

Slope Criterion for Parallel Lines

Two non-vertical lines are parallel if and only if they have
the same slope.

Vertical lines are always parallel. $\updownarrow \updownarrow$

Example 1: Write equations of parallel lines.

- a) Write an equation of the line passing through the point $(3, 4)$ that is parallel to the line with equation $y = -4x + 5$.

old slope: -4 new slope: -4

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -4(x - 3)$$

$$y - 4 = -4x + 12$$

$$y = -4x + 16$$

- b) Write the equation of the line that passes through $(3, 5)$ and is parallel to the line that passes through $(3, 3)$ and $(-3, -1)$

old slope = $\frac{-1-3}{-3-3} = \frac{-4}{-6} = \frac{2}{3}$ new slope = $\frac{2}{3}$

$$y - 5 = \frac{2}{3}(x - 3)$$

$$y - 5 = \frac{2}{3}x - \frac{6}{3}$$

$$y - 5 = \frac{2}{3}x - 2 \Rightarrow y = \frac{2}{3}x + 3$$

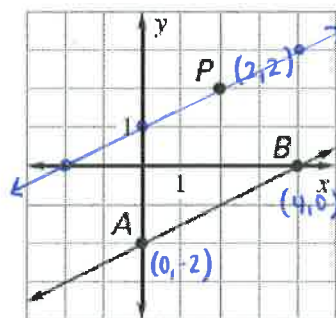
- c) Graph the line parallel to line AB that passes through point P and write its equation.

slope of $\overleftrightarrow{AB} = \frac{-2-0}{0-4} = \frac{-2}{-4} = \frac{1}{2}$ new slope = $\frac{1}{2}$

$$y - 2 = \frac{1}{2}(x - 2)$$

$$y - 2 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x + 1$$

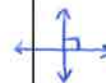


Slope Criterion for Perpendicular Lines

Two non-vertical lines are perpendicular if and only if

they have opposite reciprocal slopes.

Vertical lines and horizontal lines are always perpendicular.



Example 2: Write equations of perpendicular lines.

- a) Write an equation of the line passing through the point $(6, -3)$ that is perpendicular to the line with equation $y = 4x - 7$.

old slope = 4 new slope = $-\frac{1}{4}$

$$y + 3 = -\frac{1}{4}(x - 6)$$

$$y + 3 = -\frac{1}{4}x + \frac{6}{4}$$

$$y + 3 = -\frac{1}{4}x + \frac{3}{2}$$

$$y + \frac{6}{2} = -\frac{1}{4}x + \frac{3}{2}$$

$$y = -\frac{1}{4}x - \frac{3}{2}$$

- b) Write the equation of the line that passes through $(-2, 3)$ and is perpendicular to the line that passes through $(0, 1)$ and $(-3, -1)$

old slope = $\frac{-1 - 1}{-3 - 0} = \frac{-2}{-3} = \frac{2}{3}$

new slope = $-\frac{3}{2}$

$$y - 3 = -\frac{3}{2}(x + 2)$$

$$y - 3 = -\frac{3}{2}x - \frac{6}{2}$$

$$y - 3 = -\frac{3}{2}x - 3$$

$$y = -\frac{3}{2}x$$

- c) Graph the line perpendicular to line AB that passes through point P and write its equation.

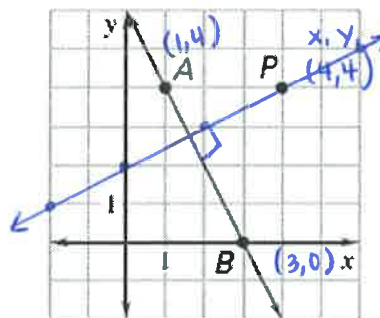
slope of $AB = \frac{4 - 0}{1 - 3} = \frac{4}{-2} = -2$

new slope = $\frac{1}{2}$

$$y - 4 = \frac{1}{2}(x - 4)$$

$$y - 4 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x + 2$$



Equations of Lines

Slope-intercept form: $y = mx + b$

Point – slope form: $y - y_1 = m(x - x_1)$

Standard form: $ax + by = c$

Example 3: Rewrite standard form in slope-intercept form.

Given $2x + 3y = 18$, rewrite the equation in slope-intercept form. Identify the slope and y-intercept.

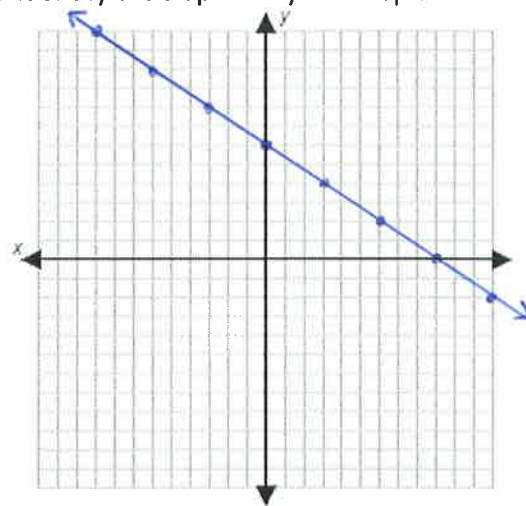
Then graph the line.

$$3y = 18 - 2x$$

$$y = \frac{18}{3} - \frac{2}{3}x$$

$$y = -\frac{2}{3}x + 6$$

$$m = -\frac{2}{3}, b = 6$$



Can we graph an equation from standard form without rewriting it in slope-intercept form first? Of course we can!!! We can use the x-intercept and y-intercept.

Example 8: Graph a line with the equation in standard form.

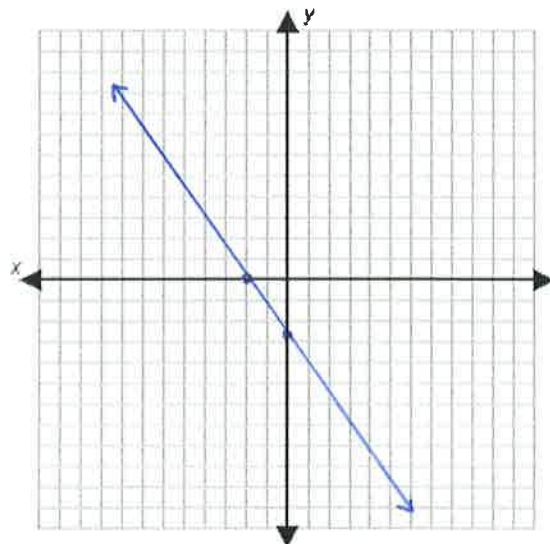
a) Given $7x + 5y = -14$, graph the line using intercepts.

To find the x-intercept, let $y = 0$, then solve for x .

To find the y-intercept, let $x = 0$, then solve for y .

$$\begin{aligned} \text{x-int: } 7x + 5(0) &= -14 \\ 7x &= -14 \\ x &= -2 \end{aligned} \Rightarrow (-2, 0)$$

$$\begin{aligned} \text{y-int: } 7(0) + 5y &= -14 \\ 5y &= -14 \Rightarrow y = -14/5 \Rightarrow (0, -2.8) \end{aligned}$$



b) Graph $3(y-2) = 5x - 12$ and $10x - 6y = 12$ on the same coordinate plane. Then use the graph to estimate how many solutions the equations share.

a. $3y - 6 = 5x - 12$

$$3y - 5x = -6$$

x-int: $3(0) - 5x = -6$

$$0 - 5x = -6 \Rightarrow (6/5, 0)$$
$$x = 6/5$$

y-int: $3y - 5(0) = -6$

$$3y - 0 = -6 \Rightarrow (0, -2)$$

$$3y = -6$$

$$y = -2$$

b. $10x - 6y = 12$

y-int: $10(0) - 6y = 12$

$$-6y = 12$$

$$y = -2$$

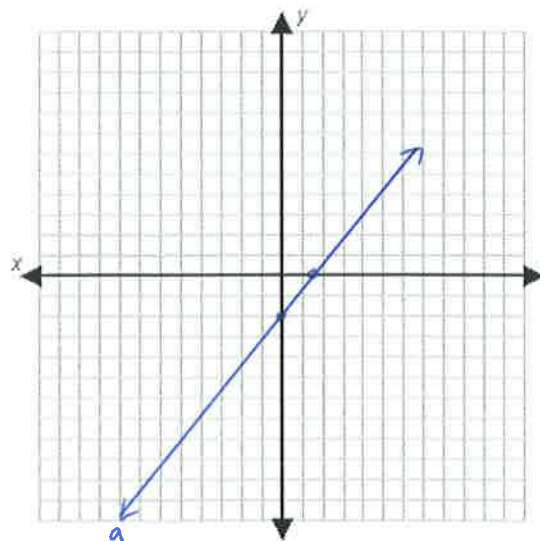
$$(0, -2)$$

x-int: $10x - 6(0) = 12$

$$10x = 12$$

$$x = \frac{12}{10} = \frac{6}{5}$$

$$(6/5, 0)$$



Same line \Rightarrow
Infinitely many
solutions