Geometry H | **Name:** Key
Notes – 4.1/4.7 Coordinate Proofs | **Date:** ____________ **Period:** ________

LEARNING TARGETS

- I can classify triangles on the coordinate plane using slope and distance formulas.

Recall:

Slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Two lines on the coordinate plane are **perpendicular** if they have opposite reciprocal slopes ; intersect at a 90° angle.

To classify triangles on the coordinate plane:

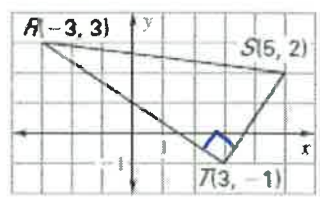
1) Use the distance formula to find the length of each side of the triangle.
   - If no sides are congruent, the triangle is scalene.
   - If two sides are congruent, the triangle is isosceles.
   - If all three sides are congruent, the triangle is equilateral.

2) Use the slope formula to determine if any sides are perpendicular to determine if the triangle is a right triangle.
   - **IF** the triangle **IS** a right triangle, the right angle will always be opposite the longest side, so...

     find the slopes of the two shortest sides

Example: Classify ΔRST by its side lengths. Then determine if the triangle is a right triangle.

Step 1: Use distance formula to find the side lengths:



$RS = \sqrt{(5+3)^2+(2-3)^2} = \sqrt{(8)^2+(-2)^2} = \sqrt{64+1} = \sqrt{65}$

$ST = \sqrt{(3-5)^2+(-1-2)^2} = \sqrt{(-2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$

$TR = \sqrt{(3+3)^2+(-1-3)^2} = \sqrt{(6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

The triangle has no congruent *sides, so it is* Scalene.

Step 2: Use slopes to determine if there is a right angle. The two shortest sides are $\overline{ST}$ and $\overline{RT}$ so find their slopes. $m_{\overline{ST}} = \dfrac{-1-2}{3-5} = \dfrac{-3}{-2} = \boxed{\dfrac{3}{2}}$ $m_{\overline{RT}} = \dfrac{-1-3}{3+3} = \dfrac{-4}{6} = \boxed{-\dfrac{2}{3}}$

opp. reciprocal slope

$\overline{ST}$ and $\overline{RT}$ are/are not perpendicular, therefore $\angle$ RTS is/is not a right angle and ΔRST is/ is not a Right triangle.

Solution: ΔRST *is* a scalene right triangle

Think you got it? Great! Try a couple on your own ☺

1) The vertices of ΔXYZ are $X(-2,3)$, $Y(-2,-7)$, and $Z(4,-5)$.

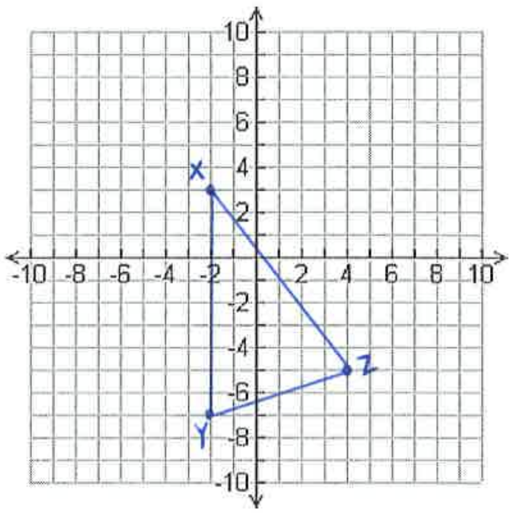
Classify ΔXYZ by its side lengths, then determine if the triangle is a right triangle.

$XY = \sqrt{(-2+2)^2+(-7-3)^2} = \sqrt{(0)^2+(-10)^2} = \sqrt{100} = 10$

$YZ = \sqrt{(4+2)^2+(-5+7)^2} = \sqrt{(6)^2+(2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$XZ = \sqrt{(4+2)^2+(-5-3)^2} = \sqrt{(6)^2+(-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$

Since two sides are $\cong$, ΔXYZ is isosceles.



$m_{\overline{XY}} = \frac{-7-3}{-2+2} = \frac{-10}{0} =$ undefined

$m_{\overline{YZ}} = \frac{-5+7}{4+2} = \frac{2}{6} = \frac{1}{3}$

$m_{\overline{XZ}} = \frac{-5-3}{4+2} = \frac{-8}{6} = -\frac{4}{3}$

No slopes are opp. reciprocals so ΔXYZ is not a right Δ

$\boxed{\Delta XYZ \text{ is an isosceles } \Delta}$

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2) The vertices of ΔPQR are $P(-3,-1)$, $Q(-4,4)$, and $R(7,1)$.

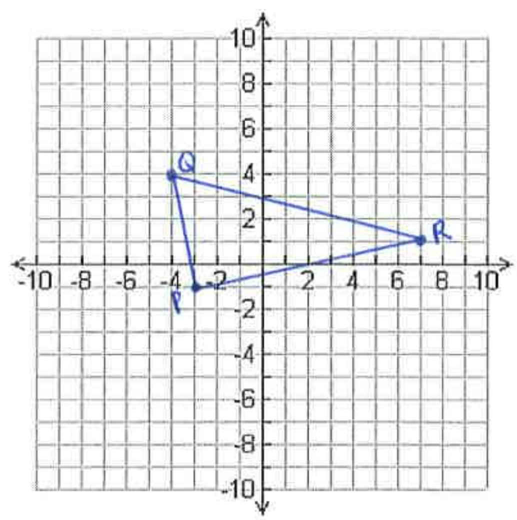
Classify ΔXYZ by its side lengths, then determine if the triangle is a right triangle.

$PQ = \sqrt{(-4+3)^2+(4+1)^2} = \sqrt{(-1)^2+(5)^2} = \sqrt{1+25} = \sqrt{26}$

$QR = \sqrt{(7+4)^2+(1-4)^2} = \sqrt{(11)^2+(-3)^2} = \sqrt{121+9} = \sqrt{130}$

$PR = \sqrt{(7+3)^2+(1+1)^2} = \sqrt{(10)^2+(2)^2} = \sqrt{100+4} = \sqrt{104}$

Since no sides are $\cong$, ΔPQR is scalene.



$m_{\overline{PQ}} = \frac{4+1}{-4+3} = \frac{5}{-1} = -5$

$m_{\overline{PR}} = \frac{1+1}{7+3} = \frac{2}{10} = \frac{1}{5}$

Since the two shortest sides have opp. reciprocal slopes, ΔPQR is a right Δ.

$\boxed{\Delta PQR \text{ is a scalene right } \Delta}$