

Please solve for the variable :

$$1) \frac{2x+5}{3} = \frac{x-5}{4}$$

$$4(2x+5) = 3(x-5)$$

$$8x+20 = 3x-15$$

$$5x+20 = -15$$

$$5x = -35$$

$$\boxed{x = -7}$$

$$2) \frac{y}{2y+3} = \frac{y-8}{y-3}$$

$$y(y-3) = (y-8)(2y+3)$$

$$y^2-3y = 2y^2+3y-16y-24$$

$$y^2-3y = 2y^2-13y-24$$

$$0 = y^2-10y-24$$

$$0 = (y-12)(y+2) \Rightarrow \boxed{y=12 \text{ or } y=-2}$$

$$3) \frac{5x}{3-x} = \frac{4}{x-3}$$

$$5x(x-3) = 4(3-x)$$

$$5x^2-15x = 12-4x$$

$$5x^2-11x-12 = 0$$

$$(5x+4)(x-3) = 0$$

$$\boxed{x = -4/5 \text{ or } x = 3}$$

- 4) Mr. Jones has taken a survey of college students and found that 1 out of 5 students are liberal arts majors. If a college has 12,000 students, what is the best estimate of the number of students who are liberal arts majors?

$$\frac{\text{Libarts}}{\text{Total}} : \frac{1}{5} = \frac{x}{12000} \Rightarrow 12000 = 5x \Rightarrow x = 2400 \text{ students are liberal arts majors}$$

- 5) A national forest service wanted to estimate the number of deer in a particular national park. They caught and tagged 72 deer and released them back into the park. Later they selected a sample of 323 deer. Of the 323 deer, 19 were tagged. Assuming that the proportion of tagged deer in the sample holds for all deer in the forest, what is the best estimate of the number of deer in the park?

$$\frac{\text{tagged}}{\text{total}} : \frac{72}{x} = \frac{19}{323} \Rightarrow 19x = 23256 \Rightarrow x = 1,224 \text{ total deer in the park}$$

- 6) A worker can build 12 lamps in 90 minutes. How many hours will it take to build 20 lamps?

$$\frac{\text{Lamps}}{\text{mins}} : \frac{12}{90} = \frac{20}{x} \Rightarrow 12x = 1800 \Rightarrow x = 150 \text{ min} \div 60 = 2.5 \text{ hours to build 20 lamps}$$

- 7) Please find the geometric means of (a) and (b) below in simplest radical form.

a) 8 and 30

$$= \sqrt{8 \cdot 30} = \sqrt{240}$$

$$= \sqrt{16 \cdot 15}$$

$$= \boxed{4\sqrt{15}}$$

b) 14 and 42

$$= \sqrt{14 \cdot 42} = \sqrt{588}$$

$$= \sqrt{196 \cdot 3}$$

$$= \boxed{14\sqrt{3}}$$

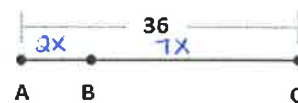
- 8) In the diagram, AB : BC is in the ratio 2:7 and AC = 36. Find AB and BC.

$$2x+7x = 36$$

$$9x = 36$$

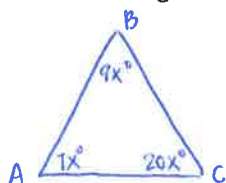
$$x = 4$$

$$\boxed{AB = 2(4) = 8 \text{ units}} \\ \boxed{BC = 7(4) = 28 \text{ units}}$$



↑
Use Segment addition postulate

- 9) The measures of the angles of a triangle are in the extended ratio of 7 : 9 : 20. Find the measures of the angles of the triangle.



$$7x + 9x + 20x = 180$$

$$36x = 180$$

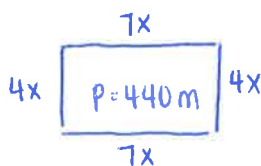
$$x = 5$$

$$m\angle A = 7(5) = 35^\circ$$

$$m\angle B = 9(5) = 45^\circ$$

$$m\angle C = 20(5) = 100^\circ$$

- 10) The perimeter of a rectangular corn field is 440 meters. The ratio of its length to its width is 7 : 4. What is the length and width of the field?



$$7x + 4x + 7x + 4x = 440$$

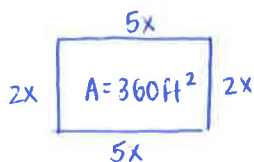
$$22x = 440$$

$$x = 20$$

$$\text{length} = 7(20) = 140 \text{ m}$$

$$\text{width} = 4(20) = 80 \text{ m}$$

- 11) The area of a rectangle is 360 square feet. The ratio of the width to the length is 2 : 5. Please find the width and the length.



$$A = l \times w$$

$$360 = (2x)(5x)$$

$$360 = 10x^2$$

$$36 = x^2$$

$$x = 6$$

$$\text{length} = 5(6) = 30 \text{ ft}$$

$$\text{width} = 2(6) = 12 \text{ ft}$$

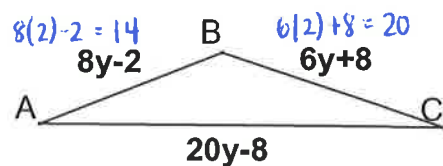
- 12) The ratio of the side lengths for $\triangle ABC$ is $AB:BC:AC = 7:10:16$. Please find the perimeter of the triangle.

$$\frac{AB}{BC} = \frac{7}{10} \Rightarrow \frac{8y-2}{6y+8} = \frac{7}{10} \Rightarrow 10(8y-2) = 7(6y+8)$$

$$80y - 20 = 42y + 56$$

$$38y - 20 = 56$$

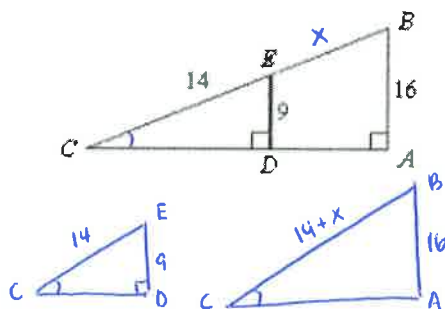
$$38y = 76 \Rightarrow y = 2$$



$$20(2) - 8 = 32$$

$$P = 14 + 20 + 32 = 66 \text{ units}$$

- 13) Find BC to the nearest tenth. NOTE : The figure is not drawn to scale.



$$\frac{CE}{CB} = \frac{ED}{BA} \Rightarrow \frac{14}{14+x} = \frac{9}{16}$$

$$\Rightarrow 9(14+x) = 224$$

$$126 + 9x = 224$$

$$9x = 98$$

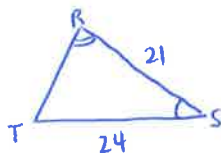
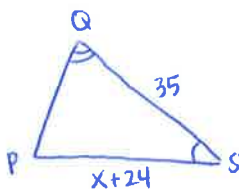
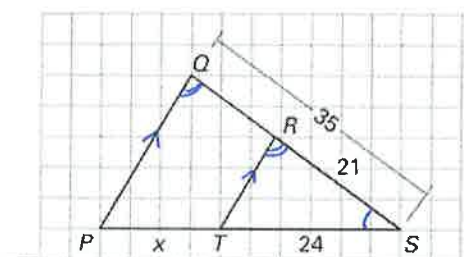
$$x = 10.9$$

$$CB = 14 + x$$

$$= 14 + 10.9$$

$$CB = 24.9$$

- 14) In the diagram, $\overline{RT} \parallel \overline{PQ}$. Find TP .



$$\frac{\text{Big } \Delta}{\text{sm } \Delta} : \frac{35}{21} = \frac{x+24}{24}$$

$$21(x+24) = 840$$

$$21x + 504 = 840$$

$$21x = 336$$

$$x = 16$$

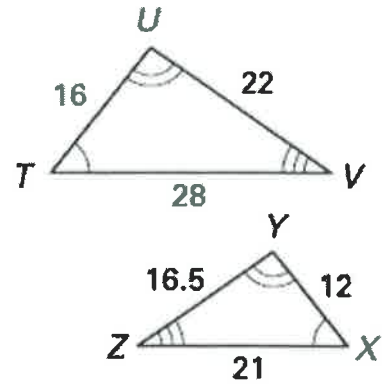
$$TP = 16 \text{ units}$$

15) Determine whether the polygons are similar. If they are, write a similarity statement and state the scale factor.

$$\frac{\Delta TUV}{\Delta ZYX} : \frac{UT}{YX} = \frac{UV}{YZ} = \frac{TV}{XZ}$$

$$\Rightarrow \frac{16}{12} = \frac{22}{16.5} = \frac{28}{21}$$

$$\Rightarrow \frac{4}{3} = \frac{4}{3} = \frac{4}{3}$$



Yes, $\Delta TUV \sim \Delta ZYX$ with a scale factor of $\frac{4}{3}$

16) Please find the values of x and y , so that ΔABC is similar to ΔDEF .

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{9}{6} = \frac{4x-1}{10} = \frac{18}{y}$$

$$\frac{9}{6} = \frac{18}{y}$$

$$9y = 108$$

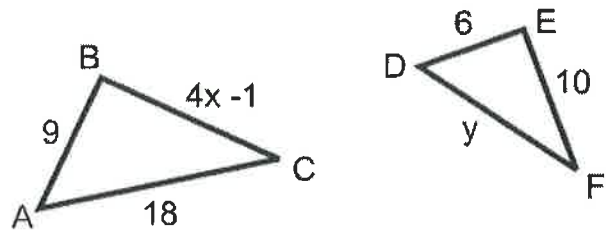
$$y = 12$$

$$\frac{9}{6} = \frac{4x-1}{10}$$

$$\Rightarrow 90 = 6(4x-1)$$

$$90 = 24x - 6$$

$$96 = 24x \Rightarrow x = 4$$



17) The ratio of one side of ΔABC to the corresponding side of similar ΔDEF is 3:5. The perimeter of ΔDEF is 48 inches. What is the perimeter of ΔABC ?

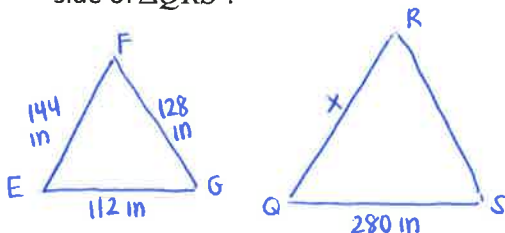
$$\frac{\Delta ABC}{\Delta DEF} : \frac{3}{5} = \frac{x}{48}$$

\uparrow scale \uparrow perimeters

$$\Rightarrow 5x = 144 \Rightarrow x = 28.8$$

Perimeter of ΔABC
is 28.8 inches

18) Triangles EFG and QRS are similar. The lengths of the sides of ΔEFG are 144 inches, 128 inches, and 112 inches. The length of the smallest side of ΔQRS is 280 inches. What is the length of the longest side of ΔQRS ?



$$\frac{\Delta EFG}{\Delta QRS} : \frac{144}{x} = \frac{112}{280}$$

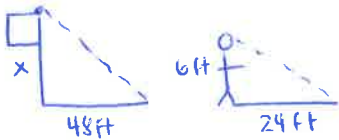
\uparrow longest \uparrow shortest

$$112x = 40320$$

$$x = 360$$

The longest side of
 ΔQRS is 360 in.

19) A flagpole casts a shadow 48 feet long at the same time that a 6-ft tall person casts a shadow 24-ft long. How tall is the flagpole?



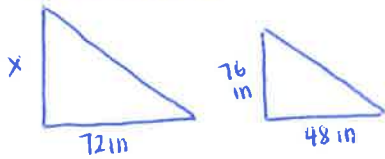
$$\frac{x}{6} = \frac{48}{24}$$

$$24x = 288$$

$$x = 12$$

The flagpole is 12 ft tall

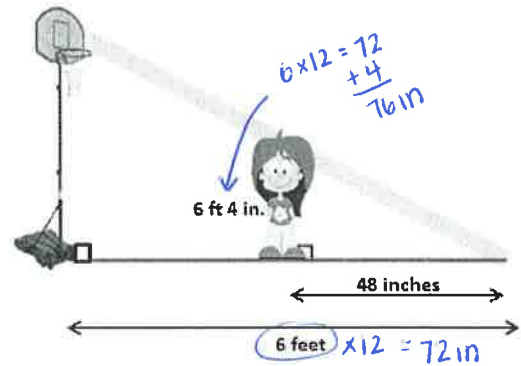
20) Liza is standing beside a basketball hoop. Liza is 6 feet 4 inches tall and casts a shadow that is 48 inches long. The basketball hoop casts a shadow that is 6 feet long. How tall is the basketball hoop (in feet and inches)?



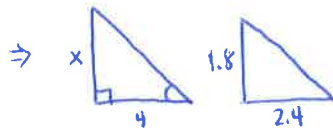
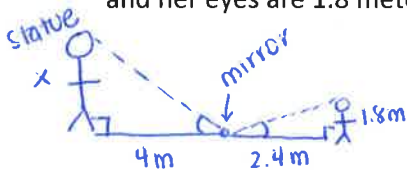
$$\frac{x}{76 \text{ in}} = \frac{72 \text{ in}}{48 \text{ in}} \Rightarrow 48x = 5472$$

$$x = 114 \text{ inches} \div 12$$

$$= 9.5 \text{ ft} = 9 \text{ ft } 6 \text{ inches}$$



21) A statue, honoring Ray Hnatyshyn (1934 – 2002), can be found on Spadina Crescent East, near the University Bridge in Saskatoon. Eli wants to determine the height of the statue. She walks 4 meters away from the statue and places a mirror on the ground. She then walks away from the mirror so that she can see the top of the statue in the center of the mirror. The distance from Eli to the mirror is 2.4 meters and her eyes are 1.8 meters above the ground. How tall is the statue? (HINT: Draw a diagram!)



$$\frac{x}{1.8} = \frac{4}{2.4}$$

$$2.4x = 7.2$$

$$x = 3$$

The statue is 3 meters tall

22) A 40 ft tree is inverted (looks upside down) when viewed through a lens. Its image through the lens is also smaller. Assume that $\overline{AB} \parallel \overline{ED}$. Please explain why the two triangles are similar, write a similarity statement, and then write a proportion and find the height of the inverted image.

Since $\angle A \cong \angle E$ (alt. int. angles) and $\angle ACB \cong \angle ECD$ (vertical angles) then

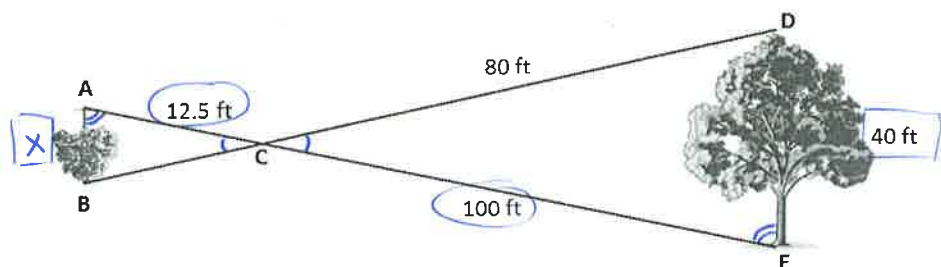
$\triangle ACB \sim \triangle ECD$ by AA \sim

$$\frac{\text{sm } \Delta}{\text{big } \Delta} : \frac{x}{40} = \frac{12.5}{100}$$

$$100x = 500$$

$$x = 5 \Rightarrow$$

The height is 5 feet



Answer Key

1) -7

2) -2, 12

3) $-\frac{4}{5}, 3$

4) 2,400 students are liberal arts majors

5) 1,224 deer in the park

6) 2.5 hours

7) a. $4\sqrt{15}$ b. $14\sqrt{3}$

8) $AB = 8, BC = 28$

9) $35^\circ, 45^\circ, 100^\circ$

10) length : 140 m, width : 80 m

11) length : 12 ft, width : 30 ft

12) $y = 2, P = 66$ units

13) $BC = 24.9$ units

14) $TP = 16$ units

15) Yes. Scale : $\frac{4}{3}$ or 4 : 3. $\triangle TUV \sim \triangle XYZ$

16) $x = 4, y = 12$

17) $P = 28.8$ inches

18) 360 inches

19) 12 feet

20) 9 feet 6 inches

21) 3 m

22) The triangles are similar since $\angle A \cong \angle E$ (or $\angle B \cong \angle D$) by the alternate interior angles theorem and $\angle ACB \cong \angle ECD$ by the vertical angles theorem, so $\triangle ACB \sim \triangle ECD$ by AA \sim . The height of the tree is 5 feet.