2\sqrt{130}$.



$a^2+14^2=(2\sqrt{130})^2$

$a^2+196=2^2(\sqrt{130})^2$

$a^2+196=4(130)$

$a^2+196=520$

$a^2=324$

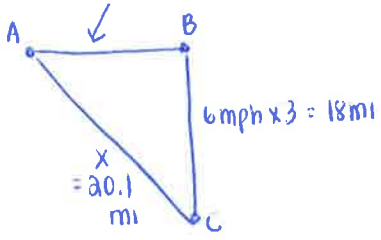
$a=18$

$A=\frac{1}{2}(14)(18)$

$=7(18)$

$A=126$ units^2

16. To avoid a large, shallow reef, a ship set a course from point *A* and traveled for 2 hours at a rate of 4.5 mph east to point *B*. The ship then turned and traveled for 3 hours at a rate of 6 mph south to point *C*. If the ship could have traveled in a straight line path from point *A* to point *C*, how many less miles would the ship have traveled? Round your answer to the nearest tenth of a mile.



$9^2+18^2=x^2$

$405=x^2$

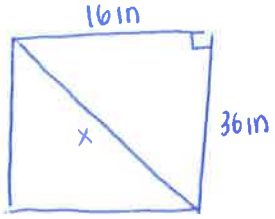
$x=\sqrt{405} \approx 20.1$ mi

A→B→C: $9+18=27$ mi

A→C: -20.1 mi

6.9 miles saved

17. The diagonals of a picture frame do not have the same length because its loose joints are not forming right angles. To make the corners of the picture frame form right angles, a wire is fastened to one corner of the frame on the longer diagonal. Then it is drawn across to the opposite corner and tightened until the frame is rectangular. The frame is supposed to be 16 inches wide and 36 inches tall. To the nearest hundredth of an inch, how long is the diagonal of the frame when its corners are square?



$16^2 + 36^2 = x^2$

$256 + 1296 = x^2$

$1552 = x^2$

$x = \sqrt{1552}$

$x = 39.40$ in

18. For each set of numbers, determine whether the numbers represent the lengths of the sides of an acute triangle, a right triangle, an obtuse triangle, or no triangle.

a) $\sqrt{38}, \sqrt{25}, \sqrt{13}$ (c, a, b)

$6.2 + 5 > 3.6$, $5 + 3.6 > 6.2$, $6.2 + 3.6 > 5$ ✓

$c^2 = a^2 + b^2$

$(\sqrt{38})^2 = (\sqrt{25})^2 + (\sqrt{13})^2$

$38 = 25 + 13$

$38 = 38$

Right

b) 3, 4, 7

$3 + 4 > 7$ ✗

not a △

c) 6, 9, 12 (a, b, c)

$6 + 9 > 12$, $9 + 12 > 6$, $12 + 6 > 9$ ✓

$c^2 = a^2 + b^2$

$12^2 = 6^2 + 9^2$

$144 = 36 + 81$

$144 > 117$

$c^2 > a^2 + b^2$

obtuse

d) 3.2, 4.2, 5.2 (a, b, c)

$3.2 + 4.2 > 5.2$, $3.2 + 5.2 > 4.2$, $4.2 + 5.2 > 3.2$ ✓

$c^2 = a^2 + b^2$

$5.2^2 = 3.2^2 + 4.2^2$

$27.04 = 10.24 + 17.64$

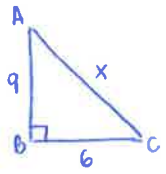
$27.04 < 27.88$

$c^2 < a^2 + b^2$

acute

19. In $\triangle ABC$, $AB = 9$ and $BC = 6$.

a) If $\triangle ABC$ is a right triangle, what are the possible lengths for $\overline{AC}$? Explain.



$9^2 + 6^2 = x^2$

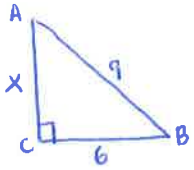
$81 + 36 = x^2$

$117 = x^2$

$x = \sqrt{117}$

$x = \sqrt{9}\sqrt{13}$

$x = 3\sqrt{13}$ hypotenuse



$x^2 + 6^2 = 9^2$

$x^2 + 36 = 81$

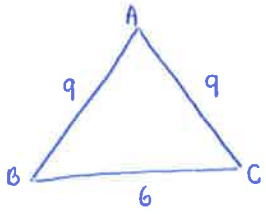
$x^2 = 45$

$x = \sqrt{45}$

$x = \sqrt{9}\sqrt{5}$

$x = 3\sqrt{5}$ Leg

b) If $\triangle ABC$ is an obtuse isosceles triangle, what are the possible lengths for $\overline{AC}$? Explain.



If AC = 9:

$c^2 > a^2 + b^2$

$9^2 > 6^2 + 9^2$

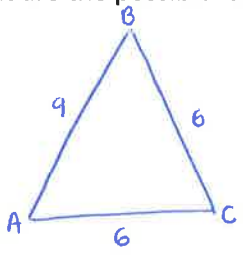
$81 > 36 + 81$

$81 > 117$

↑

not obtuse

since $c^2 < a^2 + b^2$



$c^2 > a^2 + b^2$

$9^2 > 6^2 + 6^2$

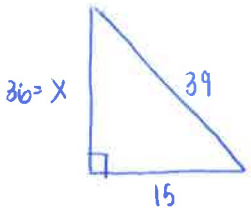
$81 > 36 + 36$

$81 > 72$

$c^2 > a^2 + b^2$

so AC = 6

20. What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet?



$x^2+15^2=39^2$

$x^2+225=1521$

$x^2=1296$

$x=\sqrt{1296}$

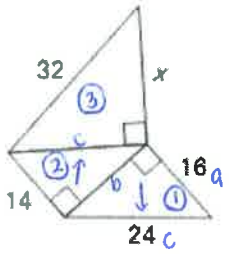
$x=36$

$A=\frac{1}{2}bh$

$=\frac{1}{2}(15)(36)$

$\boxed{A=270\ ft^2}$

21. Solve for x. Leave your answer in simplest radical form.



Triangle ①

$16^2+b^2=24^2$

$256+b^2=576$

$b^2=320$

$b=\sqrt{320}$

Triangle ②

$14^2+(\sqrt{320})^2=c^2$

$196+320=c^2$

$516=c^2$

$c=\sqrt{516}$

Triangle ③

$x^2+(\sqrt{516})^2=32^2$

$x^2+516=1024$

$x^2=508$

$x=\sqrt{508}$

$x=\sqrt{4}\sqrt{127}$

$\boxed{x=2\sqrt{127}}$

22. The sides of a triangle are x, x, and 10. (a b c)

a) What value(s) of x make the lengths form a triangle?

$x+x>10$

$2x>10$

$\boxed{x>5}$

~~$x+10>x$~~

~~$10>0$~~

~~$x+10>x$~~

~~$10>0$~~

b) Assuming that 10 is the longest side, what value(s) of x will make the triangle an obtuse triangle?

$c^2>a^2+b^2$

$10^2>x^2+x^2$

$100>2x^2$

$50>x^2$

$\sqrt{50}>x$

$5\sqrt{2}>x$

$\boxed{5<x<5\sqrt{2}}$

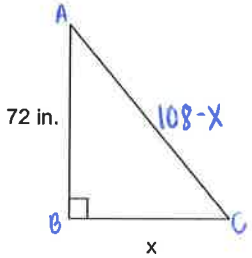
↑ from (a)

23. The perimeter of the right triangle below is 180 inches. Find the length of the two other sides, given that one leg is 72 inches long.

$x+72+AC=180$

$x+AC=108$

$AC=108-x$



$x^2+72^2=(108-x)^2$

$x^2+5184=11664-216x+x^2$

$5184=11664-216x$

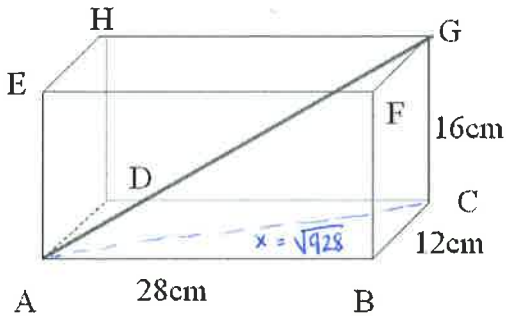
$-6480=-216x$

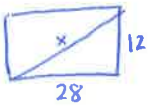
$x=30$

$\boxed{BC=30in}$

$\boxed{AC=78in}$

24. Find the length of $\overline{AG}$.



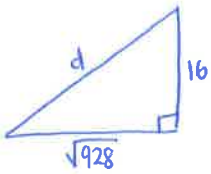


$12^2+28^2=x^2$

$144+784=x^2$

$928=x^2$

$x=\sqrt{928}$



$16^2+(\sqrt{928})^2=d^2$

$256+928=d^2$

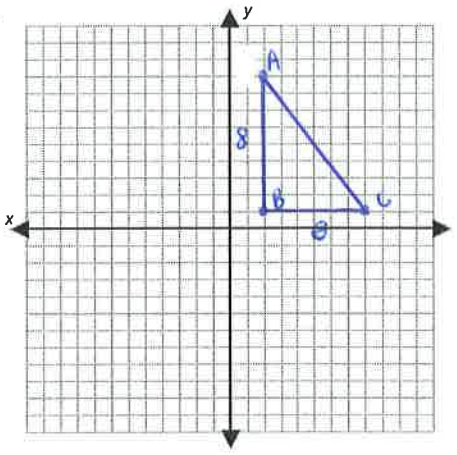
$1184=d^2$

$d=\sqrt{1184}$

$d=\sqrt{16}\sqrt{74}$

$\boxed{d=4\sqrt{74}}$

25. The vertices of $\triangle ABC$ are $A(2, 9)$, $B(2, 1)$, and $C(8, 1)$. Use the distance formula and the converse of the Pythagorean theorem to prove that $\triangle ABC$ is a right triangle.



$AB=8$

$BC=6$

$8^2+6^2=AC^2$

$64+36=AC^2$

$100=AC^2$

$AC=\sqrt{100}$

$AC=10$

a b c

Lengths: 8, 6, 10

$c^2=a^2+b^2$

$10^2=8^2+6^2$

$100=64+36$

$100=100$

$\boxed{c^2=a^2+b^2 \text{ so } \triangle ABC \text{ is a right } \triangle}$

Answer Key

1. B
2. A
3. B
4. C
5. D
6. C
7. A
8. D
9. A
10. C
11. C
12. $\sqrt{218}\,ft \approx 14.8\,ft$
13. 20.0 m
14. $\sqrt{56} = 2\sqrt{14}$
15. 126 sq. units
16. 6.9 miles
17. 39.40 inches
18. a) right
    b) not triangle
    c) obtuse
    d) acute
19. a) If $\overline{AC}$ is a leg, then $6^2 + AC^2 = 9^2$, so $AC = 3\sqrt{5}$. If $\overline{AC}$ is the hypotenuse, then $6^2 + 9^2 = AC^2$, so $AC = 3\sqrt{13}$.
    b) $\triangle ABC$ could be isosceles if AC = 6 or AC = 9. If AC = 6, then $9^2 > 6^2 + 6^2$, so the triangle would be obtuse according to the Converse of the Pythagorean Theorem. If AC = 9, then $9^2 < 6^2 + 9^2$, so the triangle would be acute according to the Converse of the Pythagorean Theorem. Therefore, $\triangle ABC$ is an isosceles obtuse triangle only if AC = 6.
20. 270 ft^2
21. $2\sqrt{127}$
22. a) $x > 5$ b) $5 < x < 5\sqrt{2}$
23. 30 in. and 78 in.
24. $4\sqrt{74}$
25. Using the distance formula, AB = 8, BC = 6, and AC = 10. Since $10^2 = 6^2 + 8^2$, the triangle is a right triangle according to the Converse of the Pythagorean Theorem.