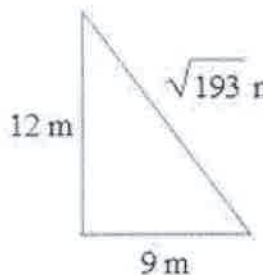


2015-2016 Geometry Review for Final Exam

Unit 5 Right Triangles

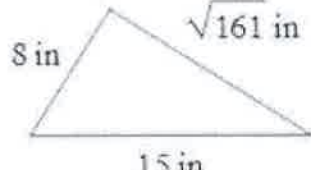
For #s 1-4, determine if the triangle is acute, right, or obtuse.

1) $(\sqrt{193})^2 - 12^2 + 9^2$
 $193 \leq 225$



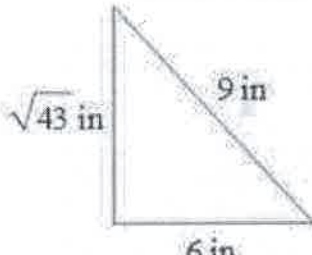
acute Δ

2) $(15)^2 - (\sqrt{161})^2 + (8)^2$
 $225 = 225$



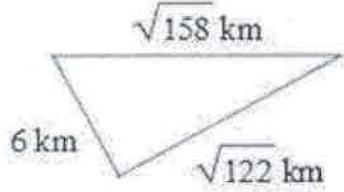
right Δ

3) $9^2 - (\sqrt{43})^2 + 6^2$
 $81 \geq 79$



obtuse Δ

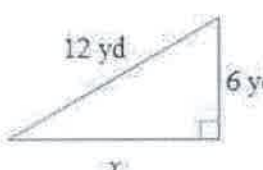
4) $(\sqrt{158})^2 - (\sqrt{122})^2 + 6^2$
 $158 = 158$



right Δ

For #5-14, find the missing side or sides of the triangle. Leave your answer in simplest radical form.

5)



$$x^2 + 6^2 = 12^2$$

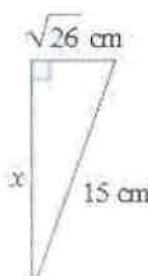
$$x^2 + 36 = 144$$

$$x^2 = 108$$

$$x = \sqrt{108}$$

$$x = \sqrt{36 \cdot 3} \Rightarrow \boxed{x = 6\sqrt{3}}$$

6)



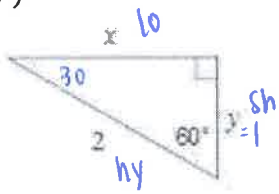
$$x^2 + (\sqrt{26})^2 = 15^2$$

$$x^2 + 26 = 225$$

$$x^2 = 199$$

$$\boxed{x = \sqrt{199}}$$

7)



$$\text{hyp} = \text{short} \cdot 2$$

$$2 = y \cdot 2$$

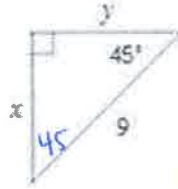
$$y = 1$$

$$\text{long} = \text{short} \cdot \sqrt{3}$$

$$x = 1 \cdot \sqrt{3}$$

$$x = \sqrt{3}$$

8)



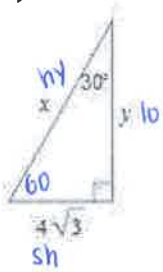
$$\text{hyp} = \text{leg} \cdot \sqrt{2}$$

$$9 = x \sqrt{2}$$

$$x = \frac{9 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{9\sqrt{2}}{2}$$

$$x = y = \frac{9\sqrt{2}}{2}$$

9)



$$\text{hyp} = \text{short} \cdot 2$$

$$x = 4\sqrt{3} \cdot 2$$

$$x = 8\sqrt{3}$$

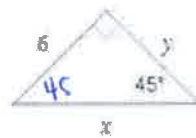
$$\text{long} = \text{short} \cdot \sqrt{3}$$

$$y = 4\sqrt{3} \cdot \sqrt{3}$$

$$y = 4 \cdot 3$$

$$y = 12$$

10)

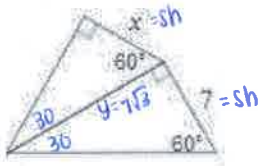


$$y = 6$$

$$\text{hyp} = \text{leg} \cdot \sqrt{2}$$

$$x = 6\sqrt{2}$$

11)



$$\text{long} = \text{short} \cdot \sqrt{3}$$

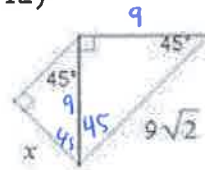
$$y = 7\sqrt{3}$$

$$\text{hyp} = \text{short} \cdot 2$$

$$\frac{7\sqrt{3}}{2} = \frac{x \cdot 2}{2}$$

$$x = \frac{7\sqrt{3}}{2}$$

12)

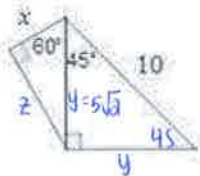


$$\text{hyp} = \text{leg} \cdot \sqrt{2}$$

$$9 = x \sqrt{2}$$

$$x = \frac{9 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{9\sqrt{2}}{2} = x$$

13)



$$\text{hyp} = \text{leg} \cdot \sqrt{2}$$

$$10 = y \sqrt{2}$$

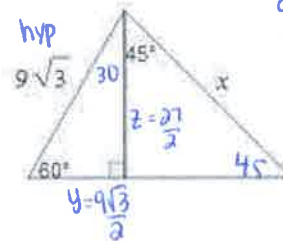
$$y = \frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$\text{hyp} = \text{short} \cdot 2$$

$$5\sqrt{2} = \frac{x \cdot 2}{2}$$

$$x = \frac{5\sqrt{2}}{2}$$

14)



$$\text{hyp} = \text{short} \cdot 2$$

$$\frac{9\sqrt{3}}{2} = \frac{y \cdot 2}{2}$$

$$y = \frac{9\sqrt{3}}{2}$$

$$\text{long} = \text{short} \cdot \sqrt{3}$$

$$z = \frac{9\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{1}$$

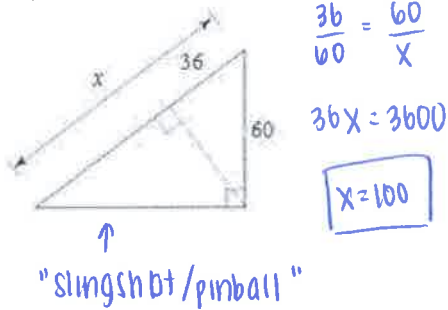
$$= \frac{9 \cdot 3}{2} = \frac{27}{2}$$

$$\text{hyp} = \text{leg} \cdot \sqrt{2}$$

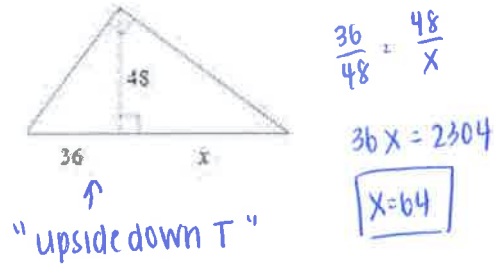
$$x = \frac{27\sqrt{2}}{2}$$

Find the missing sides.

15)

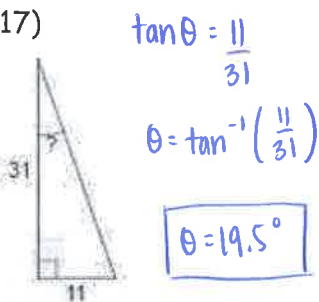


16)

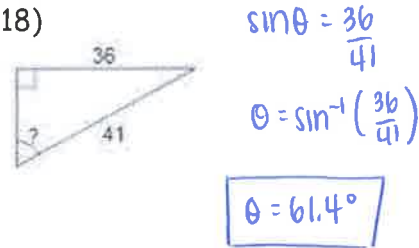


Find the indicated angle to the nearest degree.

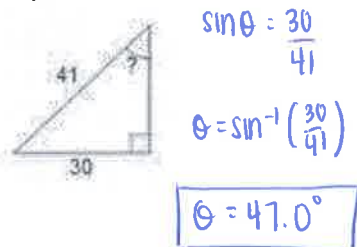
17)



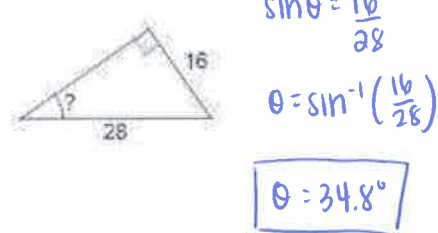
18)



19)

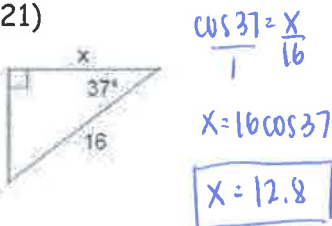


20)

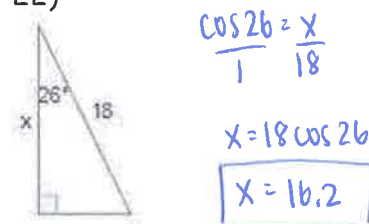


Find the missing side. Round to the nearest tenth.

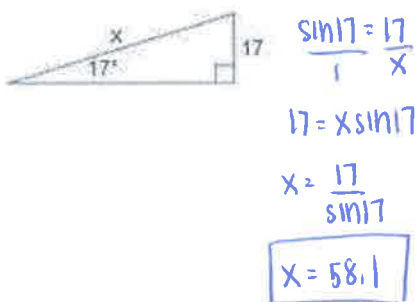
21)



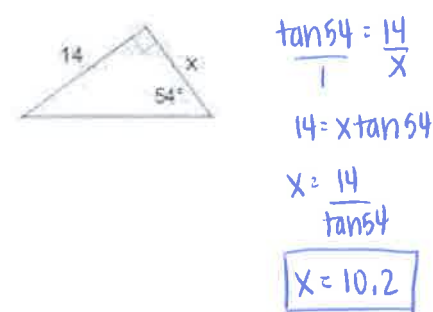
22)



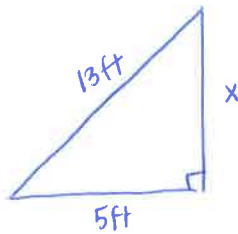
23)



24)



25) **Ladder** A ladder is leaning against a house. The ladder is 13 feet long and the foot of the ladder is 5 feet away from the base of the house. How far up the side of the house does the ladder reach?



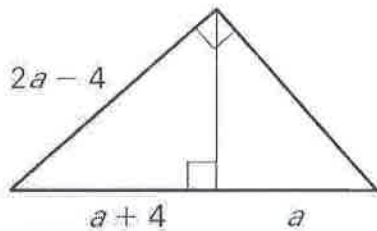
$$x^2 + 5^2 = 13^2$$

$$x^2 + 25 = 169$$

$$x^2 = 144$$

$$\boxed{x = 12 \text{ ft}}$$

26) Solve for a.



$$\frac{a+4}{2a-4} = \frac{2a-4}{2a+4}$$

$$(2a-4)(2a-4) = (a+4)(2a+4)$$

$$4a^2 - 8a - 8a + 16 = 2a^2 + 4a + 8a + 16$$

$$4a^2 - 16a + 16 = 2a^2 + 12a + 16$$

$$2a^2 - 28a = 0$$

$$2a(a-14) = 0$$

$$2a = 0 \quad a - 14 = 0$$

$$a = 0 \quad \boxed{a = 14}$$

no zero side lengths

Decide if the segment lengths form a triangle. If so, would the triangle be acute, right, or obtuse.

27) 7, 9, $\sqrt{130}$

$$(\sqrt{130})^2 = 7^2 + 9^2$$

$$130 = 130$$

$\boxed{\text{right } \Delta}$

28) 5, 13, 20

$$5 + 13 > 20 \leftarrow \text{NO}$$

$\boxed{\text{not a } \Delta}$

29) 13, 19, 29

$$29^2 = 13^2 + 19^2$$

$$841 \geq 630$$

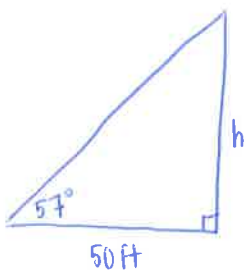
$\boxed{\text{obtuse } \Delta}$

$$13 + 19 > 29 \checkmark$$

$$13 + 29 > 19 \checkmark$$

$$19 + 29 > 13 \checkmark$$

30) **Tree height** A biologist is standing 50 feet from the base of a large oak tree. The biologist measures the angle of elevation of the tree to be 57° . Find the height h of the oak tree to the nearest foot.



$$\frac{\tan 57^\circ = h}{1 \quad 50}$$

$$h = 50 \tan 57^\circ$$

$$\boxed{h = 77 \text{ ft}}$$

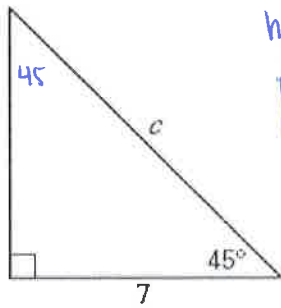
$$7 + 9 > 11.4 \checkmark$$

$$7 + 11.4 > 9 \checkmark$$

$$9 + 11.4 > 7 \checkmark$$

Find the value of each variable. Write your answers in simplest radical form.

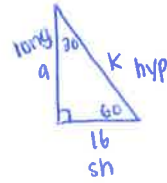
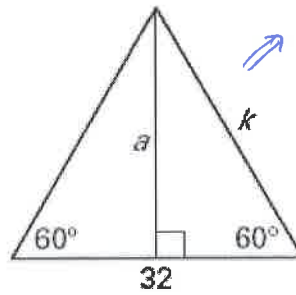
31)



$$\text{hyp} = \text{leg} \cdot \sqrt{2}$$

$$c = 7\sqrt{2}$$

32)



$$\text{hyp} = \text{short} \cdot 2$$

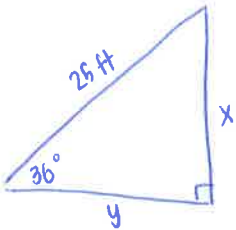
$$k = 16 \cdot 2$$

$$k = 32$$

$$\text{long} = \text{short} \cdot \sqrt{3}$$

$$a = 16\sqrt{3}$$

33) You need to determine the area of a garden which is in the shape of a right triangle. The hypotenuse measures 25 feet and one of the acute angles measures 36° . Round your answer to the nearest tenth.



$$\frac{\sin 36}{1} = \frac{x}{25}$$

$$x = 25 \sin 36$$

$$x = 14.69$$

$$\frac{\cos 36}{1} = \frac{y}{25}$$

$$y = 25 \cos 36$$

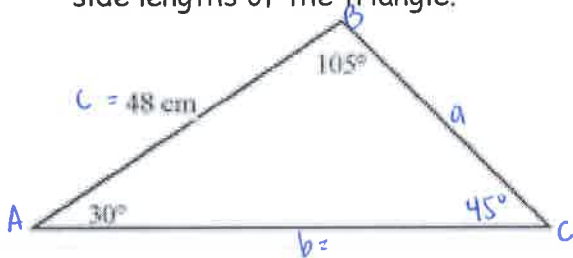
$$y = 20.23$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(14.69)(20.23)$$

$$A = 148.59 \text{ ft}^2$$

34) Use the Law of Sines to find the missing side lengths of the triangle.



$$\frac{\sin 30}{a} = \frac{\sin 105}{b} = \frac{\sin 45}{48}$$

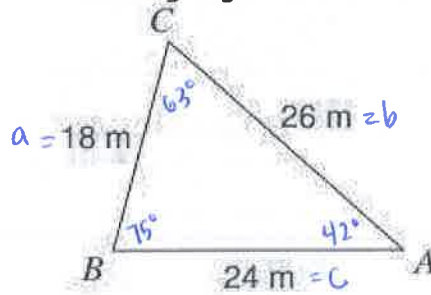
$$48 \sin 105 = b \sin 45$$

$$b = 65.6$$

$$48 \sin 30 = a \sin 45$$

$$a = 33.9$$

35) Use the Law of Cosines to find the missing angle measures.



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$26^2 = 18^2 + 24^2 - 2(18)(24) \cos B$$

$$676 = 900 - 864 \cos B$$

$$-224 = -864 \cos B$$

$$0.2593 = \cos B$$

$$m\angle B = 75^\circ$$

$$\frac{\sin A}{18} = \frac{\sin 75}{26}$$

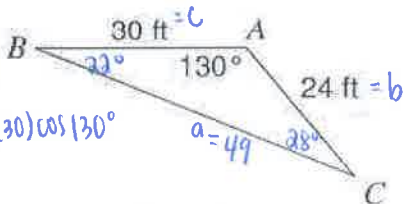
$$18 \sin 75 = 26 \sin A$$

$$0.6687 = \sin A$$

$$m\angle A = 42^\circ$$

$$m\angle C = 63^\circ$$

36) Solve the triangle.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 24^2 + 30^2 - 2(24)(30) \cos 130^\circ$$

$$a^2 = 2401.61$$

$$a = 49$$

$$\frac{\sin 130}{49} = \frac{\sin B}{24}$$

$$24 \sin 130 = 49 \sin B$$

$$0.3752 = \sin B$$

$$m\angle B = 22^\circ$$

$$m\angle C = 28^\circ$$

Unit 5.5 Transformations

37) Given point A is located at (1, 3), what is the final image of A after this series of transformations?

(1) Reflect A across the y-axis

(2) Translate the image such that $(x, y) \rightarrow (x - 4, y + 2)$

(1) y axis: $(x, y) \rightarrow (-x, y)$

(2) $(x, y) \rightarrow (x - 4, y + 2)$

$(1, 3) \rightarrow (-1, 3)$

$(-1, 3) \rightarrow (-1 - 4, 3 + 2)$

$A'(-1, 3)$

$A''(-5, 5)$

38) Graph $\triangle LMN$ with vertices L(3, -1), M(1, -5), and N(4, -3).

a. What are the coordinates of L', M', and N' after a counterclockwise rotation of 180° ?

b. Take the image from part a and perform the translation $(x, y) \rightarrow (x - 3, y - 4)$. What are the coordinates of L'', M'' and N''?

(a) Rotate 180° : $(x, y) \rightarrow (-x, -y)$

(b) $(x, y) \rightarrow (x - 3, y - 4)$

L (3, -1) \rightarrow L'(-3, 1)

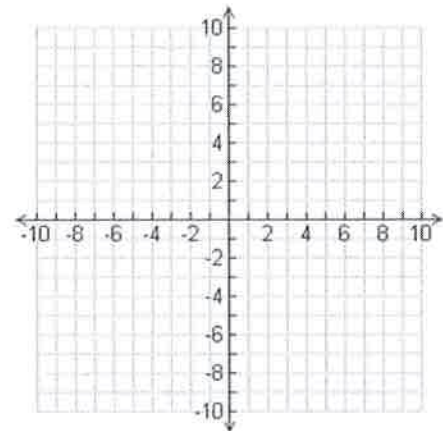
L'(-3, 1) \rightarrow L''(-6, -3)

M (1, -5) \rightarrow M'(-1, 5)

M'(-1, 5) \rightarrow M''(-4, 1)

N (4, -3) \rightarrow N'(-4, 3)

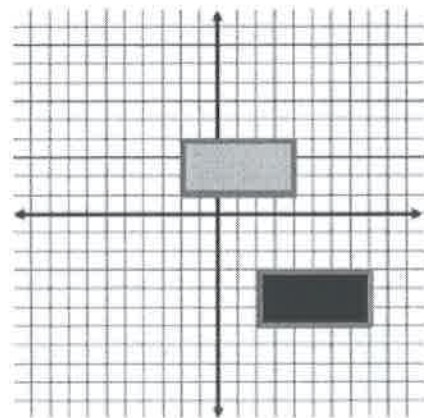
N'(-4, 3) \rightarrow N''(-7, -1)



39) What transformation will map the darker figure onto the lighter figure? Give the proper notation.

Dark \rightarrow Light: left 4 units, up 7 units

$(x, y) \rightarrow (x - 4, y + 7)$



40) The point A(4, 3) is rotated 90 degrees counterclockwise about the origin. What are the coordinates of the image?

Rotate 90° ccw: $(x, y) \rightarrow (-y, x)$

A(4, 3) \rightarrow A'(-3, 4)

41) Identify the coordinates of $(10, -20)$ after a 270° counter-clockwise rotation about the origin.

270° Rotation CCW: $(x, y) \rightarrow (y, -x)$
 $(10, -20) \rightarrow \boxed{(-20, -10)}$

42) Graph $\triangle ABC$ after a composition of the transformations in the order they are listed. You may use the grid below to help you find the coordinates of the images.

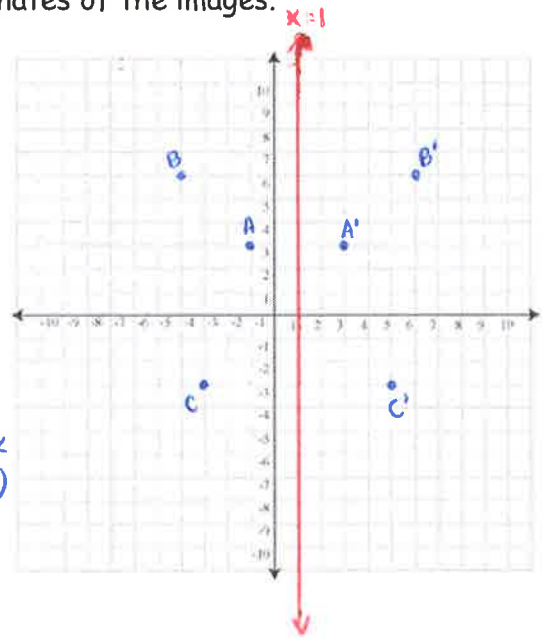
$A(-1, 3), B(-4, 6), C(-3, -3)$

a) Reflection in the line $x = 1$

$A' \underline{(3, 3)}$ $B' \underline{(6, 6)}$ $C' \underline{(5, -3)}$

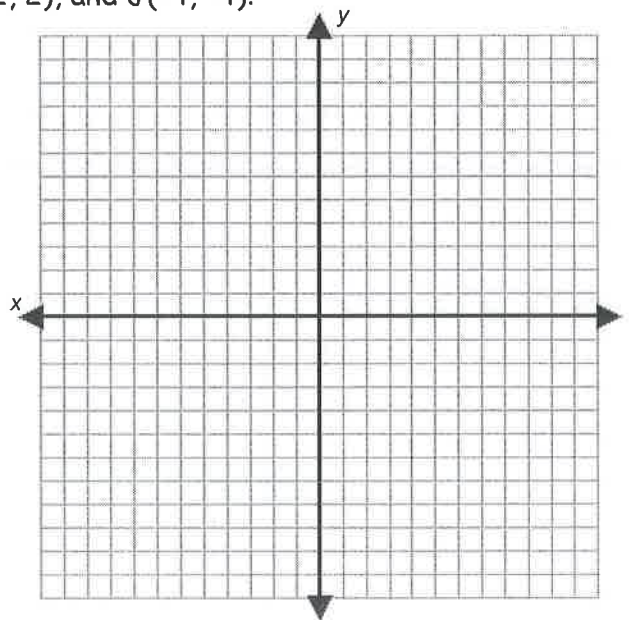
b) Rotation 270° counterclockwise about the point $(-2, 0)$

$A'' \underline{(1, -5)}$ $B'' \underline{(4, -8)}$ $C'' \underline{(-5, -7)}$ $(x, y) \rightarrow (y, -x)$
 $x+a$ to get back to $(0, 0)$



43) Find the coordinates of the dilation image of $\triangle GHJ$ centered at the point $(2, 4)$ with a scale factor of $\frac{1}{2}$ given coordinates $G(-8, 2), H(-2, 2),$ and $J(-4, -4)$.

You can skip this one; its a dilation and we did those before midterms :)



WORK for #42b.

$A'(3, 3) \xrightarrow{x+2} (5, 3) \xrightarrow{270^\circ} (3, -5) \xrightarrow{x-2} A''(1, -5)$

$B'(6, 6) \xrightarrow{x+2} (8, 6) \xrightarrow{270^\circ} (6, -8) \xrightarrow{x-2} B''(4, -8)$

$C'(5, -3) \xrightarrow{x+2} (7, -3) \xrightarrow{270^\circ} (-3, -7) \xrightarrow{x-2} C''(-5, -7)$

Unit 6 Polygons

$$\uparrow (n-2) \cdot 180$$

Find the sum of the measures of the interior angles of the indicated convex polygon.

37) 13-gon

$$(13-2) \cdot 180$$

$$= \boxed{1980^\circ}$$

38) Octagon

$$(8-2) \cdot 180$$

$$= \boxed{1080^\circ}$$

39) 22-gon

$$(22-2) \cdot 180$$

$$= \boxed{3600^\circ}$$

40) Pentagon

$$(5-2) \cdot 180$$

$$= \boxed{540^\circ}$$

41) In the figure at the right,

a.) What is the value of x ?

(Section 8.1)

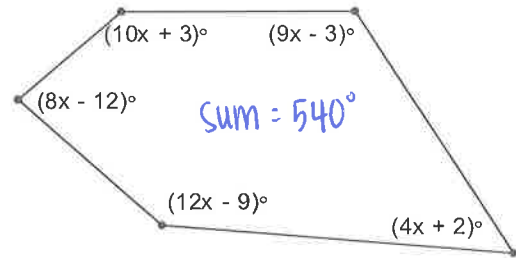
$$43x - 19 = 540$$

$$43x = 559$$

$$\boxed{x = 13}$$

b.) Find the sum of the measures of the exterior angles, one at each vertex.

$$\boxed{360^\circ}$$



Find the measure of each exterior angle in the following regular polygon.

$$\uparrow \frac{360}{n}$$

42) Quadrilateral

$$\frac{360}{4} = \boxed{90^\circ}$$

43) Octagon

$$\frac{360}{8} = \boxed{45^\circ}$$

Find the value of n for each regular n -gon described.

$$\frac{(n-2) \cdot 180}{n}$$

44) Each interior angle of the regular n -gon has a measure of 165° .

$$\frac{165}{1} = \frac{(n-2) \cdot 180}{n}$$

$$-15n = -360$$

$$\boxed{n=24}$$

$$165n = 180n - 360$$

$$\frac{360}{n}$$

45) Each exterior angle of the regular n -gon has a measure of 60° .

$$\frac{60}{1} = \frac{360}{n}$$

$$360 = 60n \Rightarrow \boxed{n=6}$$

46) The sum of the interior angles of the regular n -gon is 2700° .

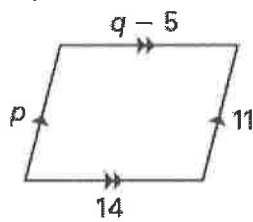
$$\frac{(n-2) \cdot 180}{180}$$

$$\frac{2700}{180} = \frac{(n-2) \cdot 180}{180}$$

$$15 = n - 2 \Rightarrow \boxed{n=17}$$

Find the values of each variable in the parallelogram.

47)

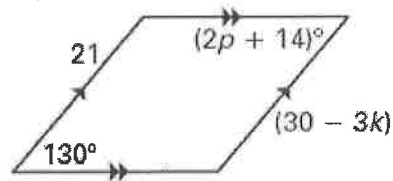


$$\boxed{p=11}$$

$$q-5=14$$

$$\boxed{q=19}$$

48)



$$2p+14=130$$

$$2p=116$$

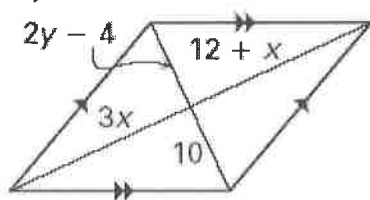
$$\boxed{p=58}$$

$$21=30-3k$$

$$-9=-3k$$

$$\boxed{k=3}$$

49)



$$3x=12+x$$

$$2x=12$$

$$\boxed{x=6}$$

$$2y-4=10$$

$$2y=14$$

$$\boxed{y=7}$$

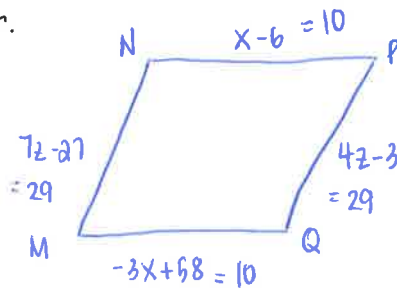
50) The sides of $\square MNPQ$ are represented by the expressions below. Sketch $\square MNPQ$ and find its perimeter.

$$MQ = -3x + 58$$

$$QP = 4z - 3$$

$$NP = x - 6$$

$$MN = 7z - 27$$



$$x - 6 = -3x + 58$$

$$4x = 64$$

$$x = 16$$

$$7z - 27 = 4z - 3$$

$$3z = 24$$

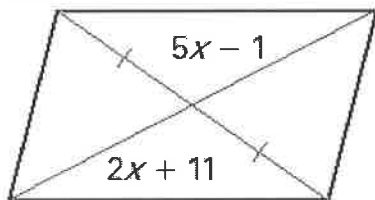
$$z = 8$$

$$P = 10 + 10 + 29 + 29$$

$$P = 78 \text{ units}$$

For what value of x is the quadrilateral a parallelogram?

51)

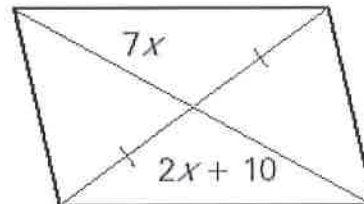


$$5x - 1 = 2x + 11$$

$$3x = 12$$

$$x = 4$$

52)

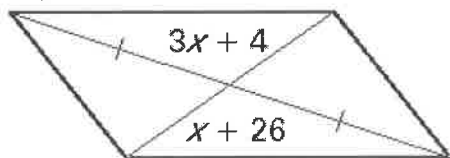


$$7x = 2x + 10$$

$$5x = 10$$

$$x = 2$$

53)



$$3x + 4 = x + 26$$

$$2x = 22$$

$$x = 11$$

The diagonals of rhombus $ABCD$ intersect at E . Given that $m\angle BAC = 50^\circ$, $AD = 13$, and $DE = 10$, find the indicated measure.

54) $m\angle ABE = 40^\circ$

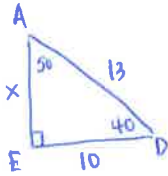
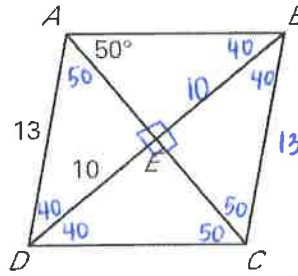
55) $m\angle DEC = 90^\circ$

56) $DB = 10 + 10 = 20$

57) $AE = \sqrt{69} = 8.3$

58) $m\angle DAC = 50^\circ$

59) $BC = 13$



$$x^2 + 10^2 = 13^2$$

$$x^2 + 100 = 169$$

$$x^2 = 69$$

$$x = \sqrt{69}$$

60) Use the diagram of the parallelogram $MNOP$ at the right to complete each statement.

Explain.

a) $\overline{MN} \cong \overline{OP}$

e) $\overline{MN} \parallel \overline{OP}$

b) $\overline{ON} \cong \overline{MP}$

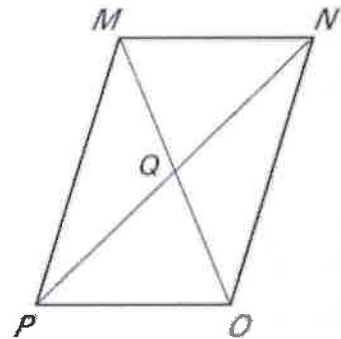
f) $\angle MQN \cong \angle OQP$

c) $\overline{PQ} \cong \overline{NQ}$

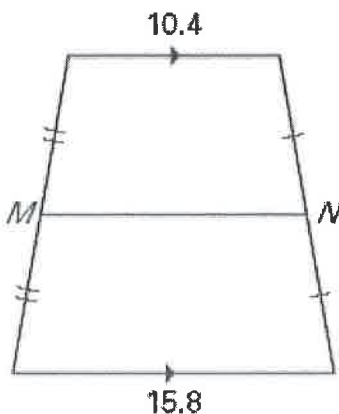
g) $\overline{MQ} \cong \overline{OQ}$

d) $\angle MQN \cong \angle OQP$

h) $\angle NPO \cong \angle PNM$



61) Find the length of the midsegment of the trapezoid.



$$MN = \frac{1}{2} (10.4 + 15.8)$$

$$MN = \frac{1}{2} (26.2)$$

$$MN = 13.1$$

Points P , Q , R , and S are the vertices of a quadrilateral. Give the most specific name for $PQRS$. Justify your answer using distance formula, slope formula, midpoint formula.

$$62) P(0, -1), Q(2, -1), R(5, -6), S(0, -3)$$

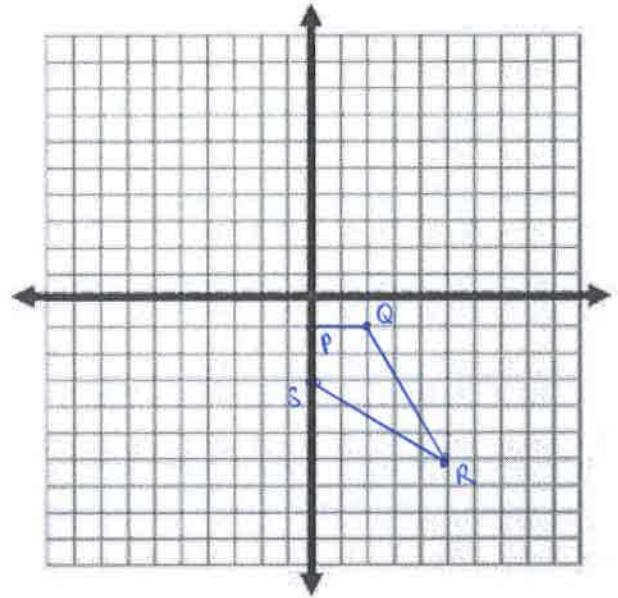
$$PS = \sqrt{(0-0)^2 + (-1+3)^2} = \sqrt{(2)^2} = \sqrt{4} = 2$$

$$PQ = \sqrt{(2-0)^2 + (-1+1)^2} = \sqrt{(2)^2} = \sqrt{4} = 2$$

$$QR = \sqrt{(5-2)^2 + (-6+1)^2} = \sqrt{(3)^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

$$SR = \sqrt{(5-0)^2 + (-6+3)^2} = \sqrt{(5)^2 + (-3)^2} = \sqrt{25+9} = \sqrt{34}$$

Since two pairs of consecutive sides are congruent ($\overline{PS} \cong \overline{PQ}$ and $\overline{QR} \cong \overline{SR}$), then quad $PQRS$ is a Kite



$$63) P(-2, 4), Q(-6, 6), R(-9, 3), S(-5, 1)$$

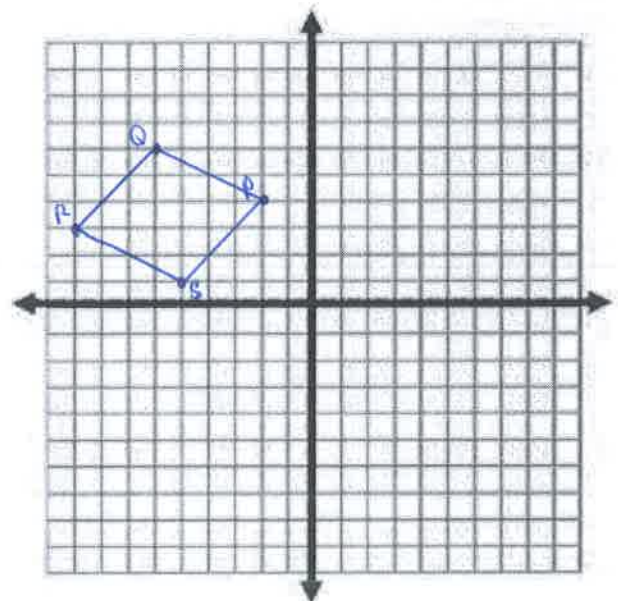
$$QR = \sqrt{(-6+9)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$PS = \sqrt{(-2+5)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$QP = \sqrt{(-2+6)^2 + (4-6)^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$RS = \sqrt{(-5+9)^2 + (1-3)^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Since both pairs of opposite sides are congruent, ($\overline{QR} \cong \overline{PS}$ and $\overline{QP} \cong \overline{RS}$), quad $PQRS$ is a parallelogram.



64) Graph the points $A(3, 2), B(1, -2), C(2, -5), D(4, -1)$ on the coordinate plane. What kind of quadrilateral is $ABCD$? Use mathematics to justify your answer.

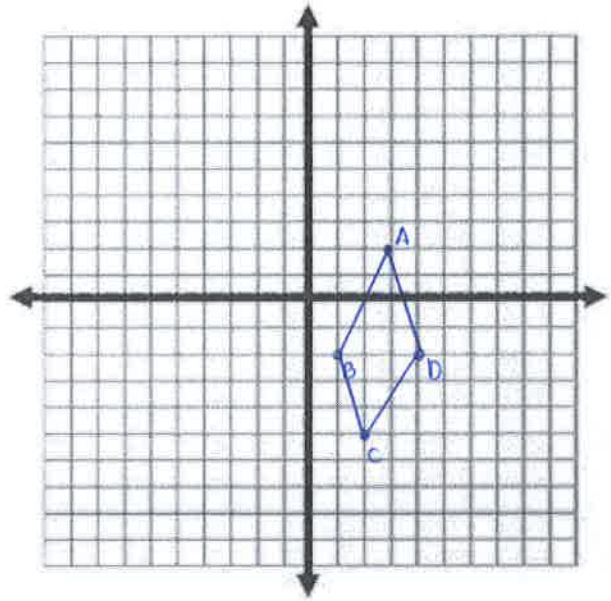
$$AB = \sqrt{(3-1)^2 + (2+2)^2} = \sqrt{(2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$CD = \sqrt{(4-2)^2 + (-1+5)^2} = \sqrt{(2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$AD = \sqrt{(4-3)^2 + (-1-2)^2} = \sqrt{(1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$$

$$BC = \sqrt{(2-1)^2 + (-5+2)^2} = \sqrt{(1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$$

Since both pairs of opposite sides are congruent, ($\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$), quadrilateral $ABCD$ is a parallelogram



Sketch a quadrilateral having the characteristics described. Mark any required congruencies.

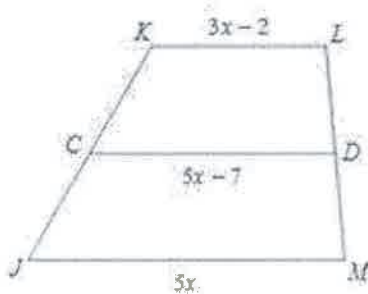
65) A rhombus that is not a square.



66) A kite



67) Find the length of the midsegment of the trapezoid.



$$5x-7 = \frac{1}{2}(3x-2+5x)$$

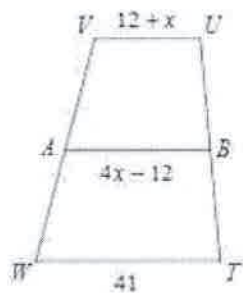
$$5x-7 = \frac{1}{2}(8x-2)$$

$$5x-7 = 4x-1$$

$$\boxed{x=6}$$

$$\text{mid} = 5(6)-7 = \boxed{23}$$

68) Find UV



$$4x - 12 = \frac{1}{2}(12 + x + 41)$$

$$4x - 12 = \frac{1}{2}(x + 53)$$

$$8x - 24 = x + 53$$

$$7x = 77$$

$$\boxed{x = 11}$$

$$UV = 12 + 11$$

$$\boxed{UV = 23}$$

Find the measurement of the angle indicated for each trapezoid.

69) Find $m\angle L$



$$9x - 19 = 7x + 3$$

$$m\angle K = 99 - 19 = 80$$

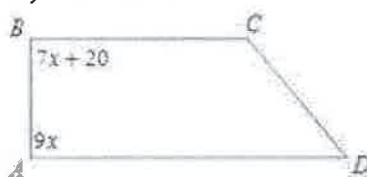
$$2x = 22$$

$$m\angle L = 180 - 80$$

$$\boxed{x = 11}$$

$$\boxed{m\angle L = 100^\circ}$$

70) Find $m\angle A$



$$9x + 7x + 20 = 180$$

$$m\angle A = 9(10)$$

$$16x + 20 = 180$$

$$\boxed{m\angle A = 90^\circ}$$

$$16x = 160$$

$$x = 10$$

71) Find the length of EG if $EG = 4x - 31$ and $DF = 3x - 19$.



$$4x - 31 = 3x - 19$$

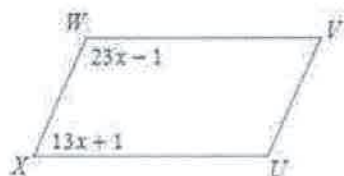
$$x = 12$$

$$EG = 4(12) - 31$$

$$\boxed{EG = 17}$$

Find the measurement indicated in each parallelogram.

72) Find $m\angle W$



$$23x - 1 + 13x + 1 = 180$$

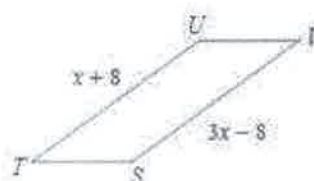
$$m\angle W = 23(5) - 1$$

$$36x = 180$$

$$\boxed{m\angle W = 114^\circ}$$

$$x = 5$$

73) Find TU



$$x + 8 = 3x - 8$$

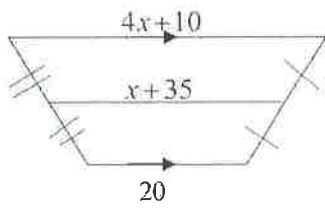
$$TU = 8 + 8$$

$$16 = 2x$$

$$\boxed{x = 8}$$

$$\boxed{TU = 16}$$

74) Find the value of x in the figure



$$x+35 = \frac{1}{2}(4x+10+20)$$

$$x+35 = \frac{1}{2}(4x+30)$$

$$x+35 = 2x+15$$

$$\boxed{20=x}$$

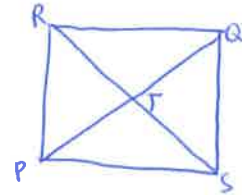
75) Quadrilateral PQRS has diagonals \overline{PQ} and \overline{RS} that intersect at point T. For the conditions given below, state the quadrilateral which they best describe. If none, write "none".

a. $\overline{PS} \parallel \overline{QR}$, $\overline{PS} \cong \overline{QR}$ *Isosceles trapezoid*

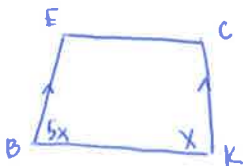
b. PQRS is a parallelogram, $\overline{PR} \perp \overline{QS}$ *Rhombus*

c. PQRS is a parallelogram, $\overline{PR} \cong \overline{QS}$, $\overline{PR} \cong \overline{QS}$, *Rectangle*

d. $\overline{QP} \parallel \overline{RS}$
none



76) In trapezoid BECK, the $m\angle B$ is 5 times the $m\angle K$ and $\overline{BE} \parallel \overline{CK}$. Find the $m\angle K$.



$$6x = 180$$

$$x = 30$$

$$\boxed{m\angle K = 30^\circ}$$

77) If rectangle JKLM has $MK = 3x - 5$, $JL = 2y + 5$, $KL = x + 5$, and $JM = 4y - 5$. Find JK and KL.

$$\begin{aligned} MK &= JL \\ 3x - 5 &= 2y + 5 \\ 3x &= 2y + 10 \\ 2y &= 3x - 10 \\ 4y &= 6x - 20 \end{aligned}$$

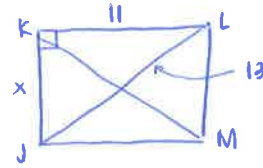
$$\begin{aligned} KL &= JM \\ x + 5 &= 4y - 5 \\ 4y &= x + 10 \end{aligned}$$

$$\begin{aligned} x + 10 &= 6x - 20 \\ 10 &= 5x - 20 \\ 5x &= 30 \end{aligned}$$

$$y = 4$$

$$x = 6$$

$$\begin{aligned} JK &= 4\sqrt{3} \\ KL &= 6 + 5 = 11 \end{aligned}$$



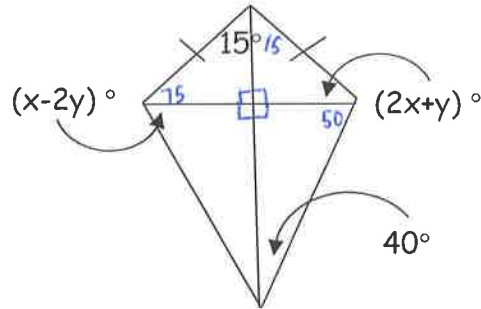
$$\begin{aligned} x^2 + 11^2 &= 13^2 \\ x^2 &= 48 \\ x &= 4\sqrt{3} \end{aligned}$$

78) If the figure is a kite, find the values of x and y

$$\begin{aligned} x &= \underline{40} \\ y &= \underline{-5} \end{aligned}$$

$$\begin{aligned} 2x + y &= 75 \\ -a [x - 2y &= 50] \Rightarrow \begin{aligned} 2x + y &= 75 \\ -x + 4y &= -100 \\ \hline 5y &= -25 \\ y &= -5 \end{aligned} \end{aligned}$$

$$\begin{aligned} 2x - 5 &= 75 \\ 2x &= 80 \\ x &= 40 \end{aligned}$$



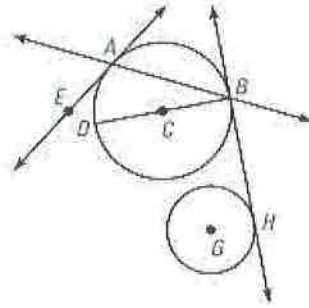
79) In quadrilateral LMNO, if $m\angle L \cong m\angle N$, how could LMNO not be classified?

- a) parallelogram b) kite c) trapezoid **d) rhombus** e) None of the above

Unit 7 Circles

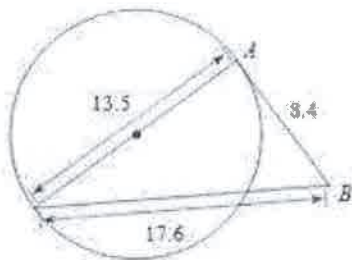
80) Identify each in the diagram.

- a. Radius $\overline{CD}, \overline{CB}$
- b. Center C, G
- c. Tangent $\overleftrightarrow{EA}, \overleftrightarrow{BH}$
- d. Chord $\overline{AB}, \overline{DB}$
- e. Common Tangent \overleftrightarrow{BH}
- f. Diameter \overline{DB}
- g. Secant \overleftrightarrow{AB}



Determine if line AB is tangent to the circle.

81)

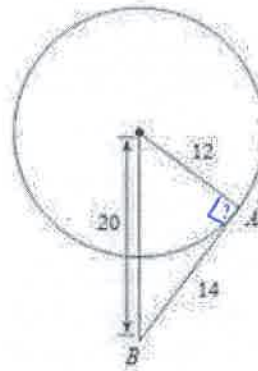


$$13.5^2 + 8.4^2 \stackrel{?}{=} 17.6^2$$

$$252.81 \leq 309.76$$

no, not tangent

82)



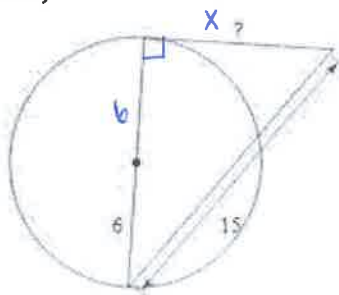
$$12^2 + 14^2 \stackrel{?}{=} 20^2$$

$$340 \leq 400$$

no, not tangent

Find the segment length indicated. Assume that lines which appear tangent are tangent.

83)



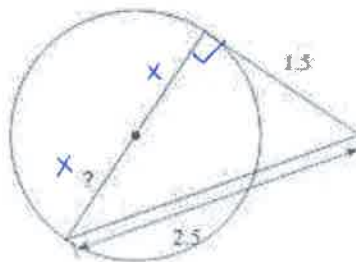
$$X^2 + 12^2 = 16^2$$

$$X^2 + 144 = 256$$

$$X^2 = 81$$

$X = 9$

84)



$$1.5^2 + (2X)^2 = 2.5^2$$

$$4X^2 + 2.25 = 6.25$$

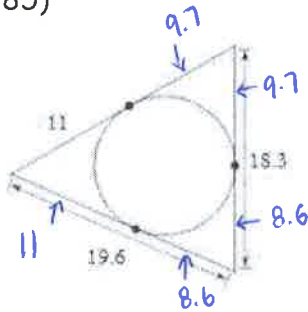
$$4X^2 = 4$$

$$X^2 = 1$$

$X = 1$

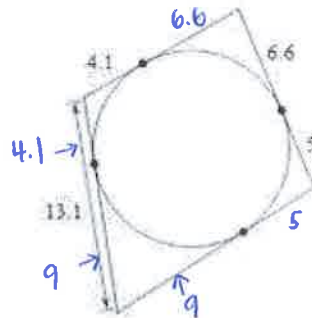
Find the perimeter of each polygon. Assume that lines which appear tangent are tangent.

85)



$$P = 58.6 \text{ units}$$

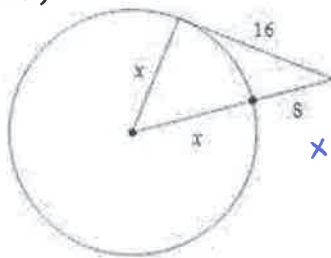
86)



$$P = 49.4 \text{ units}$$

Solve for x . Assume that lines which appear tangent are tangent.

87)



$$x^2 + 16^2 = (x + 8)^2 \quad \leftarrow \text{FOIL}$$

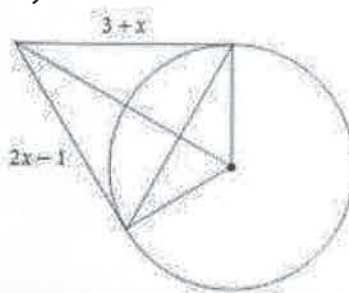
$$x^2 + 256 = x^2 + 16x + 64$$

$$256 = 16x + 64$$

$$192 = 16x$$

$$x = 12$$

88)



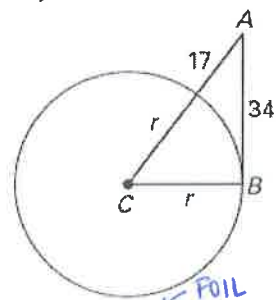
$$3 + x = 2x - 1$$

$$3 = x - 1$$

$$x = 4$$

Find the value(s) of the variable. Points B and D are points of tangency.

89)



$$r^2 + 34^2 = (r + 17)^2 \quad \leftarrow \text{FOIL}$$

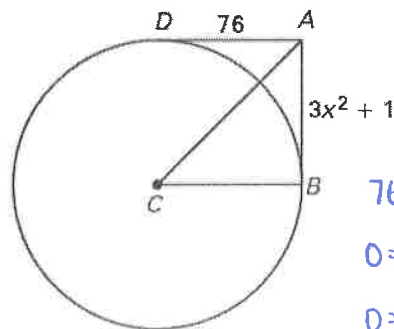
$$r^2 + 1156 = r^2 + 34r + 289$$

$$1156 = 34r + 289$$

$$867 = 34r$$

$$r = 25.5$$

90)



$$76 = 3x^2 + 1$$

$$0 = 3x^2 - 75$$

$$0 = 3(x^2 - 25)$$

$$0 = x^2 - 25$$

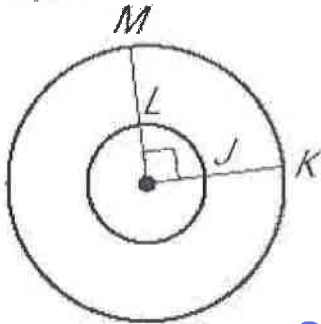
$$25 = x^2$$

$$x = \pm 5$$

Determine whether the given arcs are congruent. Explain why or why not.

91)

$\widehat{LJ}, \widehat{MK}$

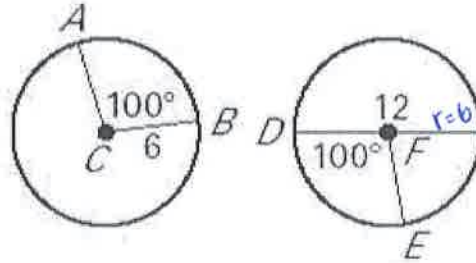


no, even though $m\widehat{LJ} = 90^\circ$ and $m\widehat{MK} = 90^\circ$, they are not in congruent circles

Find the value of the variables.

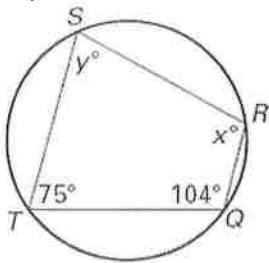
92)

$\widehat{AB}, \widehat{DE}$



yes, $m\widehat{AB} = 100^\circ$ and $m\widehat{DE} = 100^\circ$. They both are in circles with a radius of b units.

93)



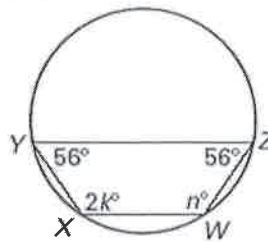
$$x + 76 = 180$$

$$x = 104$$

$$y + 104 = 180$$

$$y = 76$$

94)



$$2k + 56 = 180$$

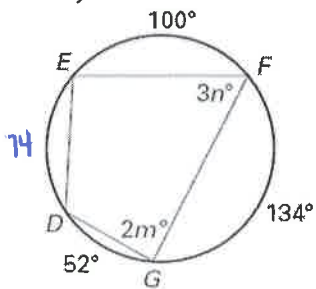
$$2k = 124$$

$$k = 62$$

$$n + 56 = 180$$

$$n = 124$$

95)



$$2m = \frac{1}{2}(74 + 100)$$

$$2m = \frac{1}{2}(174)$$

$$2m = 87$$

$$m = 43.5$$

$$3n = \frac{1}{2}(74 + 52)$$

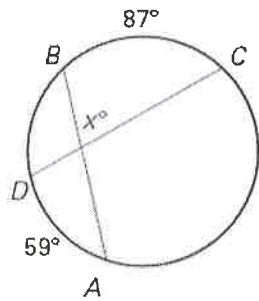
$$3n = \frac{1}{2}(126)$$

$$3n = 63$$

$$n = 21$$

Find the value of x.

96)

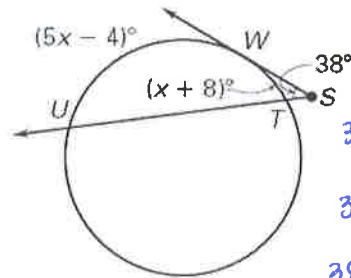


$$x = \frac{1}{2}(87 + 59)$$

$$x = \frac{1}{2}(146)$$

$$x = 73$$

97)



$$38 = \frac{1}{2}(5x - 4 - (x + 8))$$

$$38 = \frac{1}{2}(5x - 4 - x - 8)$$

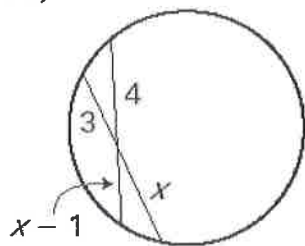
$$38 = \frac{1}{2}(4x - 12)$$

$$38 = 2x - 6$$

$$44 = 2x$$

$$x = 22$$

98)



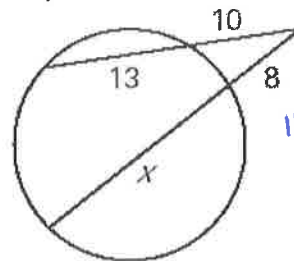
$$4(x-1) = 3x$$

$$4x - 4 = 3x$$

$$-4 = -x$$

$$x = 4$$

99)



$$10(10+13) = 8(x+8)$$

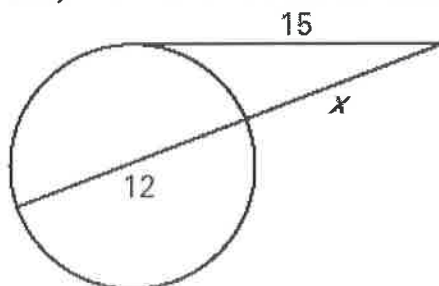
$$10(23) = 8x + 64$$

$$230 = 8x + 64$$

$$166 = 8x$$

$$x = 20.75$$

100)



$$15^2 = x(x+12)$$

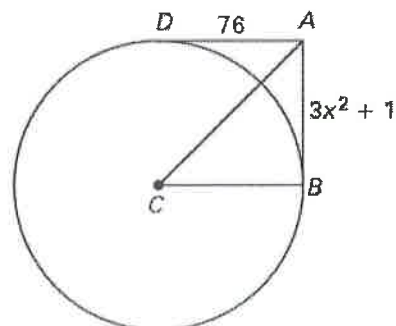
$$225 = x^2 + 12x$$

$$x^2 + 12x - 225 = 0$$

$$\frac{-12 \pm \sqrt{1044}}{2} \quad \begin{matrix} 10.16 \\ -22.16 \end{matrix}$$

$$x = 10.16$$

101)



$$76 = 3x^2 + 1$$

$$0 = 3x^2 - 75$$

$$0 = 3(x^2 - 25)$$

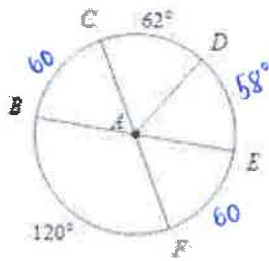
$$0 = x^2 - 25$$

$$25 = x^2$$

$$x = \pm 5$$

Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

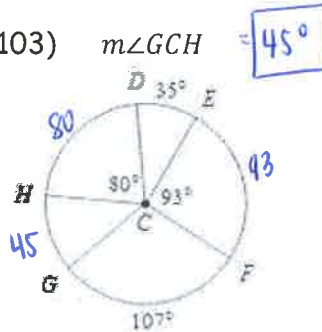
102) $m\angle DAF$



$$m\angle DAF = 58 + 60$$

$$m\angle DAF = 118^\circ$$

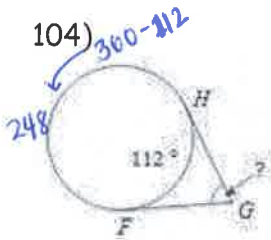
103) $m\angle GCH$



$$45^\circ$$

Find the measure of the arc or angle indicated. Assume that lines which appear to be tangent are tangent.

104)

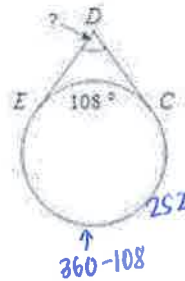


$$x = \frac{1}{2}(248 - 112)$$

$$x = \frac{1}{2}(136)$$

$$x = 68$$

105)

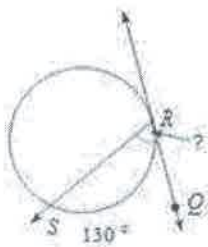


$$x = \frac{1}{2}(252 - 108)$$

$$x = \frac{1}{2}(144)$$

$$x = 72$$

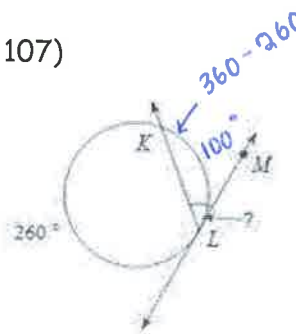
106)



$$x = \frac{1}{2}(130)$$

$$x = 65$$

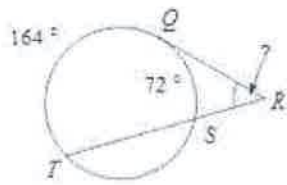
107)



$$x = \frac{1}{2}(100)$$

$$x = 50$$

108)

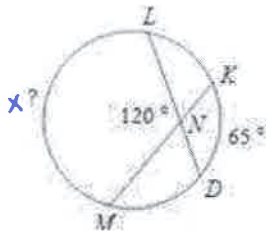


$$x = \frac{1}{2}(164 - 72)$$

$$x = \frac{1}{2}(92)$$

$$x = 46^\circ$$

110)

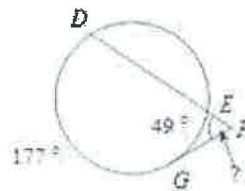


$$120 = \frac{1}{2}(x + 65)$$

$$240 = x + 65$$

$$x = 175$$

109)

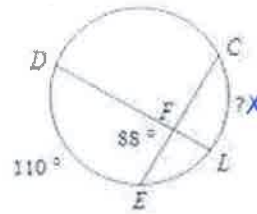


$$x = \frac{1}{2}(177 - 49)$$

$$x = \frac{1}{2}(128)$$

$$x = 64$$

111)

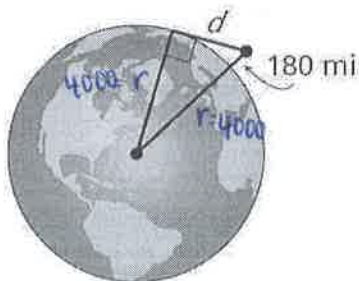


$$88 = \frac{1}{2}(110 + x)$$

$$176 = 110 + x$$

$$x = 66$$

112) Suppose a space shuttle is orbiting about 180 miles above Earth. What is the distance d from the shuttle to the horizon? The radius of Earth is about 4000 miles. Round your answer to the nearest tenth.



$$4000^2 + d^2 = (180 + 4000)^2$$

$$4000^2 + d^2 = 4180^2$$

$$d^2 = 1472400$$

$$d \approx 1213.4 \text{ mi}$$

Use the given information to write the standard equation of the circle.

113) The center is $(-3, 2)$ and a point on the circle is $(5, 2)$.

$$r = \sqrt{(5+3)^2 + (2-2)^2} = \sqrt{(8)^2} = \sqrt{64} = 8$$

$$(x+3)^2 + (y-2)^2 = 64$$

114) The center is $(6, -1)$ and a point on the circle is $(-1, 6)$.

$$r = \sqrt{(6+1)^2 + (-1-6)^2} = \sqrt{(7)^2 + (-7)^2} = \sqrt{49+49} = \sqrt{98}$$

$$(x-6)^2 + (y+1)^2 = \sqrt{98}^2$$

$$(x-6)^2 + (y+1)^2 = 98$$

Identify the center and radius of each circle. Then sketch the graph.

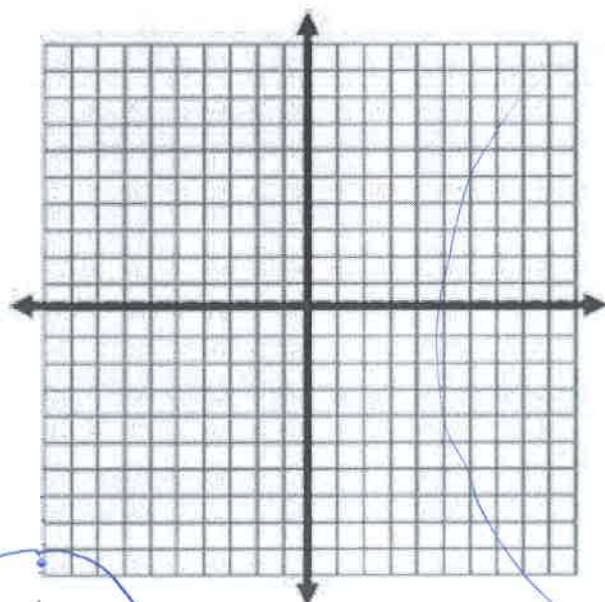
115) $x^2 + y^2 + 20x + 28y + 277 = 0$

$$x^2 + 20x + \frac{100}{\uparrow} + y^2 + 28y + \frac{196}{\uparrow} = -277 + 100 + 196$$

$\frac{1}{2}(20) = 10$ $\frac{1}{2}(28) = 14$
 $(10)^2 = 100$ $(14)^2 = 196$

$$(x+10)^2 + (y+14)^2 = 19$$

center: $(-10, -14)$ radius = $\sqrt{19} \approx 4.3$



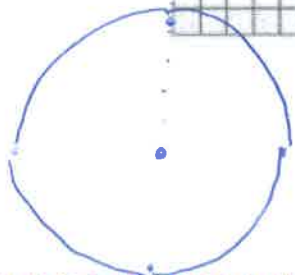
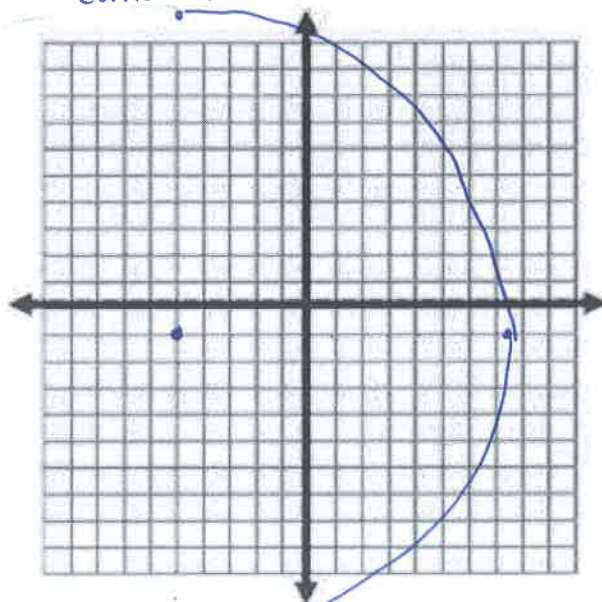
116) $x^2 + y^2 + 10x + 2y - 123 = 0$

$$x^2 + 10x + \frac{25}{\uparrow} + y^2 + 2y + \frac{1}{\uparrow} = 123 + 25 + 1$$

$\frac{1}{2}(10) = 5$ $\frac{1}{2}(2) = 1$
 $(5)^2 = 25$ $(1)^2 = 1$

$$(x+5)^2 + (y+1)^2 = 149$$

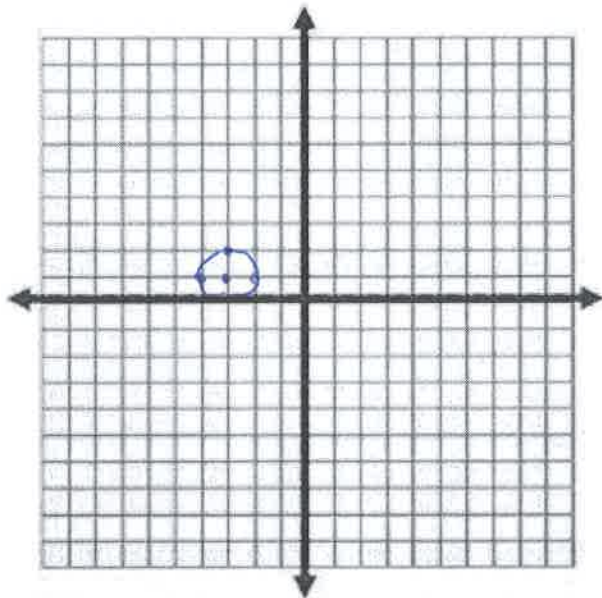
center: $(-5, -1)$ radius = $\sqrt{149} \approx 12.2$



117) $(x + 3)^2 + (y - 1)^2 = 1$

$c(-3, 1)$

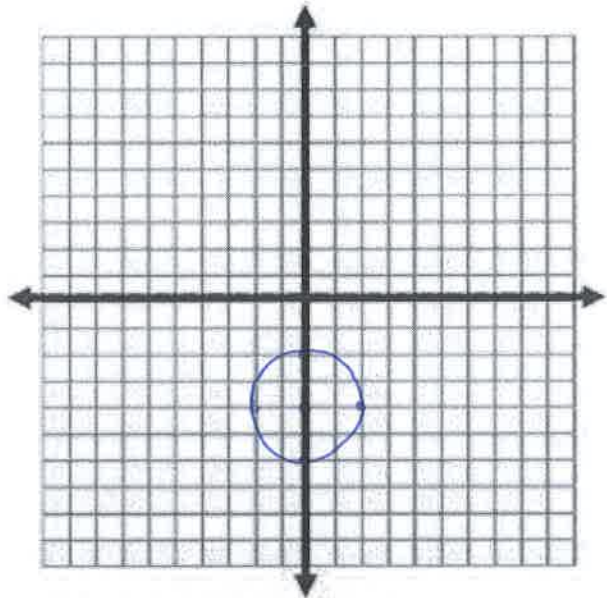
$r=1$



118) $x^2 + (y + 4)^2 = 4$

$c(0, -4)$

$r=2$



Use the information provided to write the equation of each circle.

119) Center: $(11, -13)$
Radius: 4

$(x-11)^2 + (y+13)^2 = 16$

120) Center: $(11, 9)$
Radius: 3

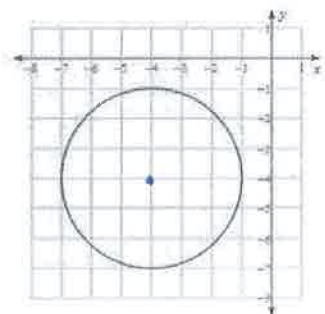
$(x-11)^2 + (y-9)^2 = 9$

121) Center: $(-2, 6)$
Point on Circle: $(-11, 7)$

$(x+2)^2 + (y-6)^2 = 82$

$r = \sqrt{(-11+2)^2 + (7-6)^2} = \sqrt{81+1} = \sqrt{82}$

123)



$c(-4, -4)$
 $r=3$

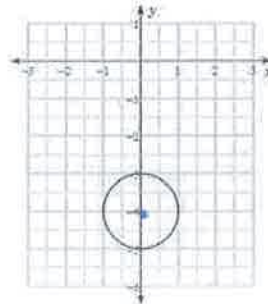
$(x+4)^2 + (y+4)^2 = 9$

122) Center: $(-8, -5)$
Point on Circle: $(-17, -2)$

$(x+8)^2 + (y+5)^2 = 90$

$r = \sqrt{(-17+8)^2 + (-2+5)^2} = \sqrt{81+9} = \sqrt{90}$

124)

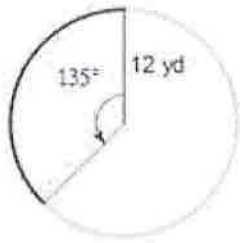


$c(-4, 0)$
 $r=1$

$(x+4)^2 + y^2 = 1$

Find the length of each arc. Round your answers to the nearest tenth.

125)

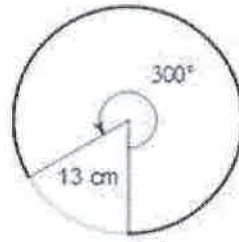


$$\frac{x}{2\pi(12)} = \frac{135}{360}$$

$$360x = 10178.7516$$

$$x = 28.3 \text{ yd}$$

126)



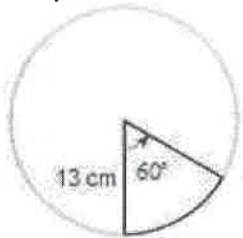
$$\frac{x}{2\pi(13)} = \frac{300}{360}$$

$$360x = 24904.402$$

$$x = 68.1 \text{ cm}$$

Find the area of each sector. Round your answer to the nearest tenth.

127)

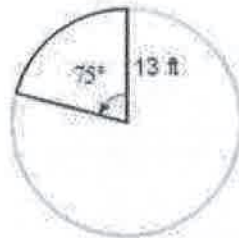


$$\frac{A}{\pi(13)^2} = \frac{60}{360}$$

$$360A = 31855.7226$$

$$A = 88.4 \text{ cm}^2$$

128)



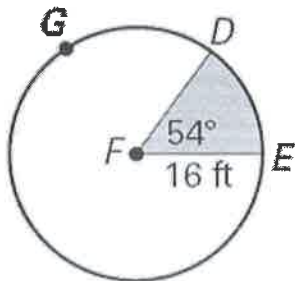
$$\frac{A}{\pi(13)^2} = \frac{75}{360}$$

$$360A = 39819.6532$$

$$A = 110.6 \text{ ft}^2$$

Find the areas of the sectors formed by $\angle DFE$. Round answers to the nearest tenth. Also find the length of arc DE .

129)



arc:

$$\frac{x}{2\pi(16)} = \frac{54}{360}$$

$$360x = 5428.66752$$

$$x = 15.1 \text{ ft}$$

$$\widehat{DE} = 15.1 \text{ ft}$$

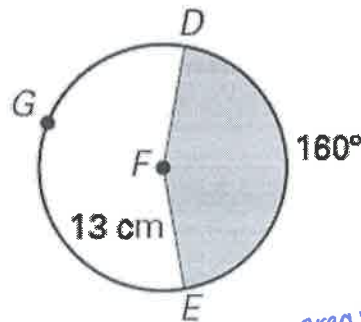
area:

$$\frac{A}{\pi(16)^2} = \frac{54}{360}$$

$$360A = 43429.3402$$

$$A = 120.6 \text{ ft}^2$$

130)



arc:

$$\frac{x}{2\pi(13)} = \frac{160}{360}$$

$$360x = 13069.0144$$

$$x = 36.3 \text{ cm}$$

$$\widehat{DE} = 36.3 \text{ cm}$$

area:

$$\frac{A}{\pi(13)^2} = \frac{160}{360}$$

$$360A = 84948.5936$$

$$A = 235.97 \text{ cm}^2$$

131) How can you convert a radian measure to degrees?

multiply by $\frac{180}{\pi}$

132) How can you convert a degree measure to radians?

multiply by $\frac{\pi}{180}$

133) Convert 40° to radians.

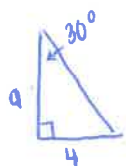
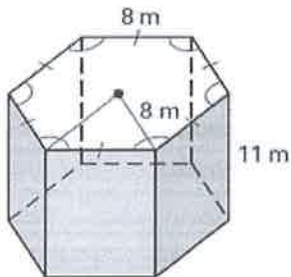
$$40 \cdot \frac{\pi}{180} = \frac{40\pi}{180} = \frac{4\pi}{18} = \boxed{\frac{2\pi}{9}}$$

134) Convert $\frac{11\pi}{3}$ radians to degrees

$$\frac{11\pi}{3} \cdot \frac{180}{\pi} = \frac{1980}{3} = \boxed{660^\circ}$$

Unit 8 Surface Area and Volume Formulas

135) Find the surface area and volume of the right hexagonal prism.



$$a = 6.92$$

$$P = 48$$

$$B = \frac{1}{2}(6.92)(48) = 166.277$$

$$h = 11$$

$$V = Bh$$

$$= (166.277)(11)$$

$$\boxed{V = 1829.06 \text{ m}^3}$$

$$SA = 2B + Ph$$

$$= 2(166.277) + (48)(11)$$

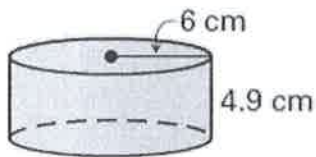
$$= 332.554 + 528$$

$$\boxed{SA = 860.55 \text{ m}^2}$$

$$\tan 30^\circ = \frac{4}{a}$$

$$a = \frac{4}{\tan 30^\circ} = 6.92$$

136) Find the surface area and volume of the right cylinder.



$$V = \pi r^2 h$$

$$= \pi (6)^2 (4.9)$$

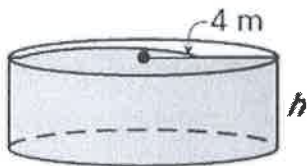
$$V = 554.18 \text{ cm}^3$$

$$SA = 2\pi r^2 + 2\pi r h$$

$$= 2\pi (6)^2 + 2\pi (6)(4.9)$$

$$SA = 410.92 \text{ cm}^2$$

137) Find the height of the right cylinder shown, which has a surface area of 168.09 square meters.



$$SA = 2\pi r h + 2\pi r^2$$

$$168.09 = 2\pi (4)h + 2\pi (4)^2$$

$$168.09 = 25.13h + 100.53$$

$$67.56 = 25.13h$$

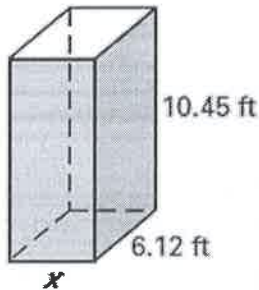
$$h = 2.69 \text{ m}$$

138) Solve for x given the surface area of the right prism is 274.39 ft². Round your answer to two decimal places.

$$B = (10.45)(6.12) = 63.954$$

$$P = 33.14$$

$$h = x$$



$$SA = 2B + Ph$$

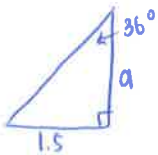
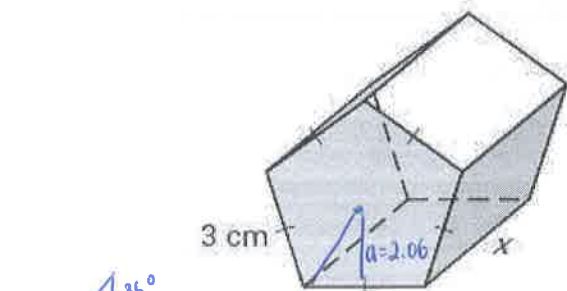
$$274.39 = 2(63.954) + 33.14x$$

$$274.39 = 127.908 + 33.14x$$

$$146.482 = 33.14x$$

$$x = 4.42 \text{ ft}$$

139) The volume of solid below is 78 cm³. Solve for x .



$$\tan 36 = \frac{1.5}{a}$$

$$a = 2.06$$

$$P = 15$$

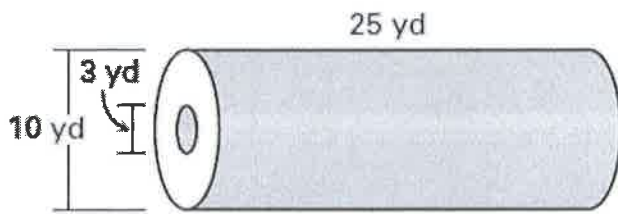
$$B = \frac{1}{2}(1.5)(2.06) = 1.545$$

$$V = Bh$$

$$78 = 15.45x$$

$$x = 5.05 \text{ cm}$$

140) Find the surface area and the volume of the solid.



$$\begin{aligned} V_{\text{outer}} &= \pi r^2 h \\ &= \pi (10)^2 (25) \\ &= 1963.495 \end{aligned}$$

$$\begin{aligned} V_{\text{inner}} &= \pi (3)^2 (25) \\ &= 176.715 \end{aligned}$$

$$\text{Total Vol} = 1963.495 - 176.715$$

$$\boxed{V = 1786.78 \text{ yd}^3}$$

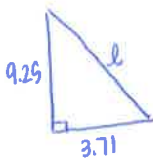
$$\begin{aligned} S_{\text{out}} &= 2\pi r^2 + 2\pi r h \\ &= 2\pi (10)^2 + 2\pi (10)(25) \\ &= 942.478 - 2\pi (1.5)^2 \\ &= 928.34 \end{aligned}$$

$$\begin{aligned} S_{\text{inner}} &= 2\pi r h \\ &= 2\pi (3)(25) \\ &= 236.619 \end{aligned}$$

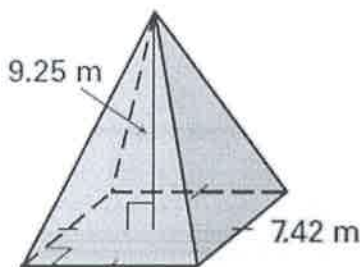
$$\text{Total SA} = 928.34 + 236.619$$

$$\boxed{\text{SA} = 1163.96 \text{ yd}^2}$$

141) Find the surface area and the volume of the pyramid.



$$\begin{aligned} 3.71^2 + 9.25^2 &= l^2 \\ 13.7641 + 85.5625 &= l^2 \\ 99.3266 &= l^2 \\ l &= 9.966 \end{aligned}$$



$$\begin{aligned} V &= \frac{1}{3} B h \\ &= \frac{1}{3} (55.0564) (9.25) \end{aligned}$$

$$\boxed{V = 169.76 \text{ m}^3}$$

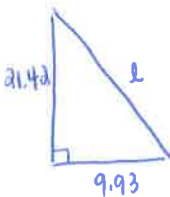
$$\begin{aligned} \text{SA} &= \frac{1}{2} P l + B \\ &= \frac{1}{2} (29.68) (9.966) + 55.0564 \\ &= 147.8954 + 55.0564 \end{aligned}$$

$$\boxed{\text{SA} = 202.95 \text{ m}^2}$$

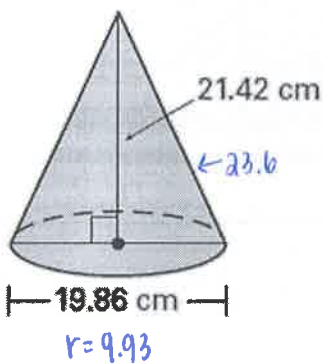
$$B = (7.42)(7.42) = 55.0564$$

$$P = 7.42 \cdot 4 = 29.68$$

142) Find the surface area and volume of the cone.



$$\begin{aligned} 9.93^2 + 21.42^2 &= l^2 \\ 557.4213 &= l^2 \\ l &= 23.6 \end{aligned}$$



$$V = \frac{1}{3} \pi (9.93)^2 (21.42)$$

$$\boxed{V = 2210.8 \text{ cm}^3}$$

$$\begin{aligned} \text{SA} &= \pi r l + \pi r^2 \\ &= \pi (9.93) (23.6) + \pi (9.93)^2 \end{aligned}$$

$$\boxed{\text{SA} = 1046 \text{ cm}^2}$$

143) Nicole is buying an aquarium in the shape of a rectangular prism and measures 3.5 feet by 4 feet by 4 feet.

What is the volume of the aquarium?

$$B = (4)(4) = 16$$

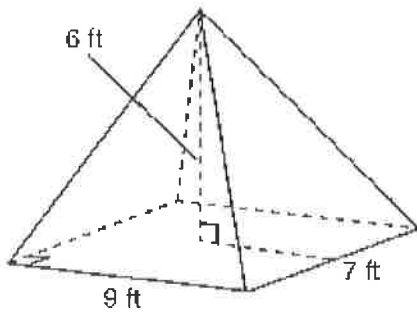
$$h = 3.5$$

$$V = B h$$

$$= (16)(3.5)$$

$$\boxed{V = 56 \text{ ft}^3}$$

144) The volume of the pyramid below is _____.



$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (63)(6)$$

$$V = 126 \text{ ft}^3$$

$$B = (9)(7) = 63$$

$$h = 6$$

145) Describe what happens to the volume of a cone if its radius is doubled while its height is halved. The volume is _____.

[A] unchanged

[B] doubled

[C] increased by a factor of $\frac{1}{3}$

[D] not able to be determined

$$r = 4$$

$$h = 10$$

$$V = \frac{1}{3} \pi (4)^2 (10)$$

$$= 167.66$$

$$r = 8$$

$$h = 5$$

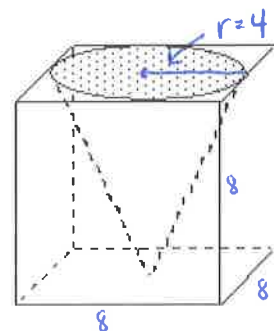
$$V = \frac{1}{3} \pi (8)^2 (5)$$

$$= 335.10$$

$$\frac{335.10}{167.55} = 2$$

146) A machinist drilled a conical hole into a cube of metal as shown. If the cube has sides of length 8 cm, what is the volume of the metal after the hole is drilled?

Use $\pi \approx 3.14$ and round to the nearest tenth.



$$V_{\text{cube}} = (8)(8)(8)$$

$$= 512$$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

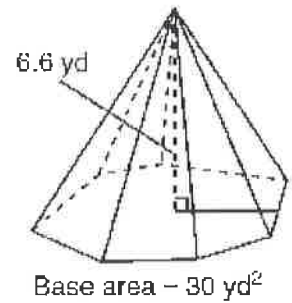
$$= \frac{1}{3} \pi (4)^2 (8)$$

$$= 134.04$$

$$\text{Total Vol} = 512 - 134.04$$

$$= 377.96 \text{ cm}^3$$

- 147) The base of the pyramid below is a non-regular heptagon with an area of 30.0 square yards. The height of the pyramid is 6.6 yards. Find the volume of the pyramid.



$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3}(30)(6.6)$$

$$= 66 \text{ yd}^3$$

- 148) Find the volume of the figure to the nearest tenth.

$$V_{\text{cyl}} = \pi r^2 h$$

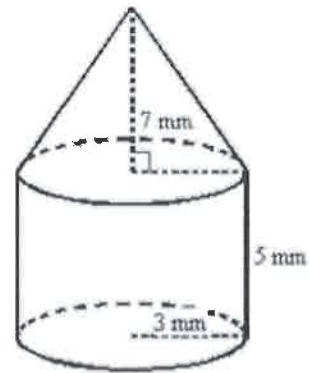
$$= \pi(3)^2(5)$$

$$= 141.37$$

$$V_{\text{conc}} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi(3)^2(7)$$

$$= 69.97$$



$$\text{Total Vol} = 141.37 + 69.97$$

$$= 207.34 \text{ mm}^3$$

- 149) Find the volume, to the nearest cubic foot, of a sphere whose surface area is 100 ft^2 .

$$SA = 4\pi r^2$$

$$100 = 4\pi r^2$$

$$7.96 = r^2$$

$$r = 2.82$$

$$V = \frac{4}{3} \pi (2.82)^3$$

$$V = 93.9 \approx 94 \text{ ft}^3$$

- 150) A sphere fits snugly inside a right cylinder as shown below. Find the volume lying outside the sphere but inside the cylinder to the nearest tenth of a cubic inch.

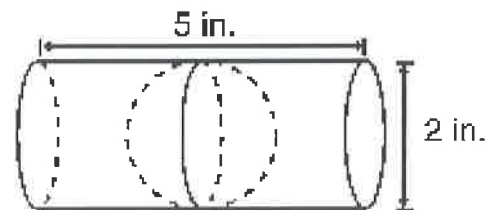
$$V_{\text{cyl}} = \pi r^2 h$$

$$= \pi(1)^2(5)$$

$$= 15.7$$

$$V_{\text{ball}} = \frac{4}{3} \pi (1)^3$$

$$= 4.19$$



$$\text{Total Vol} = 15.7 - 4.19$$

$$= 11.52 \text{ in}^3$$